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Supplementary Information

**HYBRID INTEGRAL TRANSFORMS FOR FLOW DEVELOPMENT IN DUCTS PARTIALLY FILLED WITH POROUS MEDIA**

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**Supplementary Information S1. Equivalency with the streamfunction-only formulation**

Expanding Eq. (14) in a Cartesian coordinate system like the one in Fig. 1, imposing that the only non-zero component of the base vector  is in the z-direction, and assuming the linearity of the curl operator holds for the infinite series involved, the following expression is achieved:

|  |  |
| --- | --- |
|   | (S1.1) |

where  is taken as the z-axis component of  in very much the same way as in Eq. (16). Defining the summation within the derivatives in Eq. (S1.1) as a streamfunction, the correspondence between the approaches is fully established.

**Supplementary Information S2. Derivation of the transformed problem**

Beginning with Eq. (25), expanding each term under the Cartesian coordinate system shown in Fig. 1, and employing Eq. (14) with the relation of Eq. (16) substituted on it, we then have,

*For the inertial term:*

|  |  |
| --- | --- |
|  | (S2.1) |

*For the filtered inertial terms:*

|  |  |
| --- | --- |
|  | (S2.2.a) |
|  | (S2.2.b) |

*For the inertial and dissipative terms:*

|  |  |
| --- | --- |
|  | (S2.3.a) |
|  | (S2.3.b) |
|  | (S2.3.c) |

where  is the width of the volume  in the z-direction and the coefficients are defined as in Eqs. (26.b-i), and are omitted here for the sake of brevity.

The coefficients of Eqs. (S2.3.b,c) can be further simplified using properties of the eigenvalue problem. Substituting the right-hand side of Eq. (15.a) into Eq. (S2.3.b) and employing the orthogonality property, the coefficients then become:

|  |  |
| --- | --- |
|  | (S2.4.a) |
|  | (S2.4.b) |

Gathering Eqs. (S2.1), (S2.2.a), and (S2.3.a) in accordance with Eq. (25), the transformed problem presented in Eqs. (26.a-i) is obtained.

**Supplementary Information S3. Derivation of the boundary conditions**

Let  be the vector field enclosed by the curl operator in Eqs. (27.a,b). By virtue of Eq. (16), the only non-zero component of  lies in the z-direction. Expanding the curl operation with the Cartesian coordinate system of Fig. 1, we then have,

|  |  |
| --- | --- |
|  | (S3.1.a) |
|  | (S3.1.b) |
|  | (S3.1.c) |
|  | (S3.1.d) |

where  is the z-component of the vector .

Operating Eqs. (S3.1.a,c) with  and using the boundary condition for the eigenfunction shown in Eqs. (15.b),

|  |  |
| --- | --- |
|  | (S3.2.a) |
|  | (S3.2.b) |

With the substitution of the series for vector  into Eqs. (S3.1.b,d) and (S3.2.a,b), Eqs. (28.a-d) immediately follow.