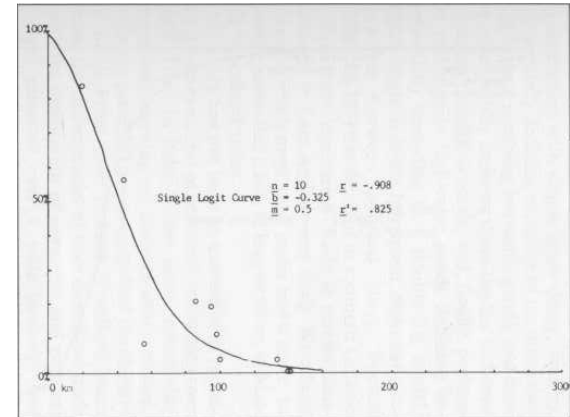
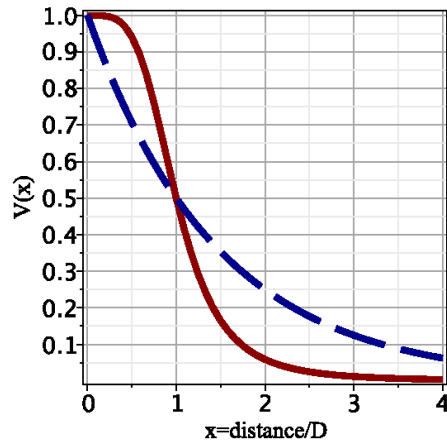


Are Data and Theory models homomorphic or complementary?

Ray Rivers (Physics, ICL)



Collaboration with

Tim Evans (Physics, ICL)

[Carl Knappett (Art, Toronto)]

EPNet, 3<sup>rd</sup> July, Barcelona 2015

**Motivation:** understanding **prehistoric** exchange networks!

**Question:** Why do we need Theory Modelling? Is data not enough!

**Unfortunately NO :** Data is often

- site-centric
- 'poor' - particularly if no written records!
- no inscriptions
- poor proxy for 'exchange'

Difficult to see the larger picture!

**Theory models can:**

- provide an understanding of agency/ large-scale narrative
- 'complete' the data

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'Explanation can be characterised by 'isomorphism' between Data Models and Theory Models'

# Talk outline:

- I. Definitions
- II. Example: planetary systems
- III. Data and Theory modelling
- IV. Theory modelling in archaeology
  - natural selection and creationism
- V. This talk: Maximum entropy modelling
- VI. Example of homomorphy: N. American lithic artefact distributions
- VII. Complementarity: 'Retail' archaeology: formation of late geometric/Archaic Greek city states
- VIII. Conclusions

# I. **Data and theory modelling: Definitions**

Definition of terms:

- **Data modelling (DM)**
  - derived from patterned observations in data

whereas

- **Theory modelling (TM)**
  - derived from theories about processes that produce these patterned observations

# I. Data and theory modelling: Definitions

Definition of terms:

- **Data modelling (DM)**
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  - phenomenological

whereas

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  - ideational

# I. Data and theory modelling: Definitions

Definition of terms:

- **Data modelling (DM)**
  - derived from patterned observations in data
  - 'bricks and mortar'

whereas

- **Theory modelling (TM)**
  - derived from theories about processes that produce these patterned observations
  - 'architecture'



# I. Data and theory modelling: Definitions

Definition of terms: **not a mathematical definition!**

## **If models are isomorphic**

- 'identical', to all intents and purposes

whereas

## **If models are homomorphic**

- structurally the 'same'
- i.e. one is a 'coarse-grained' version of the other

# I. Data and theory modelling: Definitions

Definition of terms: **not a mathematical definition!**

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- 'identical', to all intents and purposes

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## If models are homomorphic

- structurally the 'same'
- i.e. one is a 'coarse-grained' version of the other

**At best:** 'Explanation can be characterised by 'homomorphism'  
between Data Models and Theory Models'

## II. Data and theory modelling: Example

**Example:** cosmology - planetary motion (Read)

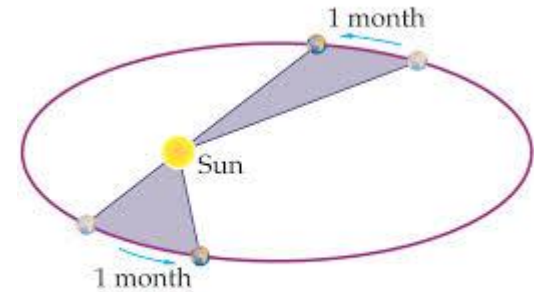
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## II. Data and theory modelling: Example

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DM: Kepler's 'laws'

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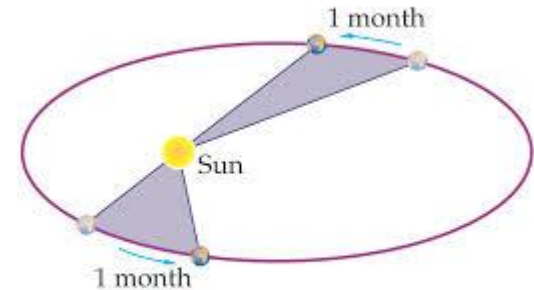
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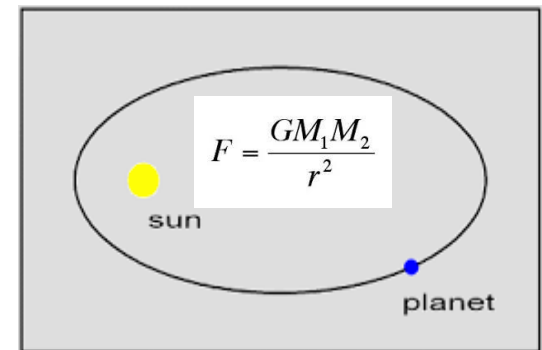
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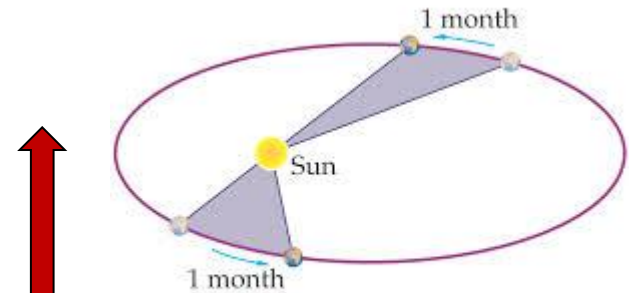
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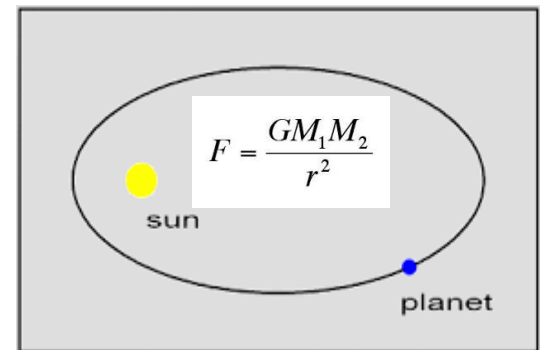
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‘homomorphism’: Newton explains comets



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## II. Data and theory modelling: Example

Arguably true for the biological domain  
and for culturally constructed idea  
systems (Read)

**Example:** cosmology - planetary motion

- **Data modelling**

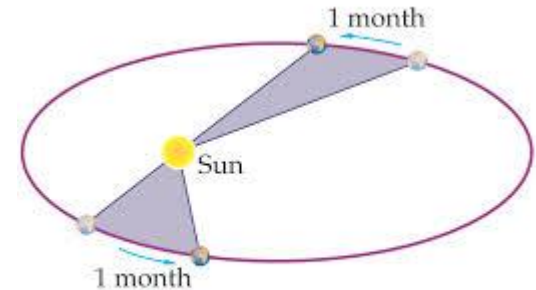
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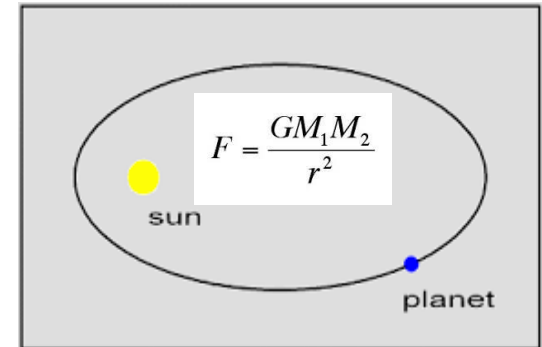
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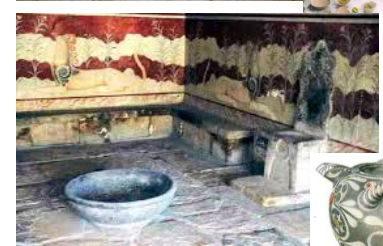


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### III. Data and theory modelling in archaeology

We cannot match that type of modelling in archaeology!

**DM:** archaeological data spans many scales





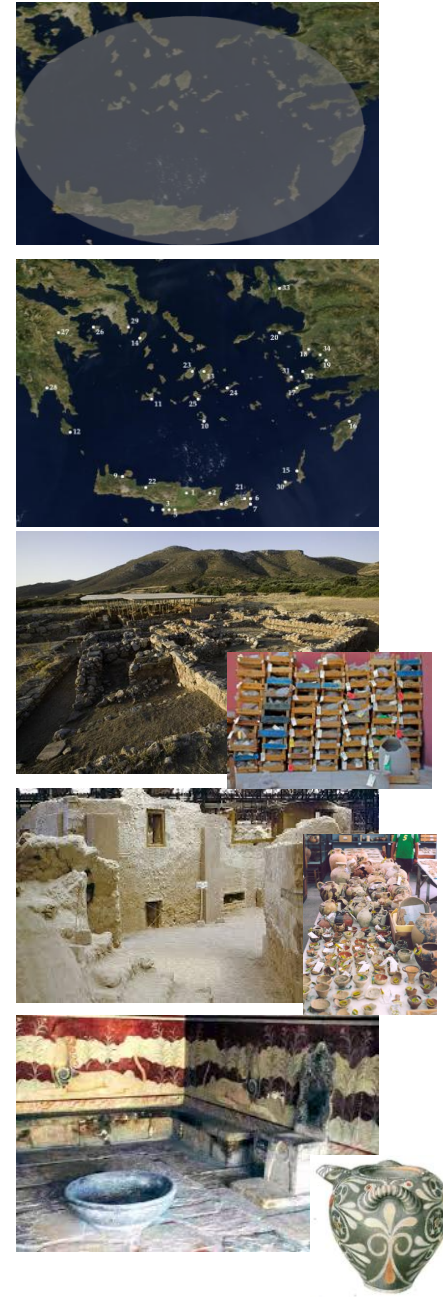
### III. Data and theory modelling in archaeology

We cannot match that type of modelling in archaeology!

**DM:** archaeological data spans many scales

**TM:** No laws for social behaviour!

- at best 'tendencies'
- 'agency'



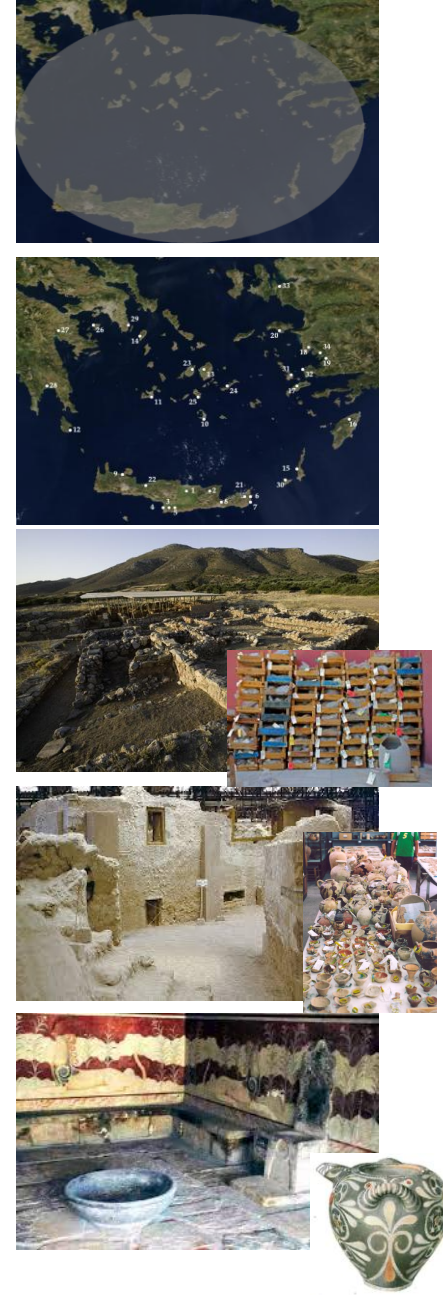
### III. Data and theory modelling in archaeology

**Data modelling:** Largely 'descriptive'

Relational: Looks for similarities between

- artefacts
  - assemblages
  - sites
- 
- Construct networks of 'similar' sites and assume they are basis for social dynamics
    - 'back door' approach
  - Construct social narratives for the system, which may include ABM

Bottom up approach!



