

Appendix A1. Tables and figures relevant to formulating, fitting and checking assumptions of the Bayesian state-space model

“Sudden collapse of a mesopredator reveals its complementary role in mediating rocky reef regime shifts”

Proceedings of the Royal Society B – DOI: 10.1098/rspb.2018-0553

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Table A1.1 Descriptions and specification for all elements within the state-space model.

Model element	Description	Specified by
Parameters Estimated by the Model		Priors
$\alpha_{O,i}$	Size-specific sea otter predation effect: The additive increase in instantaneous mortality rate of size class i associated with the presence of sea otters	Uniform(0.0001, 5)
$\alpha_{P,i}$	Size-specific <i>Pycnopodia</i> predation effect: The additive increase in instantaneous mortality rate of size class i associated with a unit increase in <i>Pycnopodia</i> biomass	Uniform(0.0001, 5)
$\bar{\delta}_1$	Mean baseline mortality parameter for small urchins ($i=1$) across sites (due to sources other than predation by sea otters or <i>Pycnopodia</i>)	Uniform(0.0001, 10)
β_i	Size-specific mortality adjustment parameter: scales baseline mortality rates (mortality from sources other than predation by sea otters or <i>Pycnopodia</i>) for medium ($i=2$) and large ($i=3$) urchins relative to small ($i=1$) urchins	Beta(1, 2)
σ_δ	Spatial variation in baseline urchin mortality (standard deviation across sites) for small urchins due to site-specific conditions or habitat	Uniform(0.001, 10)
σ_ϵ	Temporal variation in urchin mortality due to environmental stochasticity (standard deviation across years)	Uniform(0.001, 10)
θ	Observability parameter, describing the probability of detecting small urchins (1-3 cm) in surveys (“nuisance parameter”)	Beta(1.5, 1.5)
r	Overdispersion parameter for the negative binomial distribution describing observation error in urchin counts at a site	Gamma(1.1, 0.1)
Varies by Site		
$\delta_{1,j}$	Site-specific baseline mortality parameter for small urchins, $i=1$ (mortality due to sources other than predation by otters or <i>Pycnopodia</i>). Prior informed by hyperparameter σ_δ	Normal($\bar{\delta}_1, \sigma_\delta^2$)
Varies across Years		
R_t	Annual regional “effective” recruitment (urchins recruiting to the 1cm size class) in years $t = 1-4$. Weakly informed prior results in a range of possible values between 1-20 (mean ≈ 5) recruits/m ² /year, as informed by reported empirical recruitment data from B.C. (Sloan et al. 1987, Atkins et al. 2006, Zhang et al. 2011) ¹	Lognormal(1.4, 0.67)
Varies across Sites and Years		
$\epsilon_{i,j,t}$	Error in urchin mortality due to annual environmental stochasticity for size class i , at site j , in year t . Prior informed by hyperparameter σ_ϵ	Normal(0, σ_ϵ^2)
Specified Parameters		Fixed Values
G_i	Size class transition probabilities give the proportion of urchins transitioning from size class small to medium (G_1) or from medium to large (G_2)	$G_1 = 0.52$, $G_2 = 0.09$ Estimated via simulations using published urchin growth model
Variables		Data from Surveys
$C_{i,j,t,k}$	Density of urchins in size class i , at site j , in year t , in quadrat k (where urchins in $N_{i=1} = 1-3$ cm, $N_{i=2} = 4-7$ cm, and $N_{i=3} \geq 8$ cm)	Number per 1m ² quadrat
$O_{j,t}$	Sea otter presence ($O = 1$) or absence ($O = 0$) at a given site j and year t	Recorded sea otter presence
$P_{j,t}$	Biomass of <i>Pycnopodia</i> at a given site j and year t	kg per 10m ² survey area

¹ Sloan, N., Lauridsen, C.P. & Harbo, R.M. (1987). Recruitment characteristics of the commercially harvested red sea urchin *Strongylocentrotus franciscanus* in southern British Columbia, Canada. *Fish. Res.*, 5, 55–89

Atkins, M., Campbell, A., Hajas, W.C. & Tzotzos, D. (2006). Survey of red sea urchin populations in the area of Campania Island British Columbia, 2004. Canadian Manuscript Report of Fisheries & Aquatic Sciences 2750

Zhang, Z., Campbell, A., Leus, D. & Bureau, D. (2011). Recruitment patterns and juvenile-adult associations of red sea urchins in three areas of British Columbia. *Fish. Res.*, 109, 276–284

Posterior Distribution Sampling

To obtain posterior distributions we allowed the model to burn-in for 2000 samples, and then saved every 5th sample to total 1000 samples from the posterior distribution for each of 25 chains, for a total of 25,000 replicates.

Figure A1.2 Posterior distributions (grey histograms), prior distributions (black lines), and trace plots (in colour) for parameters in the Bayesian state-space model.

