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# Single shot phase retrieval via Fourier ptychographic microscopy: supplementary material

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**This document provides supplementary information to "Single shot phase retrieval via Fourier ptychographic microscopy," https://doi.org/10.1364/OPTICA.5.000976. This supplementary document consists of three sections. In section 1, we show the process to select the optimal $\Delta k_{\mathrm{i}}$ with numerical simulation. In section 2, we explain about FFT based TIE solving method with experimental results. In section 3, we describe the phase-shifting DH experiment.**

## 1. NUMERICAL SIMULATION TO SET $\Delta K_{\mathrm{I}}$

In this section, we simulate how $\Delta k_{\mathrm{i}}$ affects the reconstruction result of the SSFPM algorithm and select optimal value. Figure S1(a) plots the mean squared error (MSE) of the phase between ground truth and reconstructed results by changing the ratio of $\Delta k_{\mathrm{i}}/\Delta k_{\mathrm{m}}$. Figure S1(b) exhibits the ground truth images and the reconstructed images for several $\Delta k_{\mathrm{i}}$. The reconstructed results near $\Delta k_{\mathrm{i}}/\Delta k_{\mathrm{m}}$ = 1 and $\Delta k_{\mathrm{i}}/\Delta k_{\mathrm{m}}$ =2 show high MSE because it does not give enough object support and constrain as described in section 2.B in primary text. In this simulation, there are three minimum points showing smaller phase error near the 4/3, 3/2, and 5/3. Although near the 3/2 point represents the minimum MSE, in this case, the DC components in Fourier domain are located at the edge of the lens array and severe optical aberration in experiment (the aberration factor is not considered in this simulation). Since the bright field involve almost energy for transparent specimen, the marginal position of DC component can degrade quality of reconstruction results. Thus, we choose $\Delta k_{\mathrm{i}}/\Delta k_{\mathrm{m}}$ = 4/3 considering both reconstruction results and experimental tolerance.

## 2. PHASE RETRIEVAL ACCORDING TO SOLVING TIE

In this section, we explain about FFT based TIE solving method and experimental results. TIE [Equation (S1)] is derived from Helmholtz equation and Fresnel diffraction theory [1]

$$-\frac{2\pi}{\lambda}\frac{\partial I(x,y)}{\partial z} = \nabla\cdot\left[I(x,y)\nabla\phi(x,y)\right], \tag{S1}$$

where $I$ and $\phi$ denotes intensity pattern and phase profile respectively. To convert Equation (S1) to Poisson equation, we set temporary function $\varphi$ which satisfies Equation (S2)

$$I(x,y)\nabla\phi(x,y) = \nabla\varphi(x,y). \tag{S2}$$

Thus, Equation (S1) can be written Equation (S3) of which we call as the first Poisson equation

$$-\frac{2\pi}{\lambda}\frac{\partial I(x,y)}{\partial z} = \nabla^2\varphi(x,y). \tag{S3}$$

By taking divergence on both sides of Equation (S2), We derive the second Poisson equation corresponding to $\phi$ as shown in Equation (S4)

$$\nabla^2\phi(x,y) = \nabla\cdot\left[I^{-1}(x,y)\nabla\varphi(x,y)\right]. \tag{S4}$$

We implement FFT to Equation (S3) to solve Poisson equations as shown in Equation (S5)

$$\mathcal{F}\left\{-\frac{2\pi}{\lambda}\frac{\partial I(x,y)}{\partial z}\right\} = -\mathbf{k}^2\mathcal{F}\left\{\varphi(x,y)\right\}, \tag{S5}$$

where $\mathcal{F}$ and $\mathbf{k}$ is two-dimensional Fourier transform operator and frequency vector respectively. Thus, the temporary function $\varphi$ can be derived as Equation (S6)

$$\varphi(x,y) = \mathcal{F}^{-1}\left\{-\frac{1}{\mathbf{k}^2}\mathcal{F}\left\{-\frac{2\pi}{\lambda}\frac{\partial I(x,y)}{\partial z}\right\}\right\}. \tag{S6}$$

Utilizing FFT method again to the second Poisson equation [Equation (S4)], we can obtain the solution of the phase.

