

# A simple model of a balanced boundary layer coupled to a large-scale convective circulation

Bob Beare <sup>1</sup> and Mike Cullen <sup>2</sup>

<sup>1</sup>University of Exeter.

<sup>2</sup>Met Office, Exeter.

PDC18, July 2018

# Introduction

- ▶ Understanding the coupling of the boundary layer and convection to the large scale.

# Introduction

- ▶ Understanding the coupling of the boundary layer and convection to the large scale.
- ▶ Parametrization development often proceeds 'bottom up' from process modelling at the small scale, then testing in a large-scale model.

# Introduction

- ▶ Understanding the coupling of the boundary layer and convection to the large scale.
- ▶ Parametrization development often proceeds 'bottom up' from process modelling at the small scale, then testing in a large-scale model.
- ▶ Reverse approach- start with large-scale balances and how do the physics preserve these?

# Introduction

- ▶ Understanding the coupling of the boundary layer and convection to the large scale.
- ▶ Parametrization development often proceeds 'bottom up' from process modelling at the small scale, then testing in a large-scale model.
- ▶ Reverse approach- start with large-scale balances and how do the physics preserve these?
- ▶ Tropical troposphere follows Weak Temperature Gradient (WTG) approximation.

# Introduction

- ▶ Understanding the coupling of the boundary layer and convection to the large scale.
- ▶ Parametrization development often proceeds 'bottom up' from process modelling at the small scale, then testing in a large-scale model.
- ▶ Reverse approach- start with large-scale balances and how do the physics preserve these?
- ▶ Tropical troposphere follows Weak Temperature Gradient (WTG) approximation.
- ▶ Significant gradients of temperature within boundary layer (connected to sea-surface temperature). Associated gradients of pressure in balance with the drag.

# Climatology (Back and Bretherton 09)

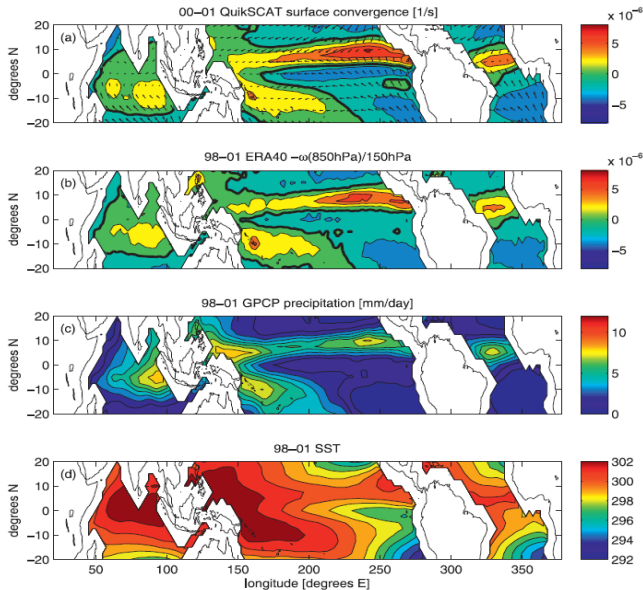
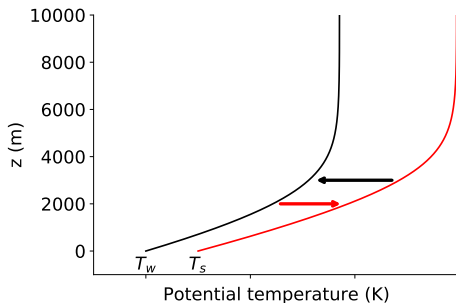


FIG. 1. (a) 2000-01 surface convergence from QuikSCAT with contours of  $2 \times 10^{-6} s^{-1}$  (heavy contour shows zero convergence). (b) 1998-2001 ERA-40  $-\omega_{850}/(150 hPa) s^{-1}$ , representative of mean convergence in the boundary layer, with the same contours as (a). The GPCP 1998-2001 (c) precipitation (contours of  $1 mm day^{-1}$ ) and (d) SST (contours of 1 K).

# Weak temperature gradient profile

## Maintenance of WTG profile

- ▶ Convection tries to relax to moist adiabat from the SST ( $T_s$ ).
- ▶ Equal and opposite relaxation back to WTG ( $T_w$ ).



## Components of simple model

1. Maintenance of WTG vertical profile
2. WTG mass balance
3. Boundary-layer balance
4. Moisture balance



## Convection layer

$$M_c = \gamma_c \frac{T_s - T_w}{\tau_c} \quad |x| \leq L_c/2,$$
$$\frac{P}{L\rho_0 H} = \gamma_q \frac{q_s - q_w}{\tau_c} \quad |x| \leq L_c/2,$$

where  $M_c$  is the mass flux divided by density,  $P$  the precipitation flux,  $\tau_c$  the relaxation timescale.

