Supporting information

Disjunctive optimization model for the production planning and blending of crude oil in a conventional oil field

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Big-M reformulation of the GDP model

To solve the problem, the GDP is reformulated as an MINLP problem replacing the Boolean variables W_{ij}, Y_{ij}, V_p, U_{mp} with the binary variables w_{ij}, y_{ij}, v_p, u_{mp} and reformulating the disjunctions (14), (29) and logic constraints (15), (16), (30), (31), (32) as the mixed-integer constraints (S1) - (S37) via the big-M technique.

The disjunction (14) which describes the production planning in wells is reformulated as follows:

$$tr_{ij} \geq tp_{ij} - BM1(1 - w_{ij}) \qquad \forall i \in I, j \in J \tag{S1}$$

$$tr_{ij} \leq tp_{ij} + BM1(1 - w_{ij}) \qquad \forall i \in I, j \in J \tag{S2}$$

$$tp_{ij} \geq t_{ij}^{o} - BM1(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S3}$$

$$tp_{ij} \leq t_{ij}^{o} + BM1(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S4}$$

$$Q_{ij} \geq q_i^{up} tp_{ij} - BM2_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S5}$$

$$Q_{ij} \leq q_i^{up} tp_{ij} + BM2_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S6}$$

$$P_{ij}^{f} \geq P_{ij}^{in} - c1_i q_i^{up}\left(\ln t_{ij}^{o} + c2_i\right) - BM3_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S7}$$

$$P_{ij}^{f} \leq P_{ij}^{in} - c1_i q_i^{up}\left(\ln t_{ij}^{o} + c2_i\right) + BM3_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S8}$$

$$P_{ij}^{f} \leq P_{ij}^{in} + BM3_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S9}$$

$$P_{ij}^{out} \geq P_{ij}^{f} - P_{ij}^{in} + P_i^{up} - \frac{\rho_i g L_i}{g_c} - \frac{L_i f_i {q_i^{up}}^2}{2A_i^2 D_i \rho_i} - C_i^{v} q_i^{up} - BM3_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S10}$$

$$P_{ij}^{out} \leq P_{ij}^{f} - P_{ij}^{in} + P_i^{up} - \frac{\rho_i g L_i}{g_c} - \frac{L_i f_i {q_i^{up}}^2}{2A_i^2 D_i \rho_i} - C_i^{v} q_i^{up} + BM3_i(1 - y_{ij}) \qquad \forall i \in I, j \in J \tag{S11}$$

$$tp_{ij} \geq t_{ij}^{c} - BM1 y_{ij} \qquad \forall i \in I, j \in J \tag{S12}$$

$$tp_{ij} \leq t_{ij}^{c} + BM1 y_{ij} \qquad \forall i \in I, j \in J \tag{S13}$$

$$Q_{ij} \geq -BM\,2_i\, y_{ij} \qquad \forall i \in I,\, j \in J \tag{S14}$$

$$Q_{ij} \leq BM\,2_i\, y_{ij} \qquad \forall i \in I,\, j \in J \tag{S15}$$

$$P_{ij}^{f} \geq \frac{P_{ij}^{in} + c1_i^{rec}\left(\ln t_{ij}^{c} + c2_i^{rec}\right) + P_i^{up} - \sqrt{\left(P_{ij}^{in} + c1_i^{rec}\left(\ln t_{ij}^{c} + c2_i^{rec}\right) - P_i^{up}\right)^2 + \delta^2}}{2} - BM3_i\, y_{ij} \qquad \forall i \in I,\, j \in J \tag{S16}$$

$$P_{ij}^{f} \leq \frac{P_{ij}^{in} + c1_i^{rec}\left(\ln t_{ij}^{c} + c2_i^{rec}\right) + P_i^{up} - \sqrt{\left(P_{ij}^{in} + c1_i^{rec}\left(\ln t_{ij}^{c} + c2_i^{rec}\right) - P_i^{up}\right)^2 + \delta^2}}{2} + BM3_i\, y_{ij} \qquad \forall i \in I,\, j \in J \tag{S17}$$

$$P_{ij}^{t} \geq P_{ij}^{f} - P_{ij}^{in} + P_i^{up} - \frac{\rho_i g L_i}{g_c} - BM\,3_i\, y_{ij} \qquad \forall i \in I,\, j \in J \tag{S18}$$

$$P_{ij}^{t} \leq P_{ij}^{f} - P_{ij}^{in} + P_i^{up} - \frac{\rho_i g L_i}{g_c} + BM\,3_i\, y_{ij} \qquad \forall i \in I,\, j \in J \tag{S19}$$

$$tr_{ij} \geq -BM\,1 w_{ij} \qquad \forall i \in I,\, j \in J \tag{S20}$$

$$tr_{ij} \leq BM\,1 w_{ij} \qquad \forall i \in I,\, j \in J \tag{S21}$$

Eq. (S22) relates the binary variables on the disjunctions (14):

$$y_{ij} \leq w_{ij} \qquad \forall i \in I,\, j \in J \tag{S22}$$

Logic propositions (15) and (16) are reformulated in the following constraints:

$$w_{ij} \geq w_{i,j+1} \qquad \forall i \in I,\, j \in J \tag{S23}$$

$$w_{ij} + w_{i,j+1} - y_{i,j+1} - y_{ij} \leq 1 \qquad \forall i \in I,\, j \in J \tag{S24}$$

$$y_{ij} + y_{i,j+1} + w_{ij} + w_{i,j+1} \leq 3 \qquad \forall i \in I,\, j \in J \tag{S25}$$

The disjunction (26) which describe part of the pooling problem are also reformulated by big-M representation:

$$b_{mp} \geq b_{mp}^{\min} - BM\,4(1 - u_{mp}) \qquad \forall m \in M,\, p \in P \tag{S26}$$

$$b_{mp} \geq -BM\,4u_{mp} \qquad \forall m \in M,\, p \in P \tag{S27}$$

$$b_{mp} \le BM4u_{mp} \qquad \forall m \in M, p \in P \tag{S28}$$

$$\sum_k x_{pk} \ge -BM4u_{mp} \qquad \forall p \in P \tag{S29}$$

$$\sum_k x_{pk} \le BM4u_{mp} \qquad \forall p \in P \tag{S30}$$

Finally, the logic propositions (27), (28) and (29) are reformulated as follows:

$$\sum_m u_{mp} \ge v_p \qquad \forall p \in P \tag{S31}$$

$$v_p \ge u_{mp} \qquad \forall m \in M, p \in P \tag{S32}$$

$$\sum_{r \ne m} u_{r,p} \ge u_{mp} \qquad \forall m \in M, r \in M, p \in P \tag{S33}$$

The tighter values of big-M constants of the abovementioned equations are calculated by the following expressions:

$$BM1 = H \tag{S34}$$

$$BM2_i = Hq_i^{up} \qquad \forall i \in I \tag{S35}$$

$$BM3_i = P_i^{up}2.5 \qquad \forall i \in I \tag{S36}$$

$$BM4_p = CP_p1.3 \qquad \forall p \in P \tag{S37}$$