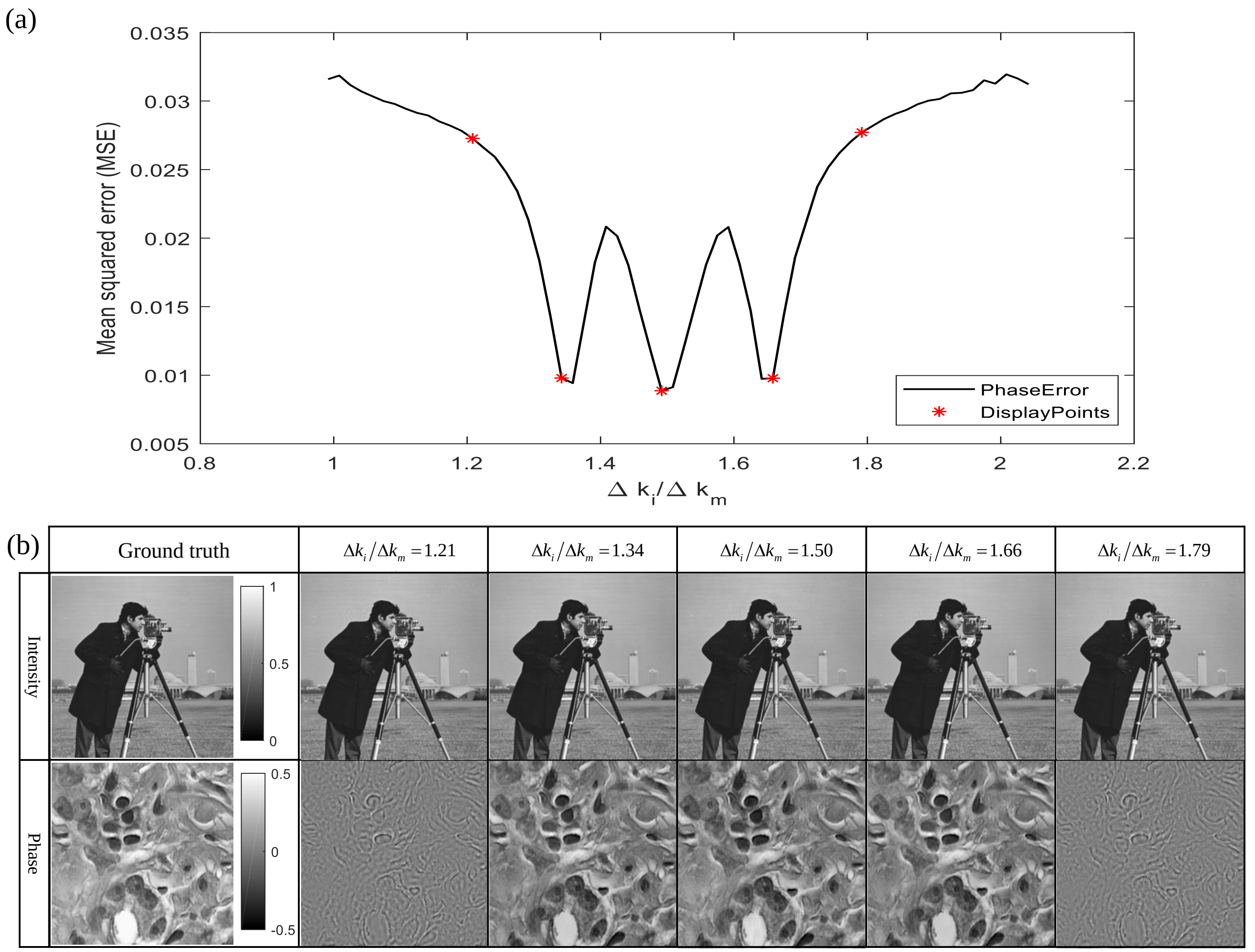


**Fig. S1.** Numerical simulation results to select $\Delta k_{\mathrm{i}}$. (a) Plot of the phase error using SSFPM method. This plot shows the MSE between the ground truth and the restored result while changing $\Delta k_{\mathrm{i}}/\Delta k_{\mathrm{m}}$ from 1 to 2. The ground truth intensity and phase images are illustrated in the leftmost column of (b). Several reconstructed results are displayed in (b).