Assume WTG and a constant radiatively-driven subsidence velocity ( $w_s$ ).

Mass balance in the Convection layer is

$$L_x w_s + L_c \langle M_c \rangle = 0$$

where  $L_x$  domain length and  $L_c$  width of convection. Angle brackets are horizontal average over the convecting region.

## Schematic of model

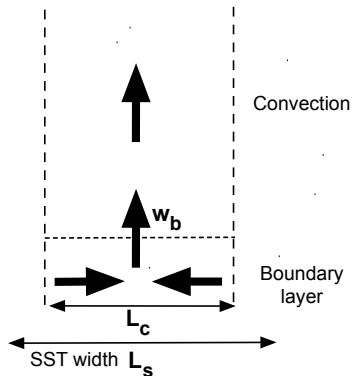
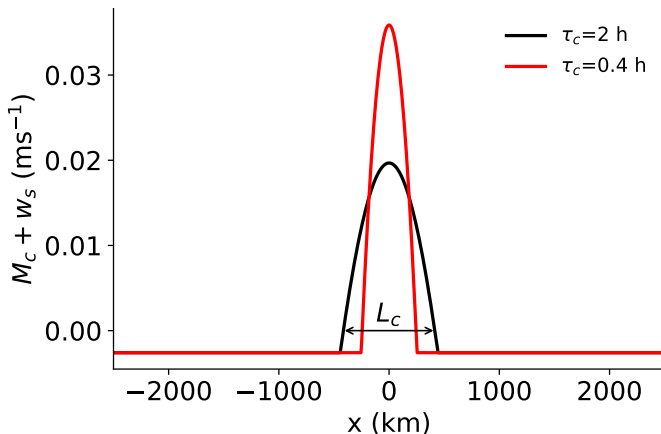


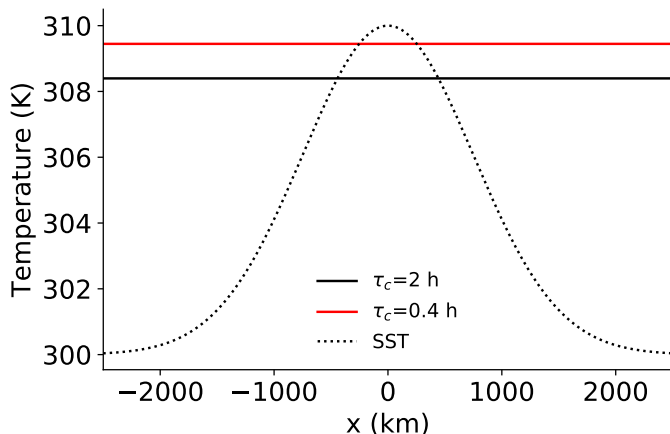
Figure: Schematics of the flows and length scales in the simple model

## Mass flux



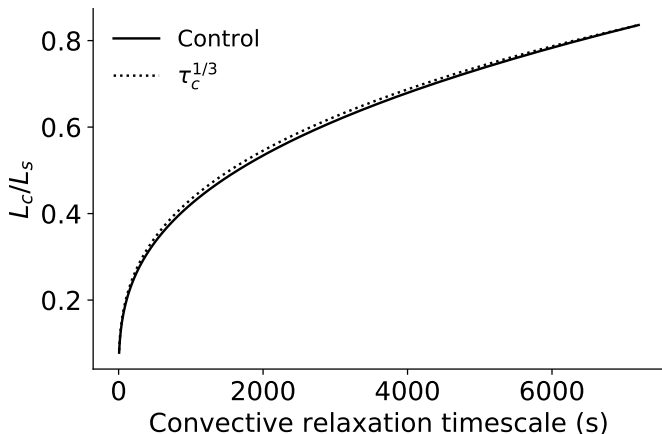
**Figure:** The sum of subsidence and mass flux for the WTG layer. Shown are profiles for the control ( $\tau_c = 2$  h, black) and  $\tau_c = 0.4$  h (red). The convective width for  $\tau_c = 2$  h is marked by the horizontal arrow.