### III. Data and theory modelling in archaeology

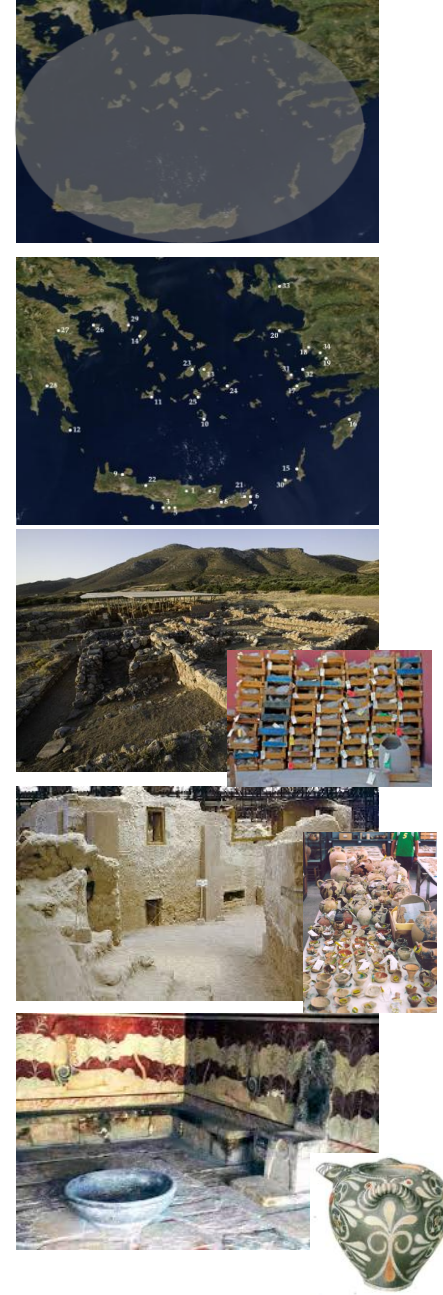
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**Bottom up approach!**

Often fails to accommodate higher levels of organisation!



### III. Data and theory modelling in archaeology

**Theory modelling:** Largely 'prescriptive'/postdictive

Takes into account

- 'geography'
- 'technology'
- agency ('front door' approach to exchange)

'Middle up'/ top down approach!





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Often fails to accommodate lowest levels of organisation!



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Large overlap: explore possible homomorphy

Small overlap: TM and DM are **complementary**  
rather than homomorphic!



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Small overlap: TM and DM merge/are complementary  
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- although possibly stitched together by ABM!



### III. Data and theory modelling in archaeology

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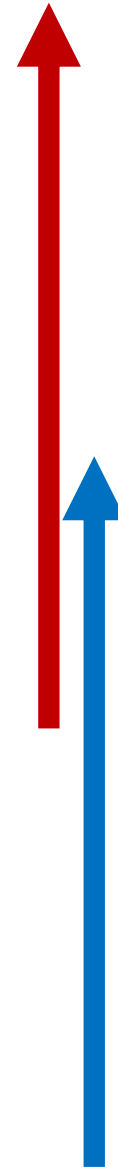
- 'geography'
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- agency ('front door' approach to exchange)

'Middle up'/ top down approach!

Large overlap: explore possible homomorphy

Small overlap: TM and DM merge/are complementary  
rather than homomorphic!

or disagreement!





### III. Data and theory modelling in archaeology: Data

**Problem:** How do we handle data?

- **DM:** Data is too much!



so much detail!



so many sherds - so little time!

### III. Data and theory modelling in archaeology: Data

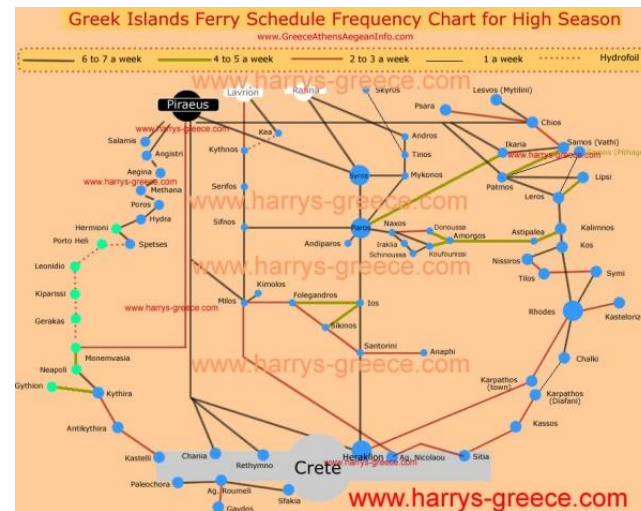
**Problem:** How do we handle data?

- **DM:** Data is too poor!



cf.

to



for exchange

### III. Data and theory modelling in archaeology: Data

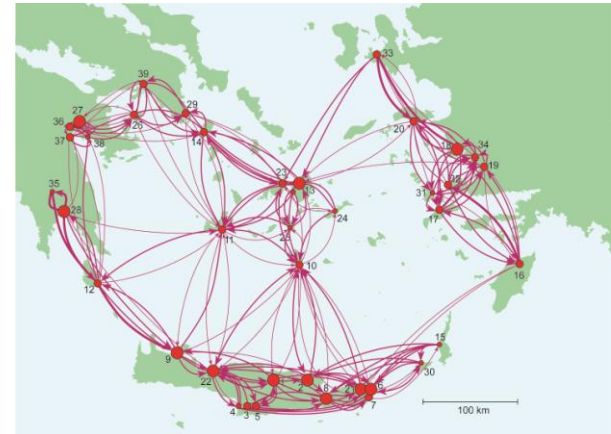
**Problem:** How do we handle data?

- **TM:** Input and output data are too simple

cf.



to links/nodes of



single label to characterise all categories  
of exchange (including trade)!

### III. Data and theory modelling in archaeology: Data

**Problem:** How do we handle data?

- **TM:** Comparing DM to TM requires
  - coarse-graining
  - block analysis
  - renormalisation

e.g.



or



### III. Data and theory modelling in archaeology: Data

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  - renormalisation
  - **literal coarse-graining!**

e.g.



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**If coarse-grained DM  $\approx$  TM at some scales then this:**

- gives insight into ‘agency’ as to why the network ‘works’
- parallels social narratives that are often part of the data modelling
- enables us to fill in gaps in DM



DM



TM



### III. Data and theory modelling in archaeology: Data

**Problem:** How do we handle data?

- **TM:** Comparing DM to TM requires
  - coarse-graining
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e.g.



or



**May not be possible to match:**

TM describes generic ('boring') behaviour

Does not easily accommodate the 'unusual'!

e.g. change in power structure!



DM



Theory models are typically for the 'good' times!

### III. Data and theory modelling in archaeology: Data

**Problem:** How do we handle data?

- **TM:** Comparing DM to TM requires
- coarse-graining
- block analysis
- renormalisation

e.g.



or



At best, do not expect too good a match, because of coarse-graining!

Models are designed to help our understanding of how the 'real world' works rather than demonstrate what happens in detailed reality.

Meanwhile, there is a parallel with pre-Keplerian/Newtonian cosmology  
- ask less!



### III. Data and theory modelling: Cosmology revisited

**Example:** cosmology - planetary motion

- **Data modelling**

- derived from patterned observations in data
- phenomenological

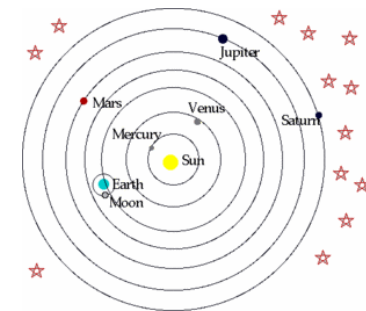
whereas

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DM: Wandering closed orbits + ....



TM: Copernican heliocentric spherical symmetry

- unmoved mover?

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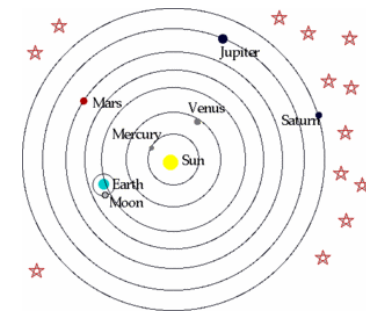
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**Heliocentric universe!**

Coarse-grained precursor  
to Kepler/Newton



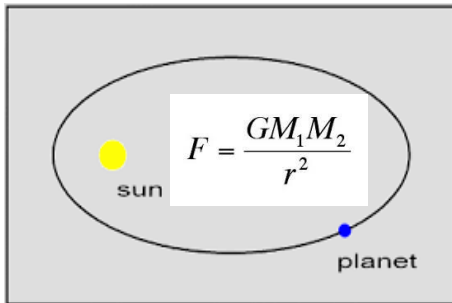
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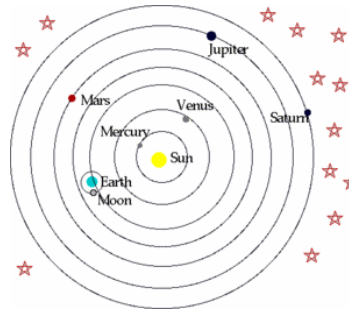
TM: Copernican heliocentric  
spherical symmetry  
- **unmoved mover?**

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**Example:** cosmology - planetary motion



and



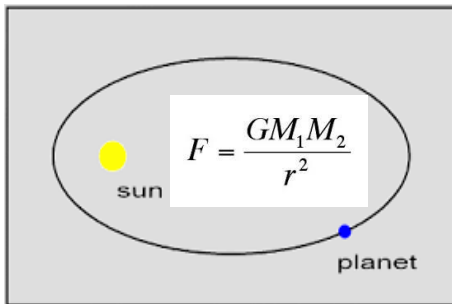
have much in common!

Forces (equations) are  
spherically symmetric

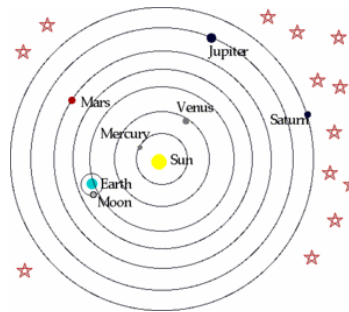
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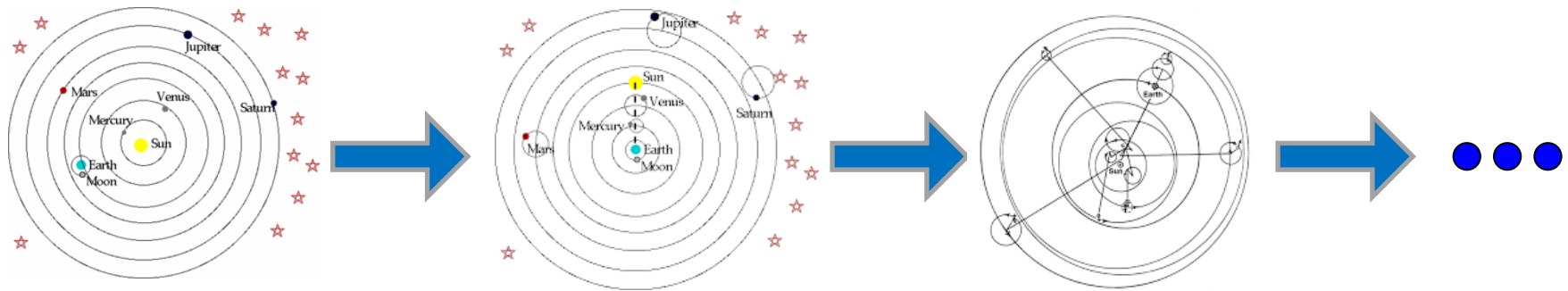
Heliocentric universe is a great achievement (in comparison to geocentric universe)  
- even though Copernican relation to data is only roughly correct!

This Copernican heliocentric universe is the level we try to achieve in archaeological model-making!

### III. Data and theory modelling: Cosmology revisited

**Problem:** To know when to stop looking for agreement with data

**Example:** pre-Newtonian planetary motion – addition of epicycles!



**TM:** Theory of unmoved mover – heliocentric spheres (Copernicus)

keep spheres  
but drop theory

‘save the phenomena’  
**DM**

#### Consequences:

- Learn nothing beyond heliocentricity
- Duhem - Quine underdeterminacy! i.e. we can get what want!

### III. Data and theory modelling in archaeology

**Problem:** How do we handle data?

Use fossil reconstruction as a metaphor !



### III. Data and theory modelling in archaeology

**Problem:** How do we handle data?

Use fossil reconstruction as a metaphor !



Coarse-grain data at the family level!

## IV. Theory modelling in archaeology: Natural selection

**Problem:** How do we choose a theory?

Use Natural Selection as a metaphor!