Figure S2 indicates the experiment results for phase retrieval according to solving TIE. We use x20 objective lens ($NA_{TIE}$ of 0.5) combined with conventional microscope (Olympus BX53F). An C-mount adapter (0.63x) is used to match up the magnification ratio of the TIE system with SSFPM. Three intensity profiles are measured by changing focus plane as $+\Delta z, 0$, and $-\Delta z$ with an LED illumination (wavelength of 532 nm). The intensities are filtered in Fourier domain to fit with NA of SSFPM (cut off frequency = $\frac{2\pi}{\lambda} NA_{SSFPM}$). Figure S2(a) is three intensity images and retrieved phase profile corresponding to the phase distribution result of Fig.6.(b) in primary text. Figure S2(b) and S2(c) show the process to find the the changing distance $\Delta z$ by applying angular spectrum propagation method to $I_2$. While changing $\delta z$, a distance focused at the same depth as $I_1$ is found. Figure S2(c) plots the line trace intensity along a blue line in red box of a micro particle. Although single shot on-axis holography can not obtain accurate holograms because twin image and noise are formed together, it is possible to find the focusing depth where the sharpest profile appear for sparse particle. Since $I_1$ is focusing on the micro particle (red box), the best focusing depth ($\delta z = -8\mu$m) is determined as considering sharpness and lateral position of minimum points of plots (Figure S2(c)). Solving TIE using FFT method, phase profile is retrieved as shown in Figure S3.

## 3. PHASE RETRIEVAL ACCORDING TO PHASE-SHIFTING DH

In this section, we describe the phase-shifting DH with experiment set-up. Figure S4 shows the optical system implementing the Mach-Zehnder interferometer. The x10 magnificent system is constructed using a tube lens disassembled in the BX53F microscope and x10 objective lens ($NA_{DH}$ of 0.4). The images are filtered in Fourier domain to fit with NA of SSFPM (cut off frequency = $\frac{2\pi}{\lambda} NA_{SSFPM}$). Using the piezo mirror, we change the phase difference of two beams as $0, \pi/2, \pi$,and $3\pi/2$. The phase profile of specimen is obtained using these four interference patterns as

$$\phi(x,y) = \arctan \frac{I(x,y;3\pi/2) - I(x,y;\pi/2)}{I(x,y;0) - I(x,y;\pi)} \qquad \text{(S7)}$$