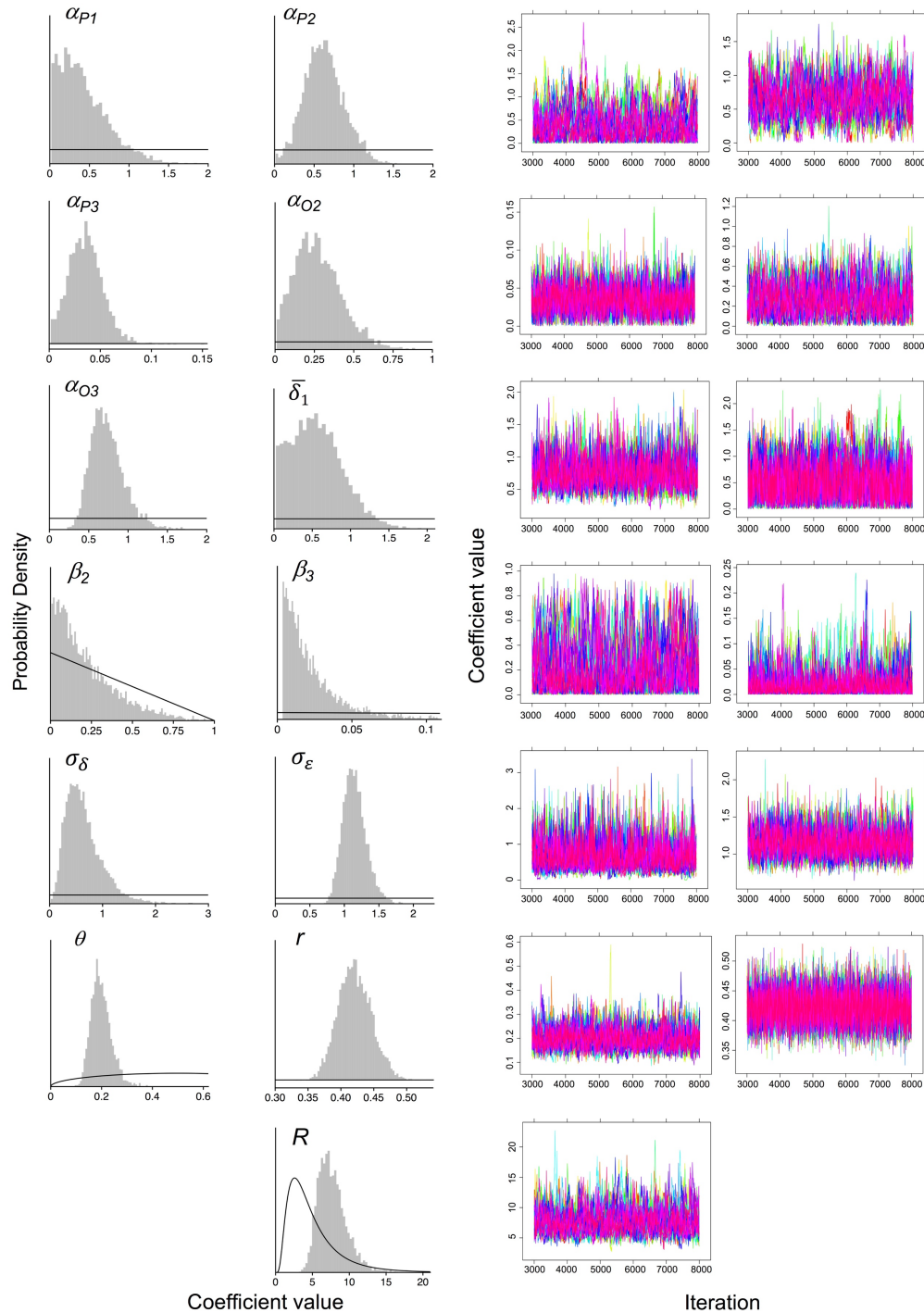
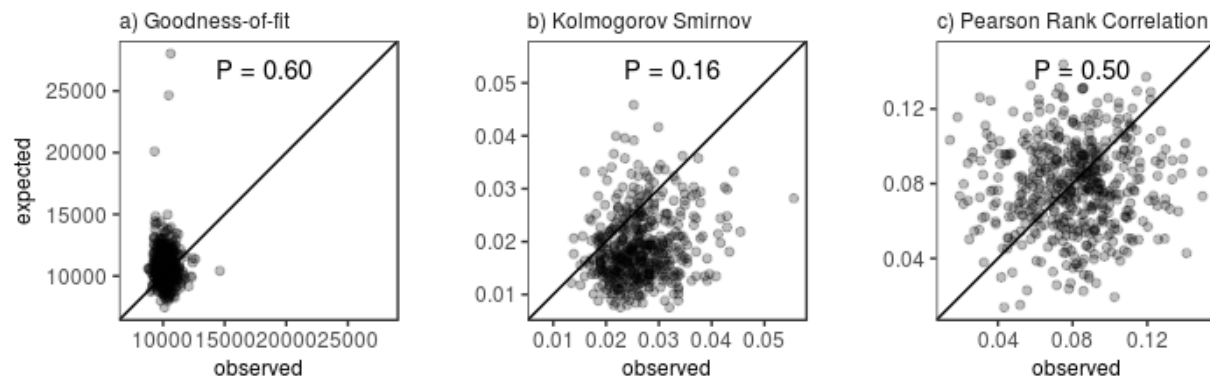