- Networks that survive are those that do something 'best' or, at least, 'well enough'.
- These different 'somethings' refer to different 'agency'
- Several 'families' of Theory Models, each aligned to a different agency
- Question then becomes one of identifying the family from the incomplete data set and the species within the family
- Possibility of a homomorphic equivalence between TM and DM





## IV. Theory modelling in archaeology: Natural selection

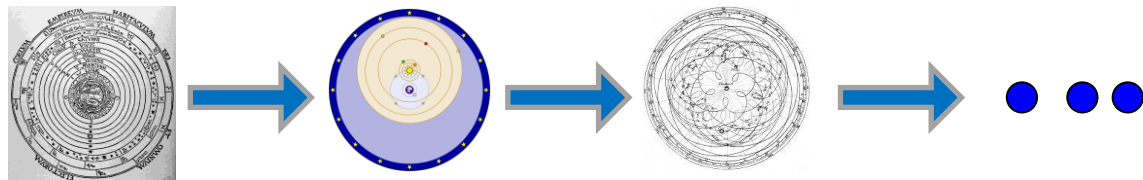
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Epicyclic approach:



**No 'optimal' behaviour!**

**Creationism!**

## IV. Theory modelling in archaeology: Natural selection

### Families:

Three main families of TM with different assumptions about agency:

- Maximum entropy models ('most likely')
- Intervening opportunity models ('easiest')
- Cost-benefit models ('best')



# IV. Theory modelling in archaeology: Natural selection

## Families:

Three main families of TM with different assumptions about agency:

- Maximum entropy models ('most likely')  
- constrained gravity models ('retail' archaeology)
- Intervening opportunity models ('easiest')  
e.g. PPA
- Cost-benefit models ('best')  
'ariadne'

Although the interpretation of the models may be very different, models can belong to more than one family!



## IV. Theory modelling in archaeology: Natural selection

### Generic behaviour:

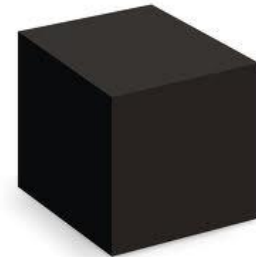
Clear that TM can only describe generic behaviour

A question of model morphology

Few calibration parameters!

e.g. Simini – Barabasi ‘radiation’ model  
for mobility/exchange has NO free calibration  
parameters (IOM).

Sole input: site positions and sizes!



Simini, F., González, M.C., Maritan, A. and Barabási, A.-L. (2012). ‘A universal model for mobility and migration patterns’, Nature, 484, 10856

## IV. Theory modelling in archaeology: Natural selection

### Disagreement with data:

Question: Have we

- chosen the 'wrong' family  
or
- the 'wrong' species in the right family  
or
- are we trying to describe a  
non-generic situation?

## IV. Theory modelling in archaeology: Natural selection

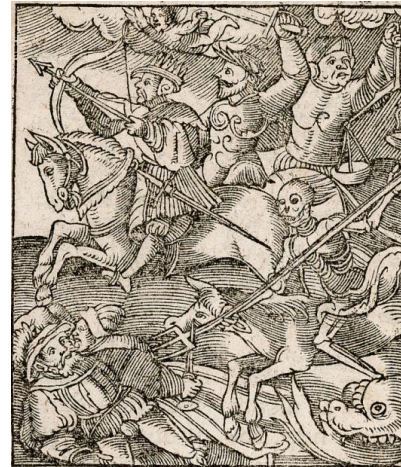
### Disagreement with data:

Question: Have we

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or
- the 'wrong' species in the right family  
or
- are we trying to describe a  
non-generic situation?

Behaviour may not be so simple!

When the 'generic' has been stripped out,  
what is left is the interesting stuff!



## V. This talk: Maximum entropy modelling:

Choose the most likely outcomes commensurate with **your** knowledge of the system.

This quantifies the statement 'All other things being equal I would expect that .....!'



What has entropy got to do with it!

### **Entropy:**

# of questions with which you need to interrogate the system to have complete knowledge of it

**Most likely state of the system is the one with maximum entropy given our limited knowledge!**

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- Laplace:  
Principle of Insufficient Reason
  - Maynard Keynes:  
Principle of indifference
  - Jaynes:  
Principle of maximum ignorance
- flat Bayseian prior!**



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### Snapshot analogy:

Take a picture of each way in which network can be implemented commensurate with (y)our knowledge and put them in a pile.

The most typical picture represents the most likely outcome.

## V. This talk: Maximum entropy modelling:

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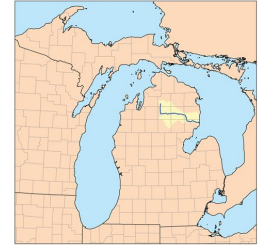
**Most likely state of the system is the one with maximum entropy given our limited knowledge!**

Why not IOP models, cost-benefit models?

**Question of distance scales!**

## VI. Example: Lithic artefact distribution in Great Lakes

**Data:** One source site, many target sites (assemblages)



**Oversimplified**, but sufficient to raise issues of

- parameter uncertainty
- model inadequacy
- data disagreement
- homomorphism

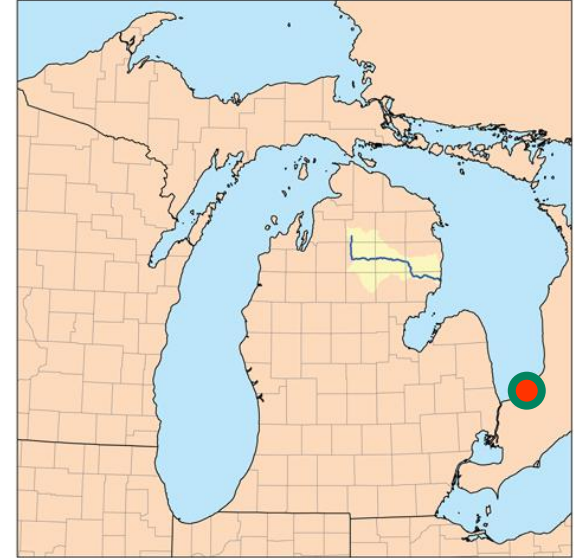
## VI. Example: Lithic artefact distribution in Great Lakes

Lithic (chert) exchange:



Archaic (6000 – 1000 BCE) and  
Woodland (1000 BCE – 500 CE) periods

Oversimplified description!



Data: One source site, many target sites (assemblages)

Artefacts (in assemblages) in several Great Lakes sites.

Look for those obtained from particular sources – specifically **Kettle Point** chert

Peter Reid: *Models for Prehistoric Exchange in the Middle Great Lakes' Basin:*

Ontario Archaeology 46, 33-44 (1986)

## VI. Example: Lithic artefact distribution in Great Lakes

**Coarse-grained data:** just count KP artefacts without discrimination and measure their numerical fraction  $F$  of the assemblages

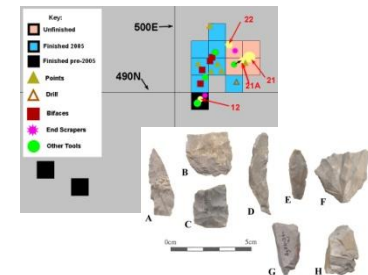
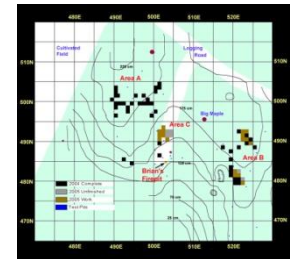
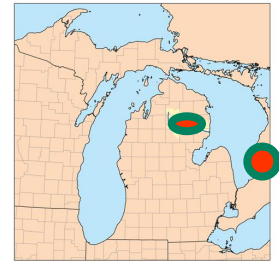
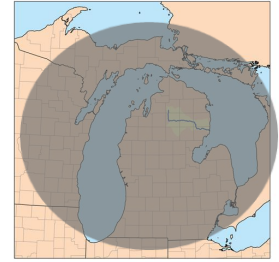
Physical parameters:

- 'Effective' lengths of journey  $d$  (mix land/water transport)
- On average artefacts travel a 'distance'  $D$

**Maximum Entropy:** Assume

- Once an artefact leaves KP it goes on single journey (Renfrew)
  - Number of artefacts fixed
  - Total effort in distributing artefacts is fixed
- } constraints

**Make NO further assumptions about social behaviour!**



# VI. Example: Lithic artefact distribution in Great Lakes

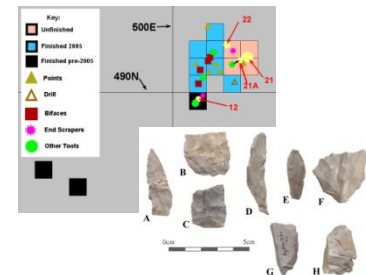
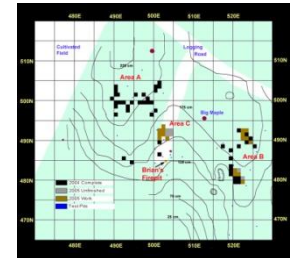
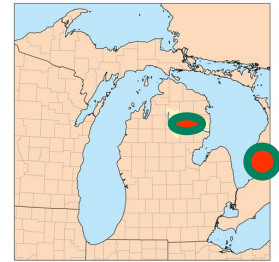
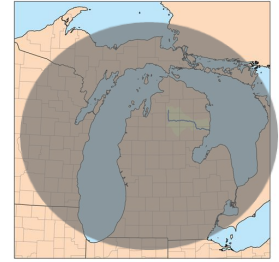
## Theory Modelling: 'Most likely' outcome

At a site  $j$ , # of KP artefacts is

$$N_j \propto \exp(-\beta c_j)$$

where  $c_j$  = effort/cost of moving artefact from KP to site  $j$

Knowledge of 'costs' poor, apart from increasing with distance!



Extremise

subject to

$$S = - \sum_j N_j (\ln N_j - 1)$$

$$\sum_j N_j = N, \quad \sum_j c_j N_j = C$$

## VI. Example: Lithic artefact distribution in Great Lakes

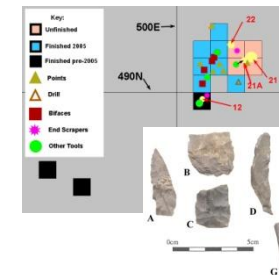
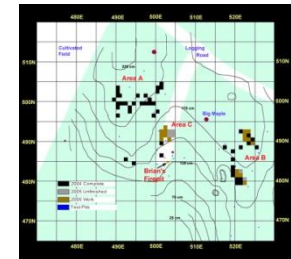
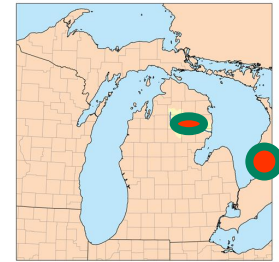
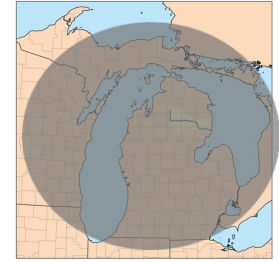
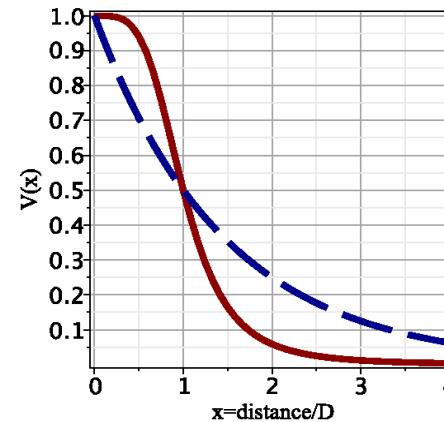
Minimal assumptions: 'Deterrence function  $V(x) = \exp(-\beta c(x))$

$$x = d/D$$

$V(x)$  – likelihood function for an artefact travelling a distance  $xD$  where  $D$  is typical distance over which artefacts are found.

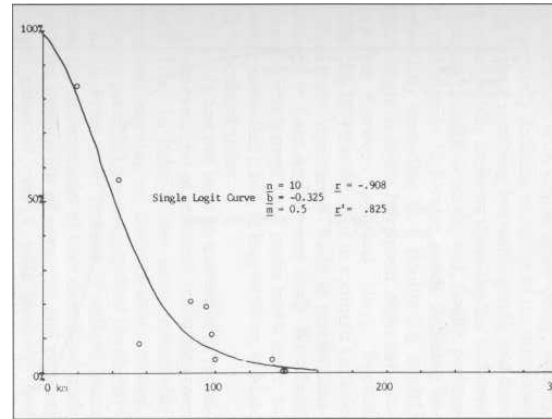
- occur in many theory models!