[2]. After that, we take one more image by blocking the refer-

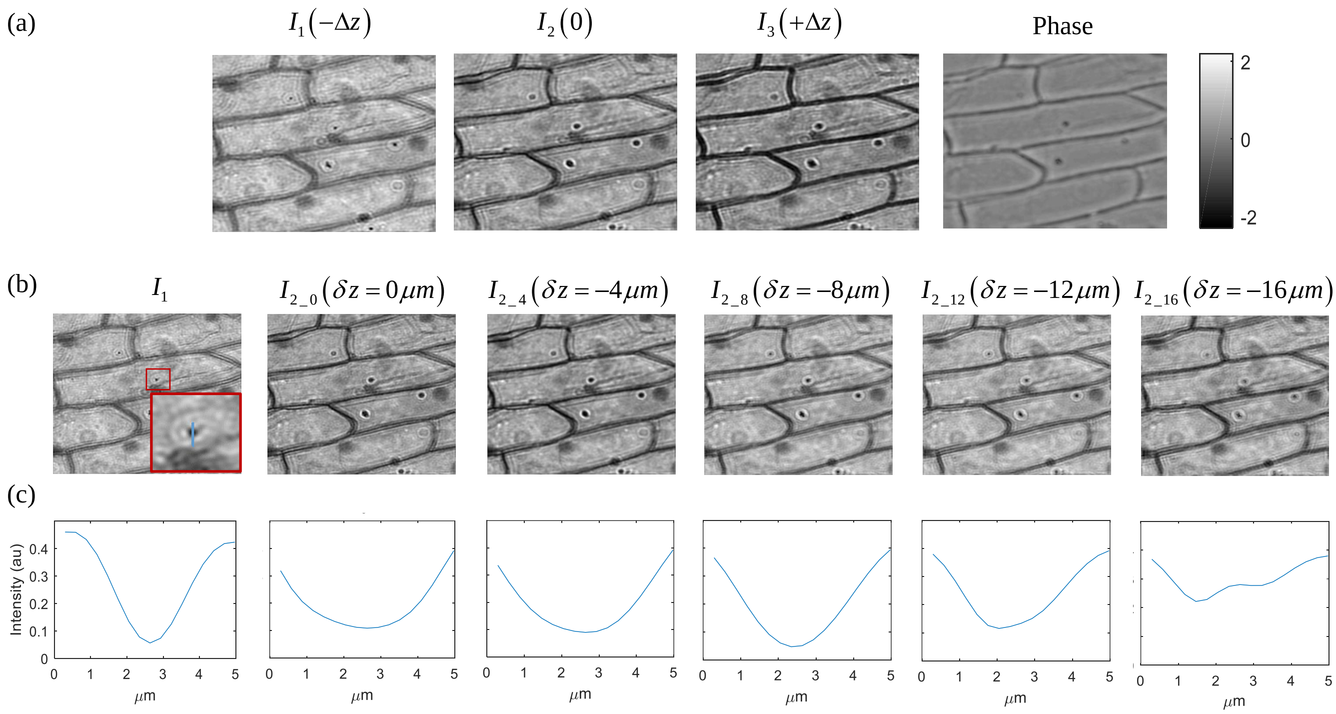


**Fig. S2.** Bio-sample imaging using TIE. (a) Measured intensity images ($I_1$, $I_2$, and $I_3$) and retrieved phase profile. (b) The leftmost image is $I_1$ where micro particle (red box) is focused, for comparison. The others are numerical propagated image of $I_2$ to find $\Delta z$. Propagating distance using angular spectrum method is $\delta z$. (c) Plots of intensity along the blue line in red box. $\Delta z$ is determined to $8\mu$m as considering sharpness and lateral position of the minimum point.

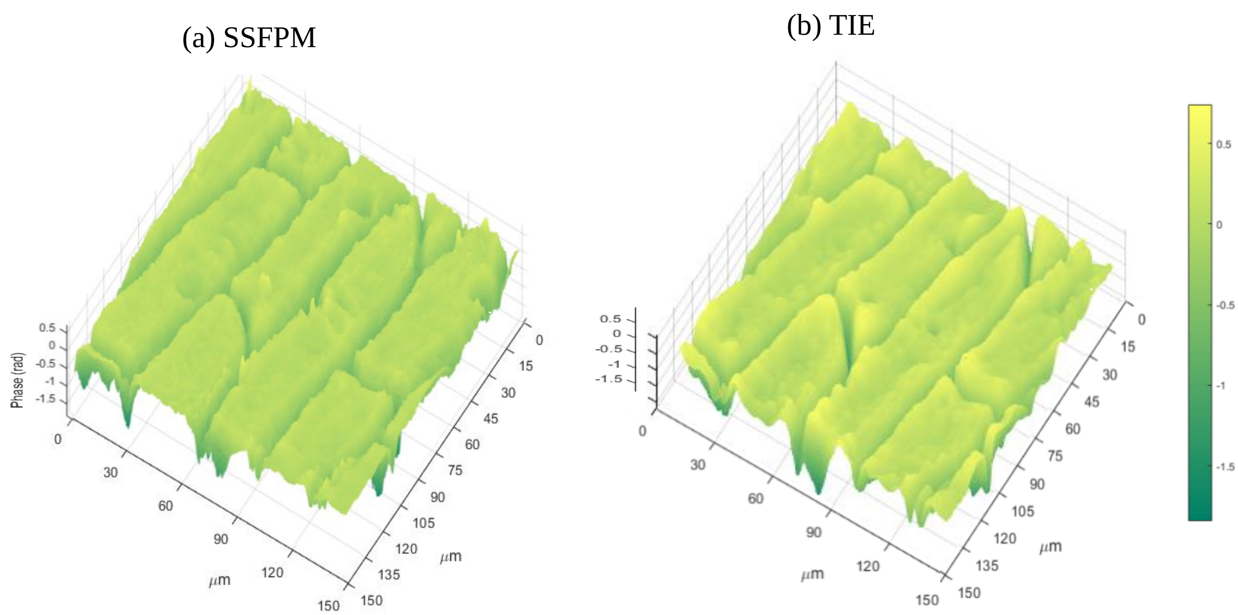


**Fig. S3.** Phase distribution of *allium cepa* epidermal cells using recovered phase profile by (a) SSFPM (b) TIE method.