Figure A1.3 Posterior predictive checks illustrating observed vs expected test-statistics for model assumptions and fit (Gelman et al.2014)¹. X-axes show the observed values, which are a single realization from the model posterior, and y-axes show expected values, which are a random draw from the model posterior prediction. Metrics include (a) χ^2 goodness of fit-test statistics, (b) Kolmogorov-Smirnov test statistics checking adequacy of the negative binomial likelihood function, and (c) Pearson rank correlation testing for correlation between residuals and predicted values. Dramatic departures from the 1:1 line illustrate more or less extreme values than assumed under the model structure and parameterization. P-values are proportions of observe test-statistic values more extreme than expected.



¹ Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A. & Rubin, D.B. (2014). *Bayesian Data Analysis*. 3rd Edn. Taylor and Francis, Florida, pp. 143-158

Table A1.4 Model-estimated values and associated statistics for fitted parameters. The mean, standard deviation (sd), and 95% credible interval (CI) are given for each parameter (refer to Table A1.1 for full parameter descriptions). Potential scale reduction factor (psrf) of the Gelman-Rubin statistic estimates the potential decrease in the between-chains variability with respect to the within-chain variability. A $\text{psrf} < 1.1$ indicates convergence was reached (Brooks and Gelman 1997).²

Model Params.	Short form description	Mean	sd	95% CI	psrf
* $\alpha_{o,2}$	Medium urchin instantaneous mortality rate (otters)	0.53	0.30	0.067 : 1.23	1.01
* $\alpha_{o,3}$	Large urchin instantaneous mortality rate (otters)	1.46	0.54	0.75 : 2.82	1.00
* $\alpha_{p,1}$	Small urchin instantaneous mortality rate (<i>Pycnopodia</i>)	0.83	0.68	0.035 : 2.52	1.01
* $\alpha_{p,2}$	Medium urchin instantaneous mortality rate (<i>Pycnopodia</i>)	1.27	0.60	0.32 : 2.66	1.01
* $\alpha_{p,3}$	Large urchin instantaneous mortality rate (<i>Pycnopodia</i>)	0.070	0.037	0.0097 : 0.16	1.00
* δ_1	Mean (across sites) baseline mortality for small urchins	1.09	0.71	0.065 : 2.72	1.01
β_2	Adjustment of baseline mortality (medium urchins)	0.21	0.18	0.00 : 0.59	1.01
β_3	Adjustment of baseline mortality (large urchins)	0.020	0.02	0.00 : 0.064	1.00
σ_δ	Variation in baseline mortality (across sites)	0.65	0.35	0.090 : 1.36	1.01
σ_ε	Variation in baseline mortality (across years)	1.15	0.17	0.83 : 1.48	1.00
θ	Probability of observing small urchins	0.20	0.041	0.13 : 0.28	1.00
r	Dispersion parameter for negative binomial	0.42	0.026	0.37 : 0.47	1.00
R	Mean (across years) urchin recruitment	7.71	1.98	4.27 : 11.60	1.01

* These parameter estimates have undergone a bias correction to account for the fact that the mean of the lognormal process error with sd σ_ε^2 is not zero but $\exp(\sigma_\varepsilon^2/2)$ (see equations 1-3 in main text). In practice the model is not estimated with this bias correction, so it is conducted following estimation. The adjusted means, sd, and CIs were calculated after multiplying each iteration in the posterior by $\exp(\sigma_\varepsilon^2/2)$.

² Brooks, S. P., and A. Gelman. 1997. General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statistics* 7: 434–455.

Figure A1.5 Comparison of model-simulated and observed mean urchin densities to examine model performance. We ran iterated simulations using our population model over the four-year study period starting with the mean values for size-specific urchin counts in 2013 (at sites without sea otters). Using parameter estimates drawn from posterior distributions, we iterated simulations of urchin dynamics using equations (1) to (3) in the main text (but without site variation in baseline mortality or environmental stochasticity) to project the abundance of urchins in each size class through time under four scenarios varying in “predator status”: 1) Both predators absent; 2) *Pycnopodia* present but otters absent; 3) Otters present but *Pycnopodia* absent; and 4) Both predators present. We report: **a)** Estimated size-specific mean urchin densities (\pm 90% credible interval) for comparison with the, **b)** Observed mean urchin densities (\pm SE) at site/year combinations corresponding to each predator status category, and **c)** A table summarizing the mean values in panels a) and b).