1. Equal effort/'cost' for equal 'distance'/time in transport of artefacts (**Blue line**)
2. Smoothed out 'so far and no further' (**Red line**)

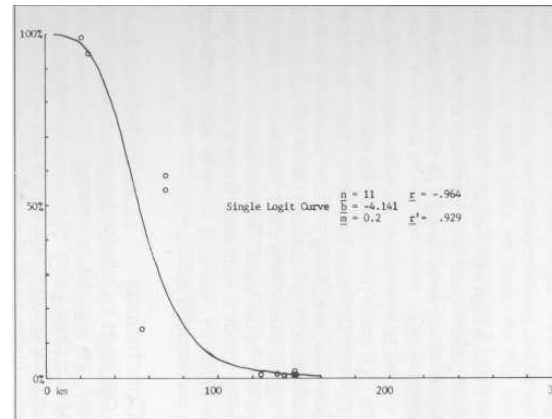


# VI. Example: Lithic artefact distribution in Great Lakes

**Data modelling (ABM):** Model data with social narratives corresponding to particular parametrisations!

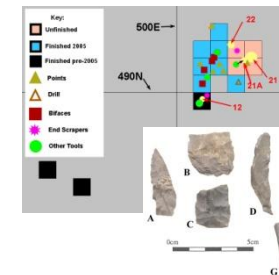
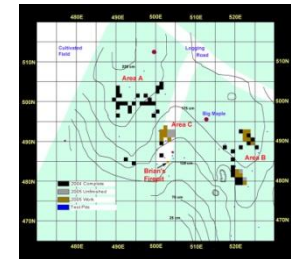
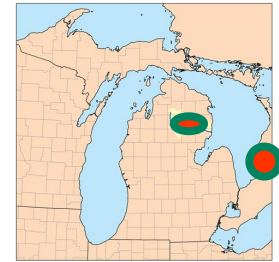
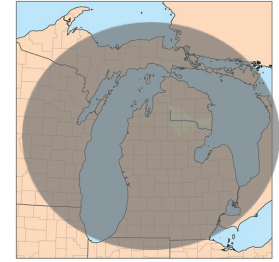


Best-fitting curve for Archaic Kettle Point chert.



Early/Middle Woodland Kettle Point chert.

% of KP artefacts v. distance  
 - 'intensity' / of the source

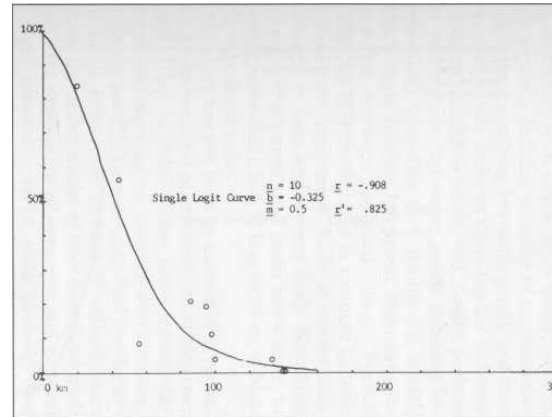




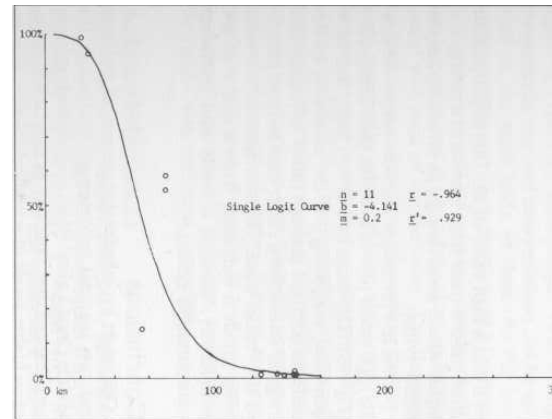
# VI. Example: Lithic artefact distribution in Great Lakes

**Data modelling (ABM):** Model data with social narratives corresponding to particular parametrisations!

“Down-the-line exchange of bulky, low value goods from a source with a low production output, through a costly, time and effort consuming transportation network.”

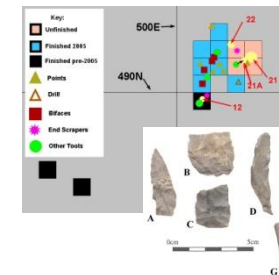
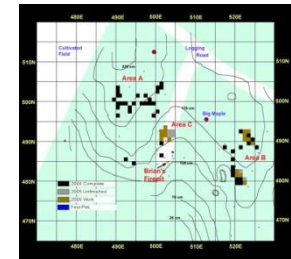
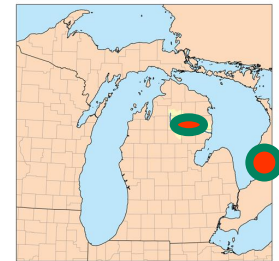
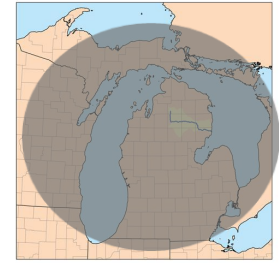


Best-fitting curve for Archaic Kettle Point chert.



Early/Middle Woodland Kettle Point chert.

% of KP artefacts v. distance  
- 'intensity' / of the source



“Infrequent exchange between groups. Perhaps very high-value goods exchanged "politically".

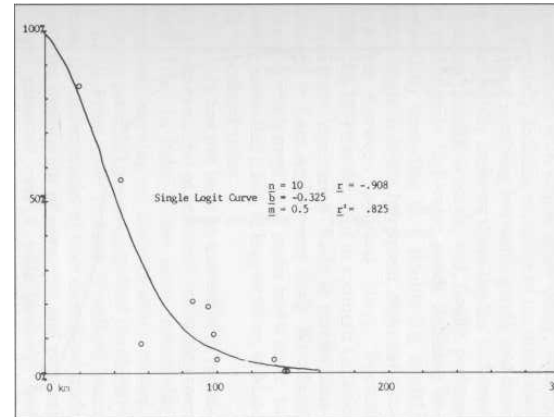
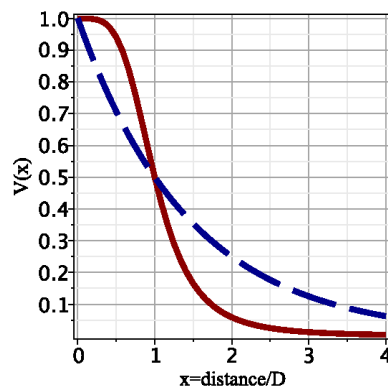
or

regular and fairly frequent contacts between communities”

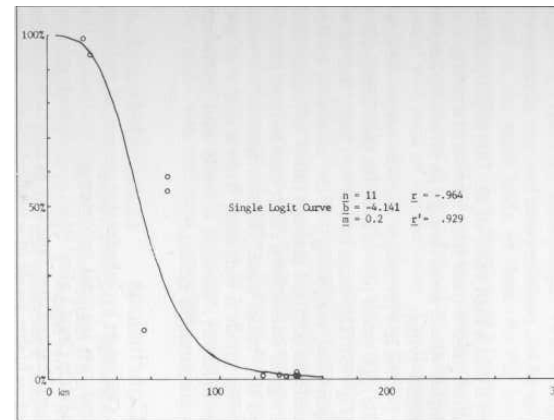
# VI. Example: Lithic artefact distribution in Great Lakes

**Data modelling (ABM):** Model data with social narratives corresponding to particular parametrisations!

“Down-the-line exchange of bulky, low value goods from a source with a low production output, through a costly, time and effort consuming transportation network.”

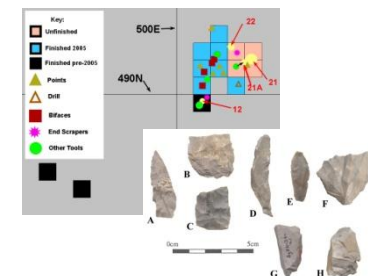
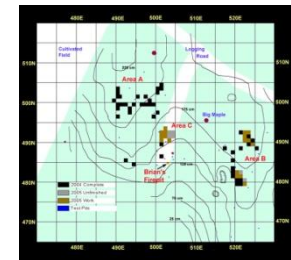
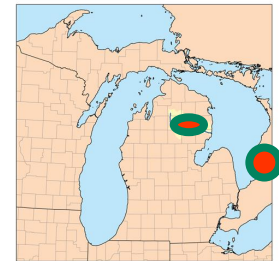
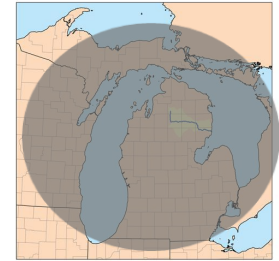


Best-fitting curve for Archaic Kettle Point chert.



Early/Middle Woodland Kettle Point chert.

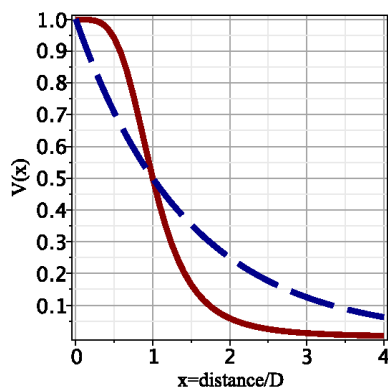
% of KP artefacts v. distance  
- ‘intensity’ / of the source



# VI. Example: Lithic artefact distribution in Great Lakes

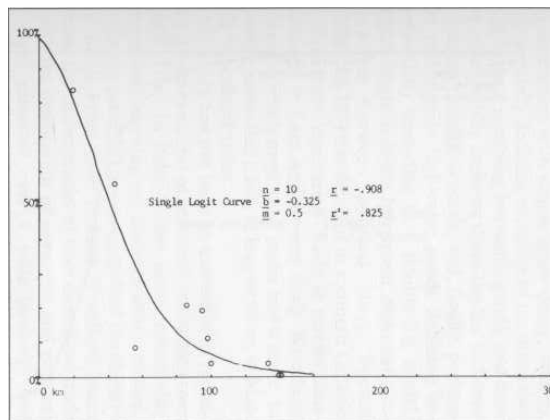
**Theory modelling:** Look for generic behaviour without need for detailed social narratives!

Most likely outcome given the facts that stuff gets made and it takes an effort to distribute it!

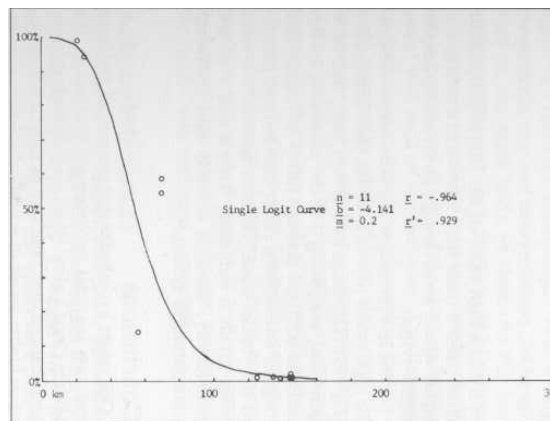


Not quite there!

$$I(x) = (1+a) V(x) / [1 + aV(x)]$$

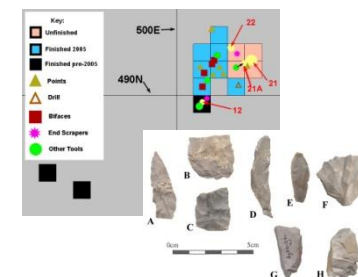
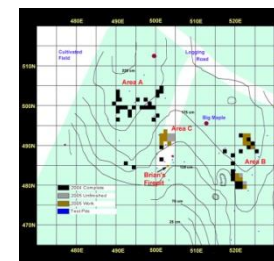
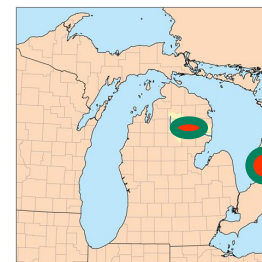


Best-fitting curve for Archaic Kettle Point chert.



Early/Middle Woodland Kettle Point chert.

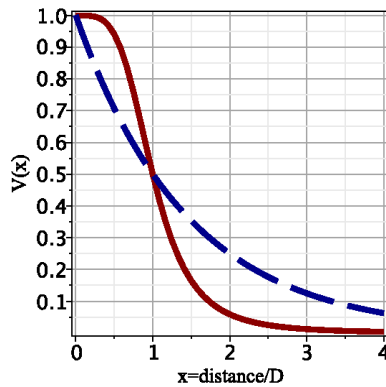
% of KP artefacts v. distance  
- 'intensity' / of the source



## VI. Example: Lithic artefact distribution in Great Lakes

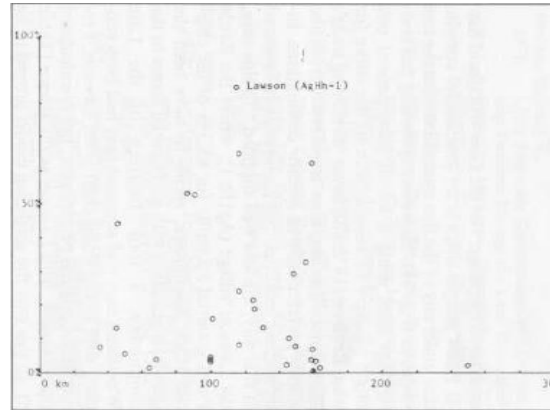
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Not quite there!

