Supplementary material

Proof of Result 1: The two budgets constraints, (8) and (9), are obviously binding. Hence, $c_p = B_0 - x$ and $c_h = F(x,\xi)$. A necessary condition for an interior solution is then

$$\left(B_0 - x^*\right)^{-r} = \frac{1}{1+\delta} E\left(F_x\left(x^*,\xi\right)\left(F\left(x^*,\xi\right)\right)^{-r}\right). \qquad (11)$$

The right hand side decreases with respect to x^* and δ whereas the left hand side increases with x^* and does not depend on δ. We thus conclude that x^* always decreases with δ.□

Proof of Result 2: The first order condition (11) characterizes x^* as a function of r. Differentiating condition (11) with respect to r leads to (we do not write the arguments):

$$r\left(B_0 - x^*\right)^{-r-1}\frac{\partial x^*}{\partial r} - \left(B_0 - x^*\right)^{-r} ln\left(B_0 - x^*\right)$$

$$= \frac{1}{1+\delta} E\left(F_{xx}F^{-r} - r\left(F_x\right)^2 F^{-r-1}\right)\frac{\partial x^*}{\partial r} - \frac{1}{1+\delta} E\left(F_x F^{-r} ln(F)\right).$$

Rearranging and using condition (11), we obtain:

$$\left(r\left(1+\delta\right)\left(B_0 - x^*\right)^{-r-1} + E\left(-F_{xx}F^{-r} + r\left(F_x\right)^2 F^{-r-1}\right)\right)\frac{\partial x^*}{\partial r} = E\left(F_x F^{-r} ln\left(\frac{B_0 - x^*}{F}\right)\right).$$

The first term on the left hand side is positive and the right hand side decreases with x. Thanks to Result 1, this is sufficient to finish the proof.