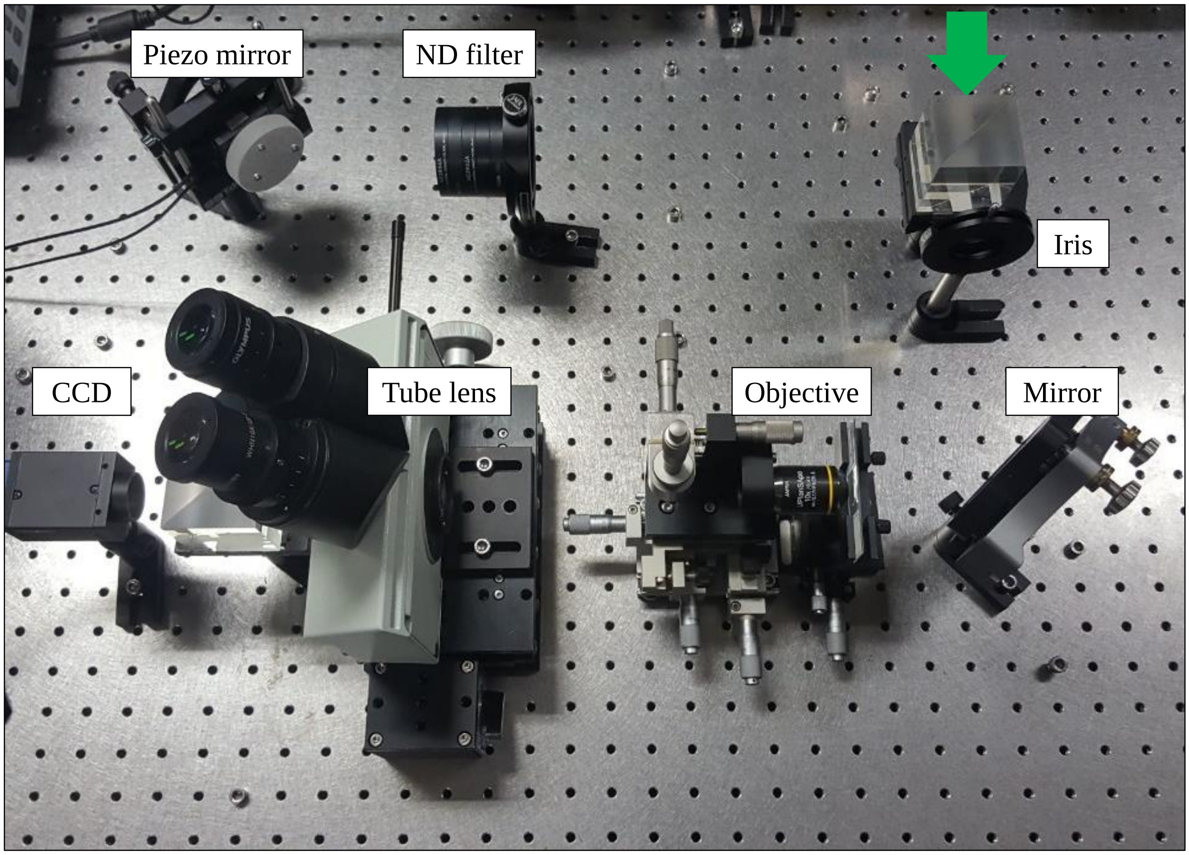


**Fig. S4.** Photograph of experimental setup for phase-shifting DH.

ence beam to acquire the amplitude information of the signal beam. Focused phase distribution is acquired by applying angular spectrum method to the retrieved complex wavefront.

## REFERENCES

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