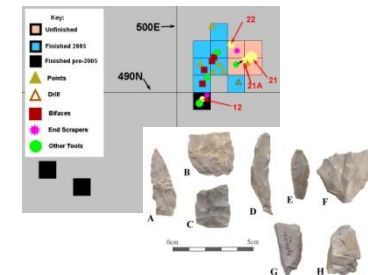
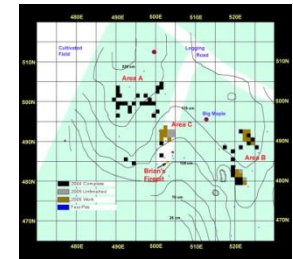
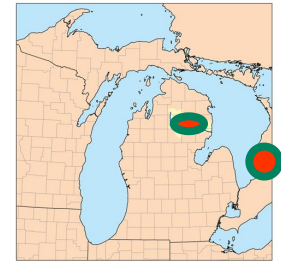
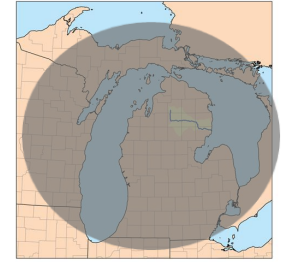
$$I(x) = (1+a) V(x)/[1 + aV(x)]$$



### Scattergram for Late Woodland Kettle Point chert.

Total disagreement with TM!

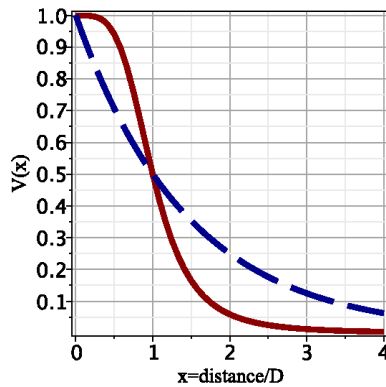
- Is this a sign of social agency?  
or
- incorrect choice of model?



## VI. Example: Lithic artefact distribution in Great Lakes

**Theory modelling:** Look for generic behaviour without need for detailed social narratives!

Most likely outcome given the facts that stuff gets made and it takes an effort to distribute it!

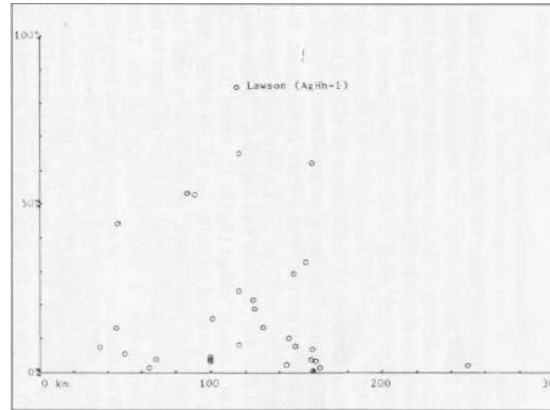


Not quite there!

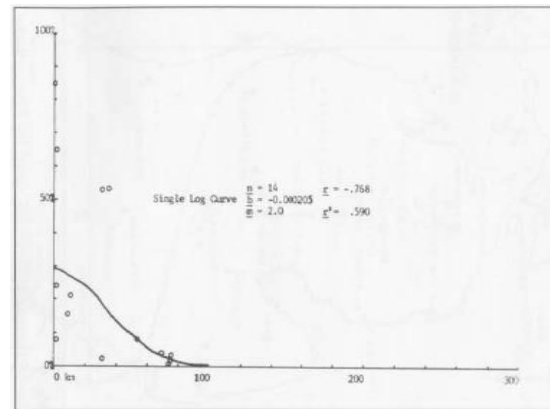
$$I(x) = (1+a) V(x)/[1 + aV(x)]$$

## Social agency:

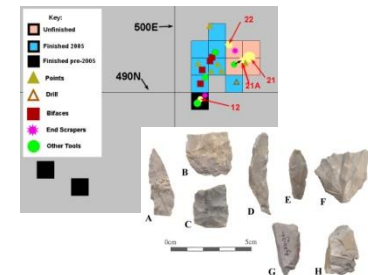
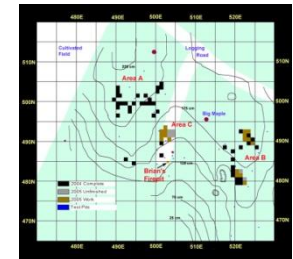
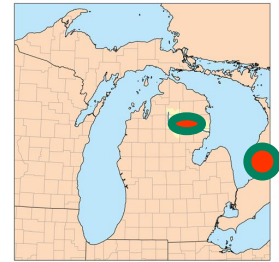
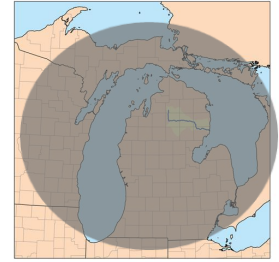
## Distribution centre away from KP



Scattergram for Late Woodland Kettle Point chert.



Scattergram for Late Woodland Kettle Point  
chert, using limited data.





# VI. Example: Lithic artefact distribution in Great Lakes

## Limiting uncertainty:

More information – less uncertainty!

e.g. 'radiation' model!

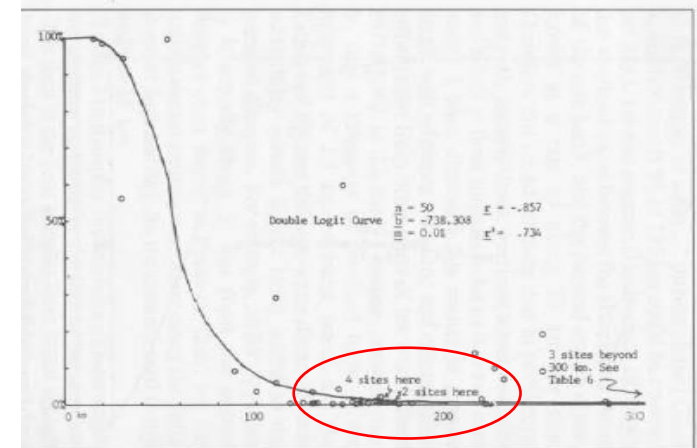
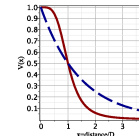
Knowing site distributions as well as distances fixes  
deterrence function  $V(x)$  and hence distribution

e.g. linear distribution of sites gives red curve  
- equifinality?

Tale is in the tail!

Rare events the best discriminators

This is where data is worst!



Best fitting curve for Late Prehistoric Bayport chert,

# VI. Example: Lithic artefact distribution in Great Lakes

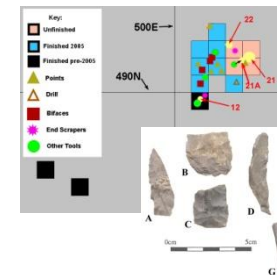
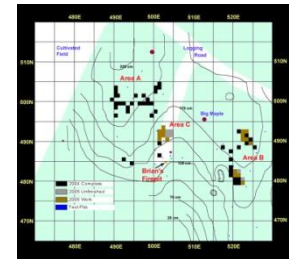
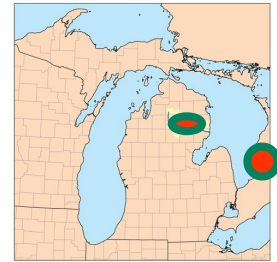
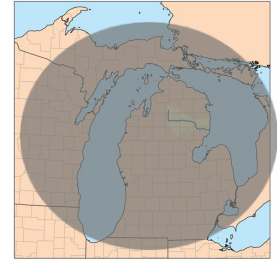
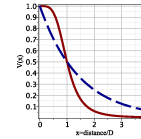
## Conclusions:

**TM:** The most likely distribution of artefacts  
given simple constraints

is commensurate with

**DM (ABM):** detailed social narratives based on coarse-grained data

Data modelling and Theory modelling are homomorphic!



# VI. Example: Lithic artefact distribution in Great Lakes

## Conclusions:

**TM:** The most likely distribution of artefacts  
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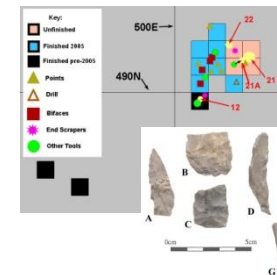
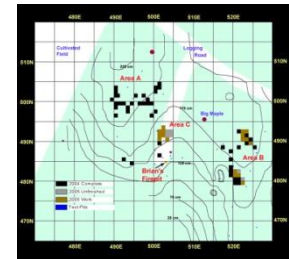
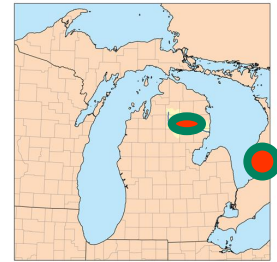
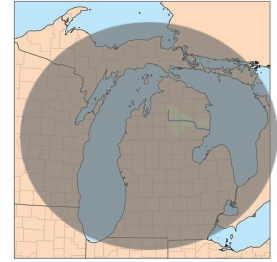
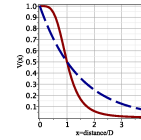
is commensurate with

**DM (ABM):** detailed social narratives based on coarse-grained data

Data modelling and Theory modelling are homomorphic!

Don't look for detailed social narratives:

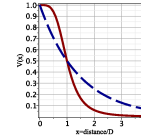
The 'generic' has occurred!  
- all things being equal, what I would expect!





## VI. Example: Lithic artefact distribution in Great Lakes

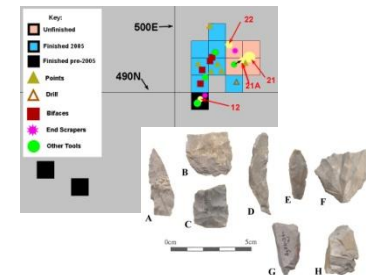
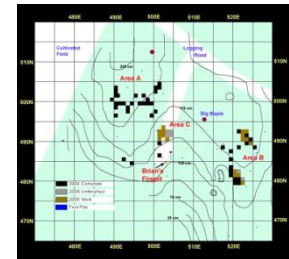
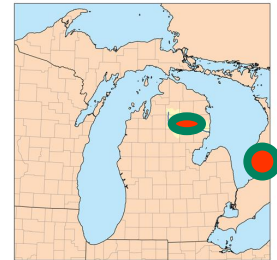
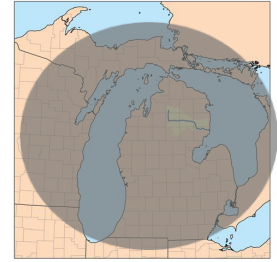
## Major uncertainty:



## Model inadequacy over $V(x)$ :

Not using the data to 'predict'  $V(x)$  because of uncertainties in:

- missing data
- 'effective' distance
- conversion of # into %



# VI. Example: Lithic artefact distribution in Great Lakes

## Major uncertainty:

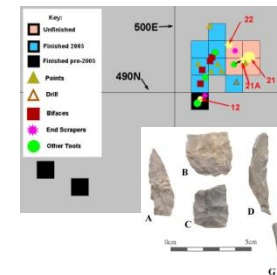
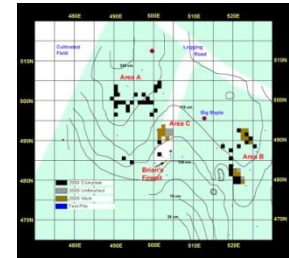
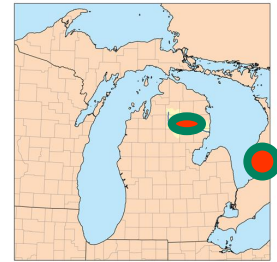
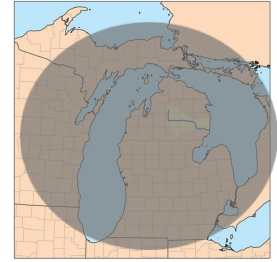
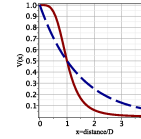
### Model inadequacy over $V(x)$ :

Not using the data to 'predict'  $V(x)$  because of uncertainties in:

- missing data
- 'effective' distance
- conversion of # into %

In this case it doesn't matter, but (unpredictable) deterrence functions are common ingredients in TM

- to deal with richer data sets need networks!



# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

Rihll, T.E. & A.G. Wilson, 1987.

Spatial interaction and structural models in historical analysis: some possibilities and an example, *Histoire & Mesure* 2: 5-32.

- Urbanisation – emergence of dominant settlements
- Synoikism – surrendering of local sovereignty

In particular, rise of Thebes, Corinth, Athens  
as dominant city states

## Data Modelling:

No relational analysis (known to me) but data does exist!

