Population Dynamics in a Changing Environment: Random versus Periodic Switching:

The Effect of Varying the Noise Intensity

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In this document, we provide some further details on the effect of varying the noise intensity in the periodic and random switching case. In ecology, there is a great interest in studying how environmental variations can effect species diversity. This led to the debate on the 'Intermediate Disturbance Hypothesis' ([1, 2, 3]), which looks at the evolutionary effects of the frequency and amplitude of external disturbances. In this broad context, and without direct relevance for the IDH, it is interesting to now study how ϕ depends on amplitude of the noise, γ, keeping ν fixed as a parameter. It is found that the effect of varying γ is different dependent on the asymmetry of the noise, δ: when $\delta > 0$ Figure A1 shows that the fixation probability and mean fixation time (MFT) are non-monotonic in γ, while for $\delta \leq 0$, ϕ is monotonically increasing and the MFT is monotonically decreasing. Hence, noise may (slightly) prolong species coexistence, but make conditions less favourable for the slow growing species. For ϕ, this can be understood analytically in the fast and slow switching regimes.

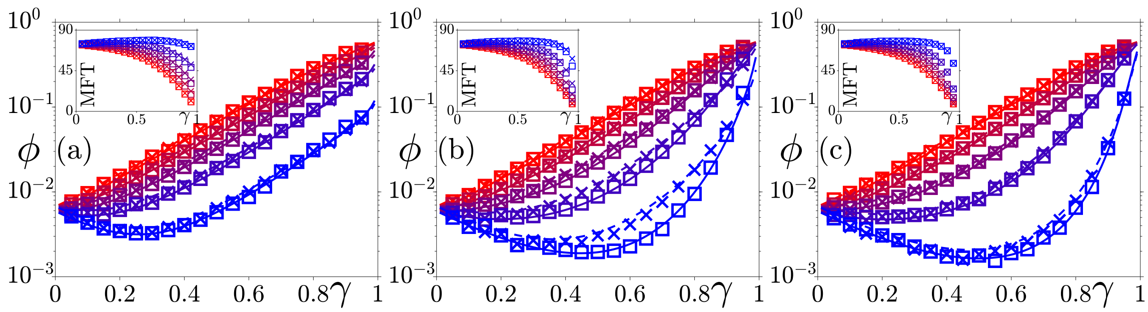

Figure A1: Dependence of ϕ (main panels) and Mean Fixation Time (insets) on γ and δ for $(K_0, s, x_0) = (250, 0.05, 0.6)$ for $\nu = (0.01, 0.1, 1)$ in (a,b,c). In all panels, x-axis shows γ and results for $\delta = (-0.8, -0.4, 0, 0.4, 0.8)$ are shown by red to blue. Squares/crosses represent simulation results for periodic/random variation. Solid/dashed lines in main panels are theoretical results from Eq. (3) of the main text for periodic/random variation. See text

When $\nu/s \gg 1$ (Figure A1(c)) the N-QSD for both periodic and random switching is peaked roughly around $\mathcal{K}_\delta = K_0(1-\gamma^2)/(1-\delta\gamma)$, which sets the fixation probability as Eq. (2) of the main text evaluated at $\mathcal{K}_\delta$. Hence when $\gamma < \delta$, $\mathcal{K}_\delta > K_0$, and $\phi|_{\mathcal{K}_\delta} < \phi|_{K_0}$, while the opposite is true for $\gamma > \delta$. For simplicity, we work in the realm of the diffusion approximation, $s \ll K_0^{-1/2} \ll 1$, which means Eq. (2) of the main text is well approximated as $\phi(x_0)|_K \approx e^{-Ks(1-x_0)}$. When $\nu \ll 1$ (Figure A1(a)) one can use this in the formula for the fixation probability under very slow switching, given by: $\phi_{\nu\to 0} \approx \frac{1}{2}\left[(1-\delta)\,\phi(x_0)|_{K_-} + (1+\delta)\,\phi(x_0)|_{K_+}\right]$ (see main text). Together, this gives:

$$\phi(\gamma) \approx \frac{1}{2}\mathrm{e}^{-K_0 s(1-x_0)}\left[(1-\delta)\mathrm{e}^{\gamma K_0 s(1-x_0)} + (1+\delta)\mathrm{e}^{-\gamma K_0 s(1-x_0)}\right]. \tag{A1}$$

Writing $y = \mathrm{e}^{\gamma K_0 s(1-x_0)}$ and setting $\phi(\gamma) = \phi|_{K_0}$ allows us to ascertain when the external noise promotes the slow growing species with respect to a constant environment, yielding the quadratic equation for the point where $\phi(\gamma)$ is the same that for a constant environment with $K = K_0$: $(1-\delta)y^2 - 2y + (1+\delta) = 0$. This has solutions $y = 1$, corresponding to the trivial solution $\gamma = 0$, and $y = (1+\delta)/(1-\delta)$, corresponding to $\gamma = \gamma^* = (K_0 s(1-x_0))^{-1} \ln\left[(1+\delta)/(1-\delta)\right]$, which is only physically realistic if $\delta > 0$. Furthermore, note that differentiat-

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ing (A1) with respect to γ yields:

$$\frac{d}{d\gamma}\phi(\gamma) \approx \frac{1}{2}\mathrm{e}^{-K_0 s(1-x_0)} K_0 s(1-x_0) \left[(1-\delta)\mathrm{e}^{\gamma K_0 s(1-x_0)} - (1+\delta)\mathrm{e}^{-\gamma K_0 s(1-x_0)}\right]. \tag{A2}$$

The term outside the bracket is always greater than zero, and expanding the term inside the bracket around $\gamma = 0$ yields:

$$\frac{d}{d\gamma}\phi(\gamma) \approx \mathrm{e}^{-K_0 s(1-x_0)} K_0 s(1-x_0) \left[-\delta + \gamma K_0 s(1-x_0)\right]. \tag{A3}$$

Hence if $\delta > 0$, $\phi(\gamma)$ initially decreases, then increases, with $\phi(\gamma) = \phi|_{K_0}$ when $\gamma \approx \gamma^*$. Furthermore, for larger values of the switching asymmetry, δ, larger values of noise intensity γ are needed to increase the fixation probability relative to the constant environment.

References

[1] M.A. Brockhurst, A. Buckling and A. Gardner, *Current Biology* **17**, 761-765 (2007)

[2] P.S. Petraitis, R.E. Latham, and R.A. Niesenbaum, *The Quarterly Review of Biology* **64**, 393-418 (1989)

[3] J.W. Fox, *Trends in Ecology and Evolution* **28**, 86-92, (2013)