DM and TM complementary with common overlap!

Cf. Mark Altaweel – ABM parallel analysis

No ABM here!



# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

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In particular, rise of Thebes, Corinth, Athens  
as dominant city states

Coarse-grained data:

- site sizes
- site 'importance'
- some understanding of exchange



## VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

Theory Modelling: Top down ME

First step: Generalise lithic analysis

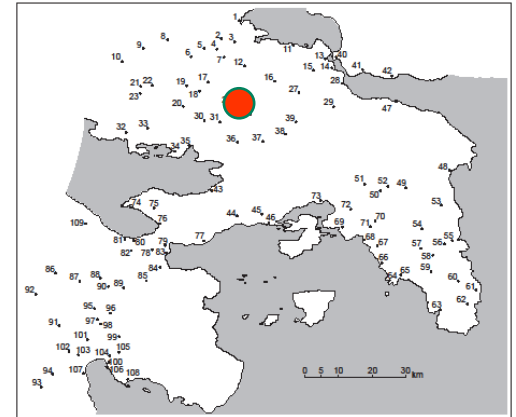
Exchange from i to j:  $T_{ij}$

Extremise

$$S = - \sum_j T_{ij} (\ln T_{ij} - 1)$$

subject to

$$\sum_{ij} T_{ij} = T, \quad \sum_j c_{ij} T_j = C$$





# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

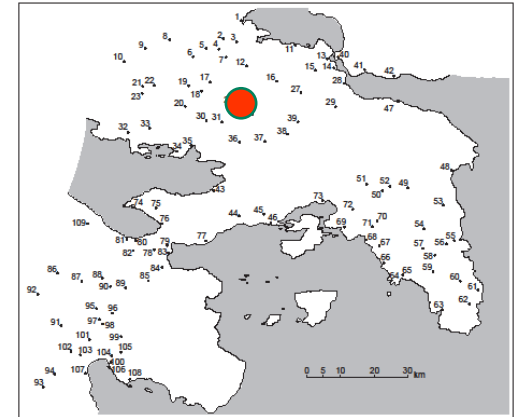
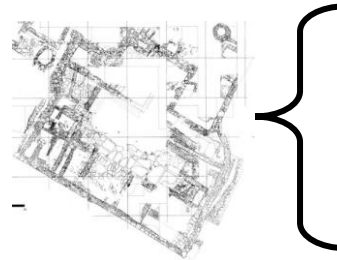
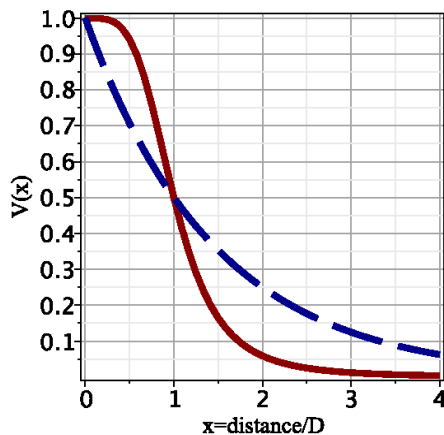
## Theory Modelling: Top down ME

First step: Generalise lithic analysis

Exchange from site  $i$  to  $j$ :  $T_{ij}$

Outcome:

$$T_{ij} \propto \exp(-\beta c_{ij}) = V(x_{ij})$$



Not yet networked!

Simple gravity model!

# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

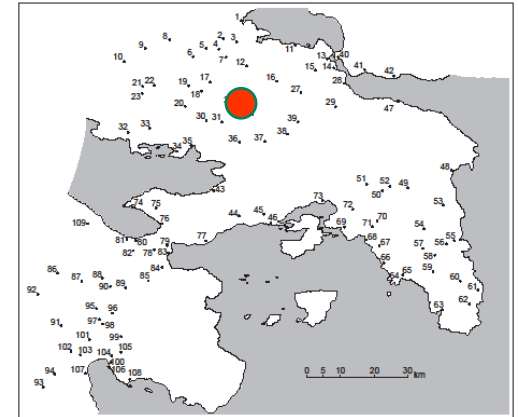
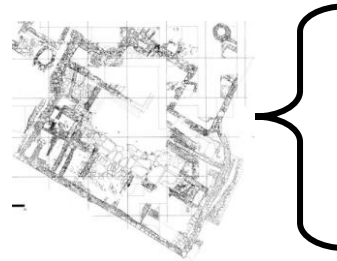
## Theory Modelling: Top down ME

Second step: incorporate benefits of concentrated resources

## Urban Planning:

Benefits of aggregating retail outlets  
in shopping malls!

Collapse of the High Street!



# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

## Theory Modelling: Top down ME

Second step: incorporate benefits of concentrated resources

Further constraint on inflows  $I_i = \sum_j T_{ij}$

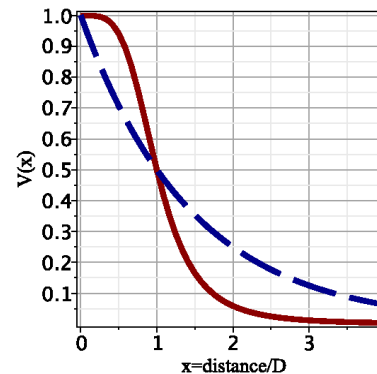
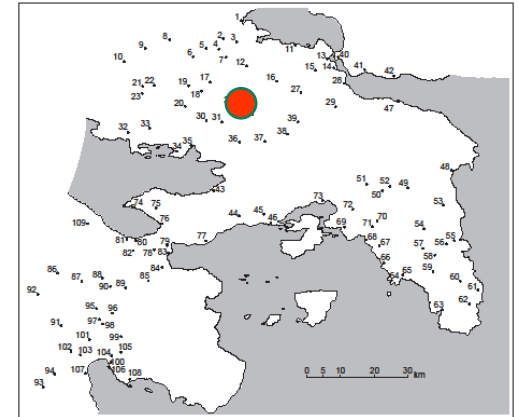
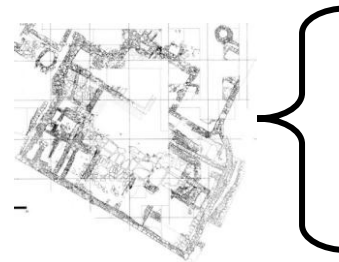
$$\sum_j I_j (\ln I_j - 1) = X$$

Outcome:

$$T_{ij} = A_i O_i (I_j)^Y \exp(-\beta c_{ij})$$

where  $\sum_j T_{ij} = O_i$  are inputs.

$A_i, I_i$  determined self-consistently:

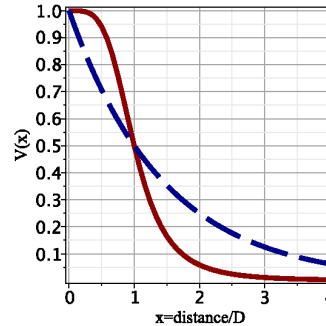




# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

## Inputs:

blue curve:





## 2-dim. calibration space ( $D$ , $Y$ )

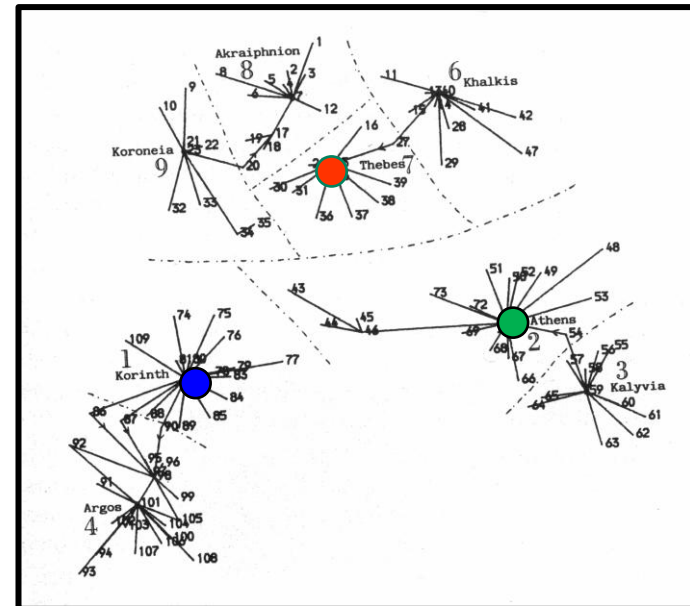
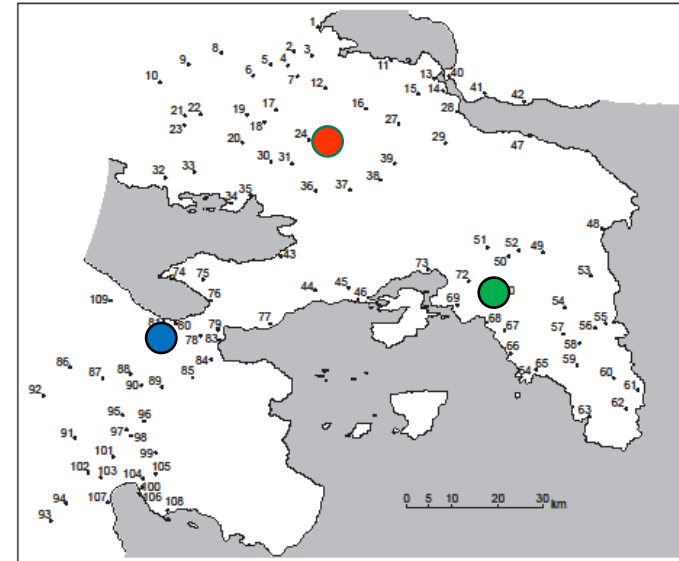
- low sensitivity to  $D$
- high sensitivity to  $Y$
- ~ 150 sites
- A few important sites grow at the expense of small sites
- identifiable 'regional structure'
- Key sites are 'in good accord' with historical record!



In particular:

- Thebes 
- Corinth 
- Athens 

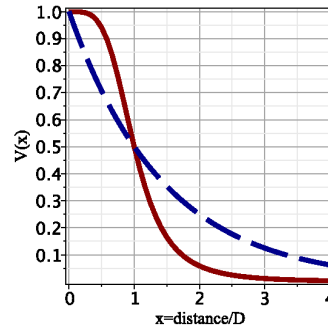
Thebes, Athens, Corinth as the Tesco, Auchan, Carrefour of geometric/archaic Greece!



## VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

Inputs:

red curve:



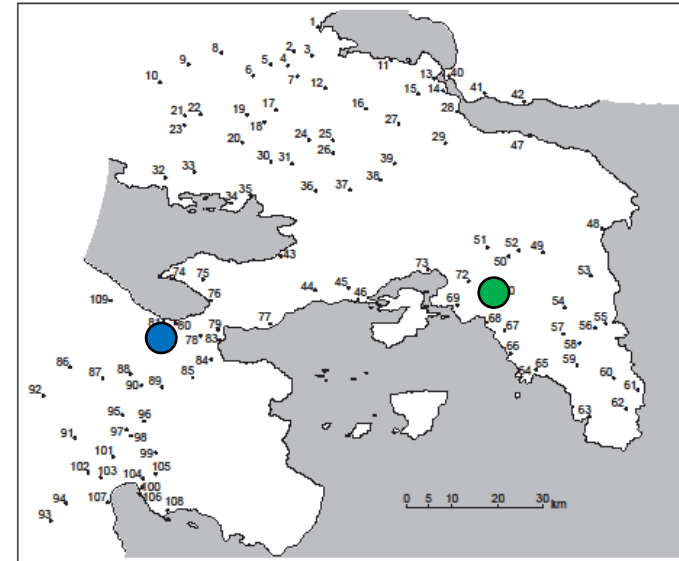
2-dim. calibration space ( $D$ ,  $Y$ )

- low sensitivity to  $D$
  - high sensitivity to  $Y$
  - ~ 150 sites
- 
- A few important sites grow at the expense of small sites
  - identifiable 'regional structure'
  - Other key sites are 'in good accord' with historical record!



In particular:

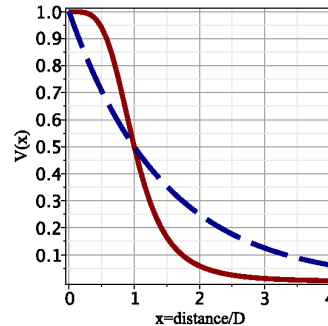
- NO Thebes
- Corinth  still important!
- Athens  still important!



# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

Inputs:

red curve:



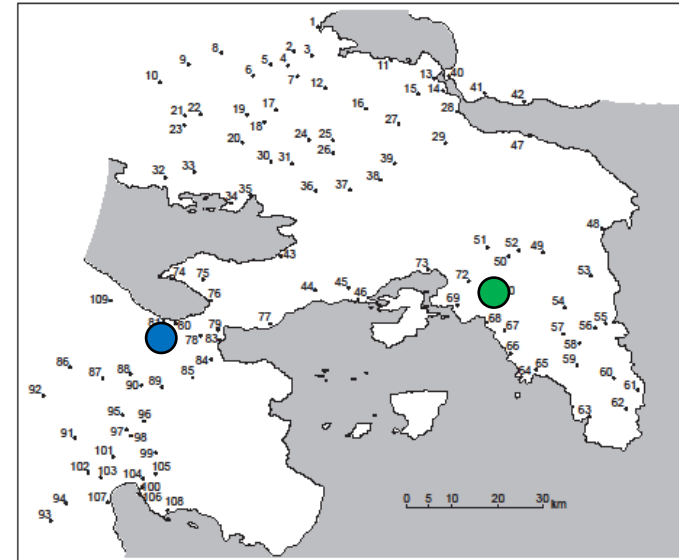
2-dim. calibration space ( $D$ ,  $Y$ )

- low sensitivity to  $D$
- high sensitivity to  $Y$
- ~ 150 sites
- A few important sites grow at the expense of small sites
- identifiable 'regional structure'
- Other key sites are 'in good accord' with historical record!



In particular:

- NO Thebes
- Corinth  still important!
- Athens  still important!



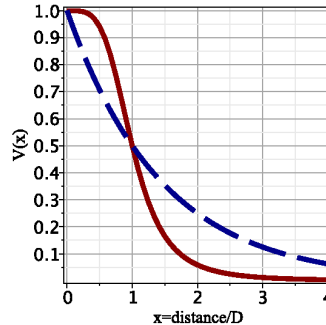
- Athens and Corinth secure because of their positions and connectivity (obvious!)
- Thebes (in central plain) with less obvious connectivity is contingent on social forces –
- No necessity for Thebes!



# VII. 'Retail' Archaeology: Settlement structure in 9<sup>th</sup>/8<sup>th</sup> C BC Greece

Inputs:

red curve:



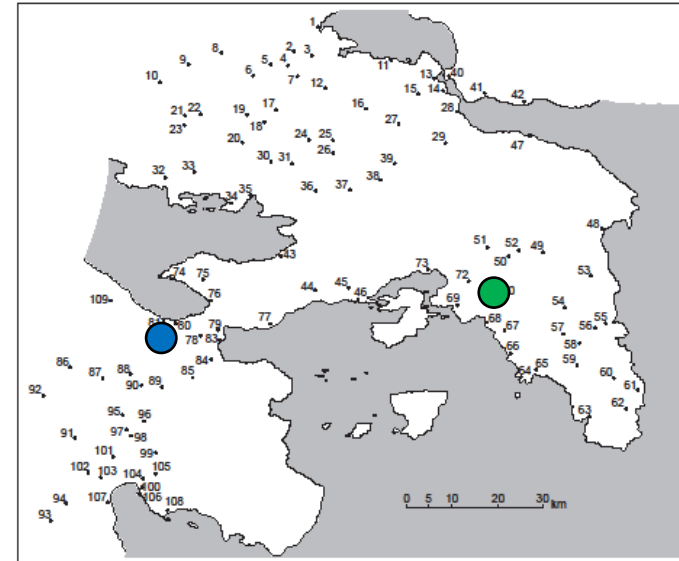
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In particular:

- NO Thebes
- Corinth  still important!
- Athens  still important!



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- Thebes (in central plain) with less obvious connectivity is contingent on social forces –
- No necessity for Thebes!

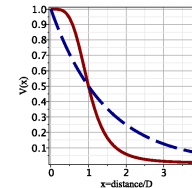


## VII. Conclusions: re-examination of Data and Theory modelling

- Read article a provocation for re-examination of modelling rather than a useful routemap
- **DM:** Relational DM and/or ABM?
- **TM:** Natural selection v. Creationism
  - structured attempt to avoid 'epicycles'
- Choice of family not straightforward! ME v. IOM v. CB
- Generic behaviour!
- Example: Success for simple models:  $DM \approx TM$ 
  - the generic ('obvious') has occurred
- Don't look for detailed social narratives:
  - tale lies in the tail
- More often than not, DM and TM occur at different scales
  - do they merge, or not? 'horses for courses'



v.



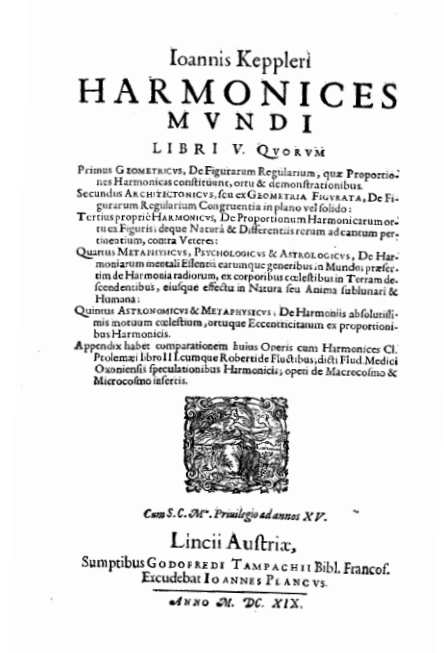
# VII. Conclusions: re-examination of Data and Theory modelling

## Further metaphor for modelling the 'good times':

- TM provides the ground bass/basso ostinato to which society provides the melodic line

Present in Keplerian cosmology:

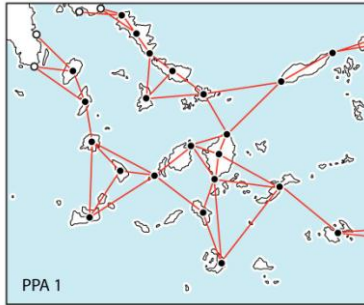
- modification of 'music of the *spheres*' to accommodate elliptic motion!



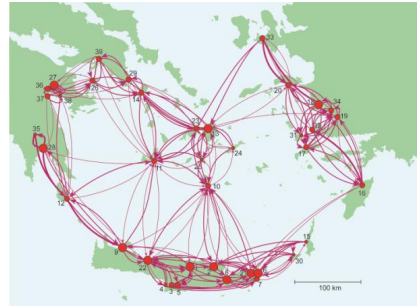
**Thank you!**

**In practice:** TM for large scale exchange networks is useful:

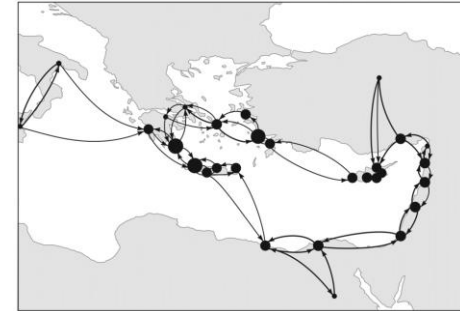
Bronze Age E. Mediterranean maritime networks:



**EBA**

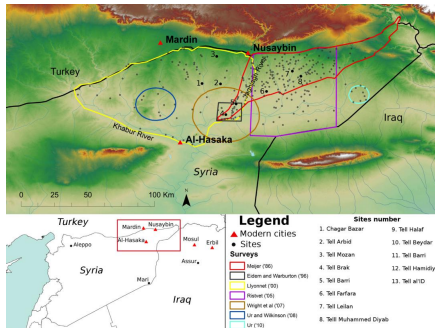


**MBA**



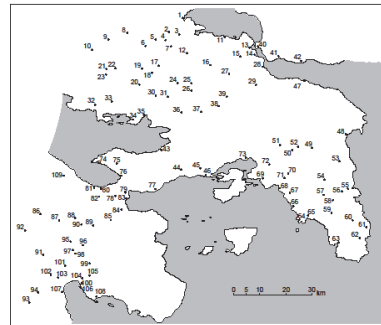
**LBA**

Settlement formation and structure:



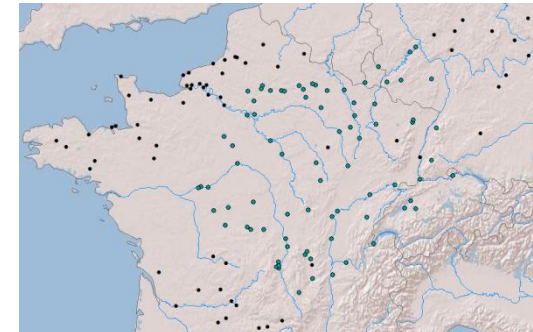
**LBA/IA**

**Khabur triangle**



**9th – 8th C BC**

**Greek city states**



**4th – 1st C BC**

**Celtic W. Europe**



# Appendix I: Modelling prehistoric exchange

*Models for Prehistoric Exchange in the Middle Great Lakes' Basin:*  
Peter Reid (1986) Ontario Archaeology 46, 33-44

- No Decay:  $y = a$ :

Nomadic groups visit source seasonally.  
No exchange between groups.

- Linear:  $y = a + bx^1$

Settled groups visit source directly.  
No exchange between groups.

- Single Log:  $\log y = a + bx^m$

Down-the-line exchange between groups to source.

$b$  varies directly with 'cost' of exchange, inversely  
with value of goods.

$m$  varies directly with value of goods and scale of production at source, and  
also with efficiency of exchange system.

- Single Logit:  $\log (y/1-y) = a + bx^m$

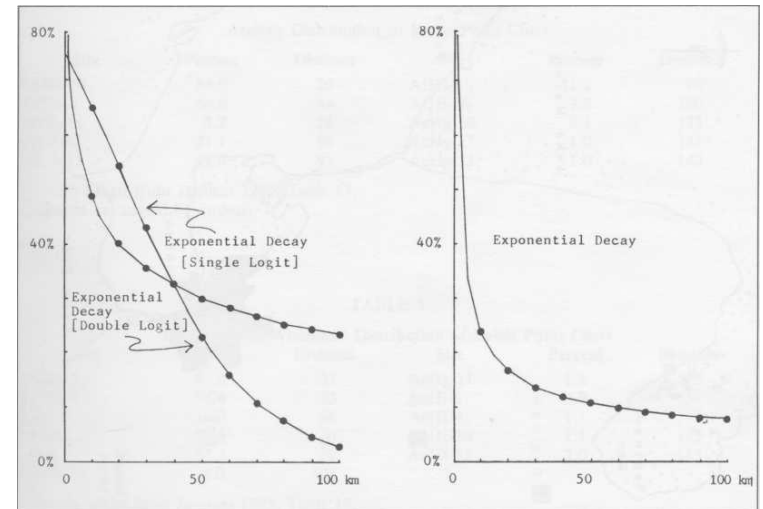
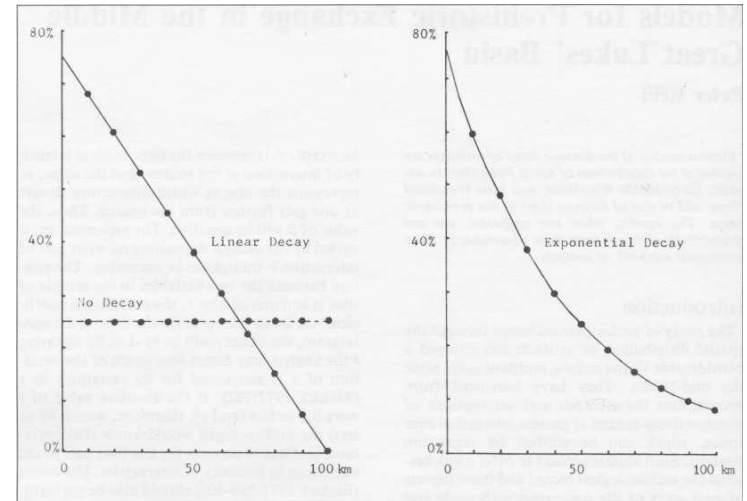
As above, when amount of exchange is represented fractionally.

- Double Log:  $\log y = a + b \log x^m$

Infrequent exchange between groups. Perhaps very high-value goods  
exchanged "politically".

- Double Logit:  $\log (y/1-y) = a + b \log x^m$

As above, when amount of exchange is represented fractionally

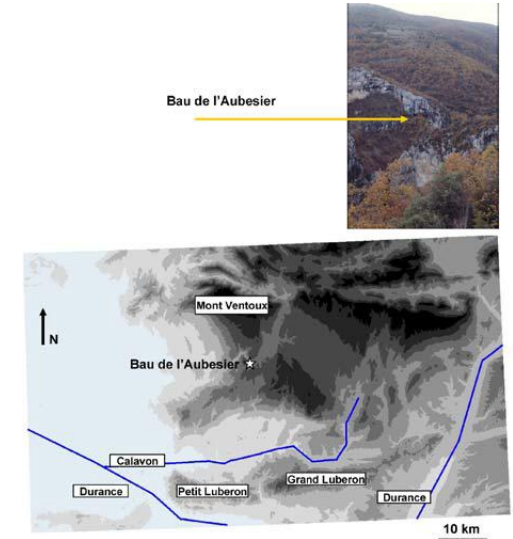


## Further example: Lithic artefact distribution in Vaucluse

**Data:** One target site, many source sites

New information/data.      **Need to weight sources!**

Source  
attractiveness:  $A = \frac{(\text{quality})(\text{extent of source})}{(\text{cost of extraction})(\text{size})(\text{scarcity})}$



Maximise (relative) entropy with this additional information

$$T_i \propto A_i \exp(-\beta c_i)$$

*Understanding Prehistoric Lithic Raw Material*

*Selection: Application of a Gravity Model*

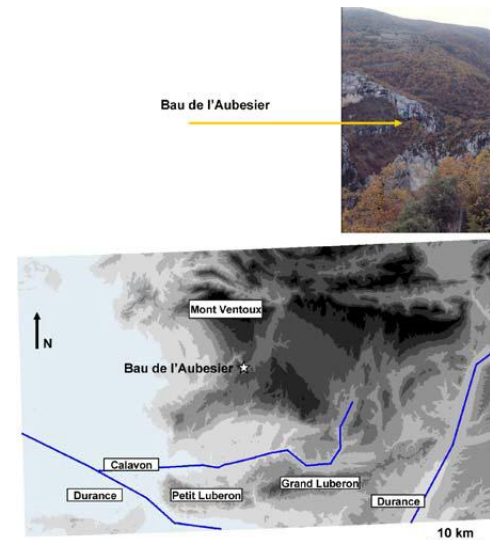
*Lucy Wilson, J. Archaeol. Method Theory (2007) 14:*

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Maximise (relative) entropy with this additional information

$$T_i \propto A_i \exp(-\beta c_i)$$

Wilson proposed deterrence function -  
'Newtonian'

'Unnatural' in the context of 'all things  
being equal' - problematic at small  $d$ !

*Understanding Prehistoric Lithic Raw Material*

*Selection: Application of a Gravity Model*

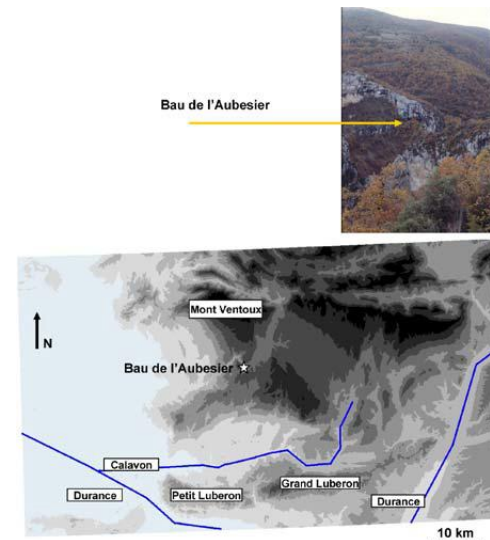
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Maximise (relative) entropy with this additional information

$$T_i \propto A_i \exp(-\beta c_i)$$

**Not able to match data in the same way  
as for KP chert!**

**Homomorphism?**

*Understanding Prehistoric Lithic Raw Material*

*Selection: Application of a Gravity Model*

*Lucy Wilson, J. Archaeol. Method Theory (2007) 14:*

# Conclusions?

Very simple systems!

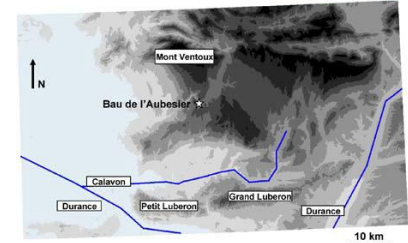
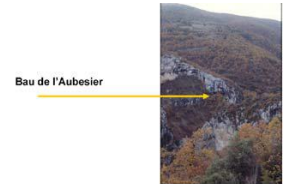
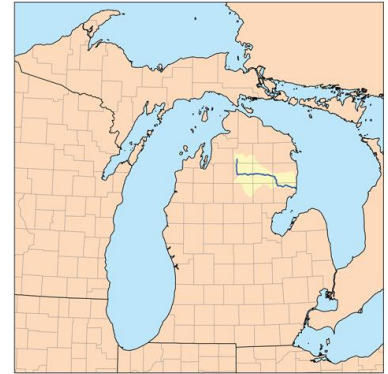


- N. America: no relationships between targets
- France: no relationships between sources

Binary relationships typical of simple gravity models:

To deal with richer data sets need **networks**

**Borrow from models for urban development!**



## Appendix II: Historical networks

**Example:** Bilateral trade in age of High Imperialism (1870 - 1913)

*Trade and Empire, Michener and Wiedenmier (2008)*

**Aim:** Understand the importance of 'empire' on colonies below

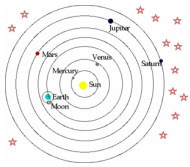
- Aden, Algeria, Australia (New South Wales, Western Australia,
- Queensland, South Australia, Tasmania, Victoria), Bahamas, Barbados, Belgium Congo, Bermuda, British Guiana,
- British Honduras, Brunei, Canada, Ceylon, Cuba, Cyprus, Djibouti, Dutch Guiana, Egypt, Falkland Islands, Fiji,
- French Guiana, French Indochina, Gambia, German East Africa, German SW Africa, German West Africa,
- Gibraltar, Gold Coast, Guadeloupe, Hawaii, Hong Kong, India, Jamaica, Labuan, Lagos, Madagascar, Maldives,
- Malta, Martinique, Mauritius, Morocco, Netherlands East Indies, New Caledonia, New Hebrides, New Zealand,
- Newfoundland, Nyasa, Philippines, Portonovo, Portuguese West Africa, Puerto Rico, Reunion, Sarawak, Senegal,
- Seychelles, Sierra Leone, Somalia, South Africa (Natal Province, Cape Province, and Transvaal), Southern Nigeria,
- St. Helena, St. Pierre/Miquelon, Straits Settlement, Togo, Trinidad and Tobago, Tunis, Uganda, UK East Africa, and Zanzibar.

## Appendix II: Historical networks

Generalised Gravity Model:

$F_{ij}$  = trade from site  $i$  to site  $j$  (single label!)

$$F_{ij} = [(P_i P_j)^a / (r_{ij})^b]$$



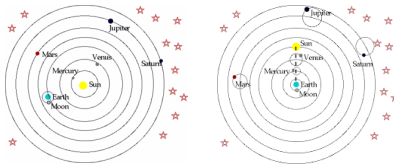
- $P_i$ : population of  $i$
- $r_{ij}$ : distance in miles between  $i$  and  $j$

# Appendix II: Historical networks

Generalised Gravity Model:

$F_{ij}$  = trade from site  $i$  to site  $j$  (single label!)

$$F_{ij} = [(P_i P_j)^a / (r_{ij})^b] \cdot \exp[cE_{ij}].$$



- $P_i$ : population of  $i$
- $r_{ij}$ : distance in miles between  $i$  and  $j$
- $E_{ij}$  is a binary variable which is unity if both countries are part of the same empire

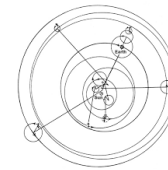
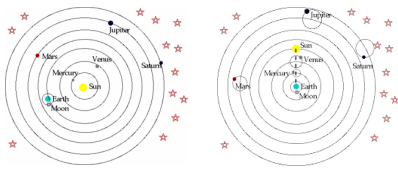


# Appendix II: Historical networks

Generalised Gravity Model:

$F_{ij}$  = trade from site  $i$  to site  $j$  (single label!)

$$F_{ij} = [(P_i P_j)^a / (r_{ij})^b] \cdot \exp[cE_{ij}] \cdot \exp[dR_i R_j] \cdot \exp[eL_{ij}] \cdot \exp[fB_{ij}] \cdot \exp[gW_{ij}] \cdot \exp[hG_{ij}] \cdot \exp[jC_{ij}] \cdot \exp[kV_{ij}] \cdot \exp \varepsilon_{ij}$$



- $P_i$ : population of  $i$
- $r_{ij}$ : distance in miles between  $i$  and  $j$
- $E_{ij}$  is a binary variable which is unity if both countries are part of the same empire
- $R_i$  is railroad track miles of  $i$
- $L_{ij}$  is the number of landlocked countries in the country-pair dyad (0, 1, or 2)
- $B_{ij}$  is a binary variable which is unity if  $i$  and  $j$  share a border, zero otherwise
- $W_{ij}$  is a binary variable which is unity if countries  $i$  and  $j$  are at war
- $G_{ij}$  is a binary variable which is unity if  $i$  and  $j$  both are on the gold standard
- $C_{ij}$  is a binary variable which is unity if both countries are part of either the Latin or Scandinavian currency unions
- $V_{ij}$  is mutual exchange rate volatility
- $\varepsilon_{ij}$  is a white noise error term capturing other influences on bilateral trade.

Can almost get what we want!

Duhem-Quine thesis!

# Appendix III. 'Retail' Archaeology:

Theory Modelling: **Middle up**

Second step: incorporate benefits of concentration of resources

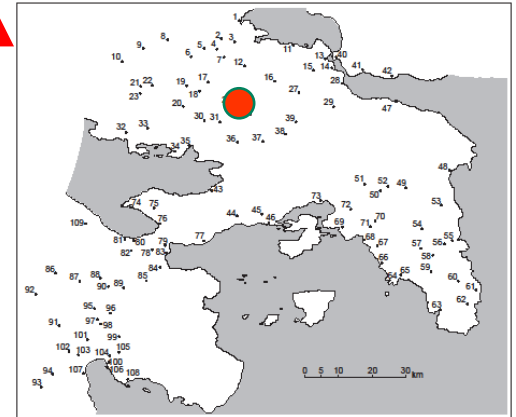
Alternative approach: **Strong 'retail' narrative**

- i. Interaction between two places is proportional to the size of the origin zone and the importance and distance from the origin zone of all the other sites which compete as destination zones
- ii. the 'importance' of a place is proportional to the interaction it attracts from other places
- iii. 'size' of a site  $\sim$  site 'importance'

– **NO mention of entropy, but same results!**

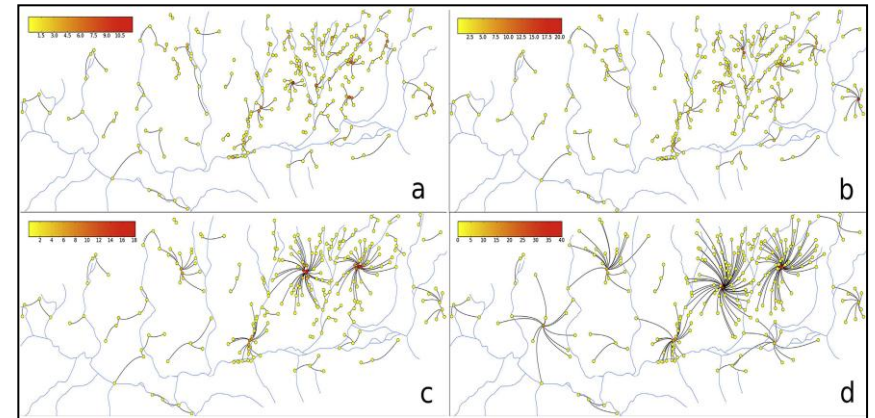
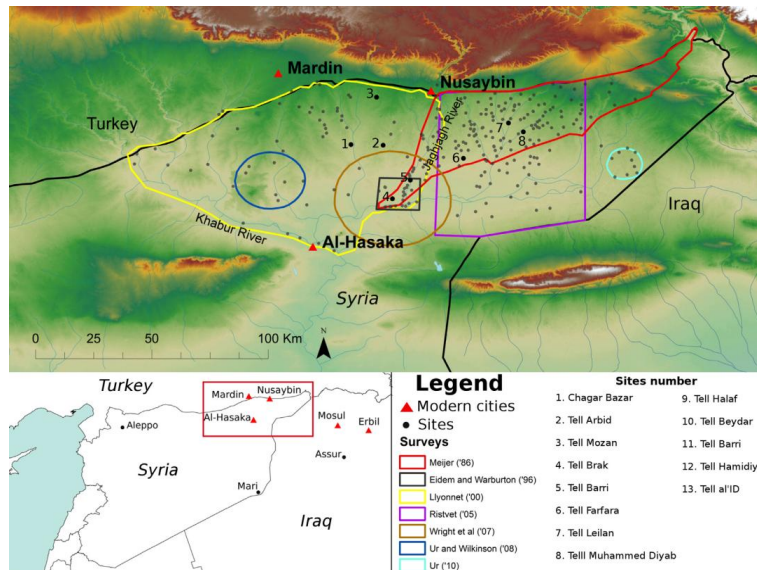
Solved with Lotka-Volterra equation

– **predator-prey equation!**



# Appendix III. 'Retail' Archaeology:

## Example: Khabur triangle



*'Application of an entropy maximizing and dynamics model for understanding settlement structure: the Khabur Triangle in the Middle Bronze and Iron Ages'*

Davies et al. (2014), Journ. Arch. Sci. 43, 143-154