## SST and WTG temperatures



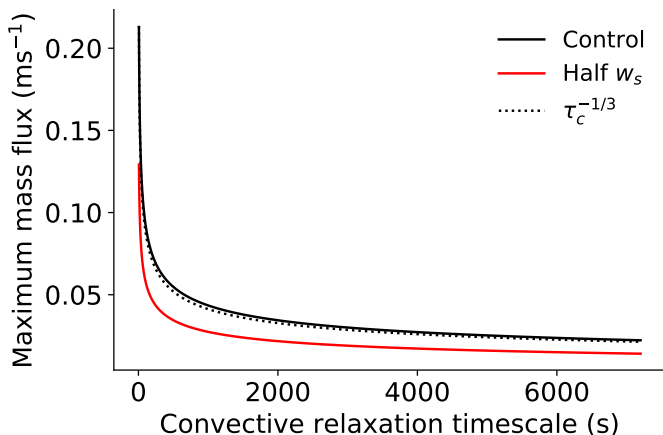
**Figure:** The SST and WTG temperatures for the control ( $\tau_c = 2$  h, black horizontal line) and  $\tau_c = 0.4$  h (red horizontal line).

## Contraction of convection width



**Figure:** The convective width (normalised by width of SST) plotted against convective relaxation timescale. Also shown is a  $\tau_c^{1/3}$  power law (dotted).

## Maximum mass flux



**Figure:** Maximum mass flux plotted against convective relaxation timescale for the control (black) and half subsidence (red) cases. Also shown is a  $\tau_c^{-1/3}$  power law (dotted).

# Coupling to boundary layer

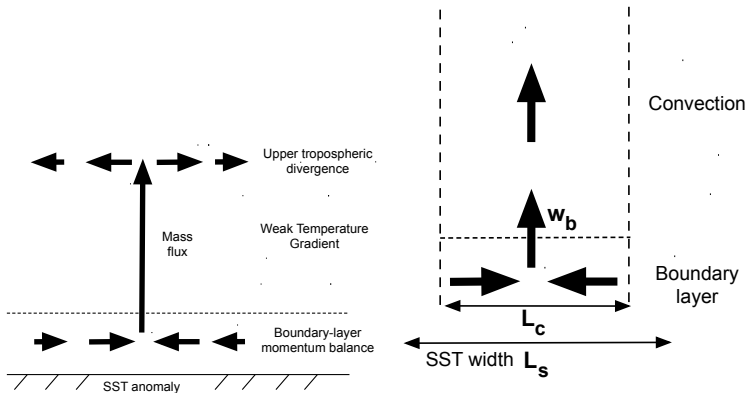


Figure: Schematics of the flows and balances in the simple model

## Coupling to boundary layer

In contrast to the WTG convection layer, thermal gradients are significant within the boundary layer, so we need a momentum balance

$$\overbrace{\frac{d\phi_b}{dx}}^{\text{Pressure gradient}} = - \overbrace{\frac{u_b}{\tau_b}}^{\text{Drag}},$$

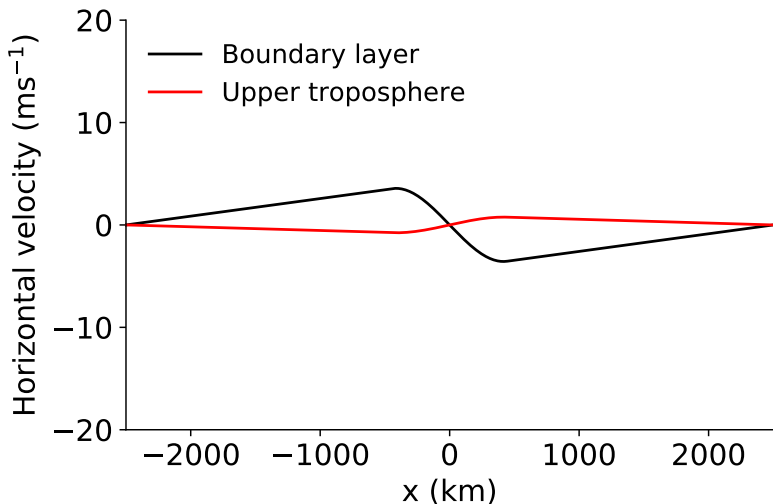
where  $u_b$  is boundary-layer wind,  $\phi_b$  geopotential,  $\tau_b$  the Rayleigh boundary-layer timescale. Boundary-layer top vertical velocity ( $w_b$ ) is calculated using continuity and hydrostatic balance is given by

$$w_b = -\frac{du_b}{dx}h, \quad \phi_b = -\frac{h}{2} \frac{g(\theta_b - \theta_0)}{\theta_0}$$

where  $h$  is the boundary-layer depth. The boundary layer potential temperature matches the ascent in the convection region.

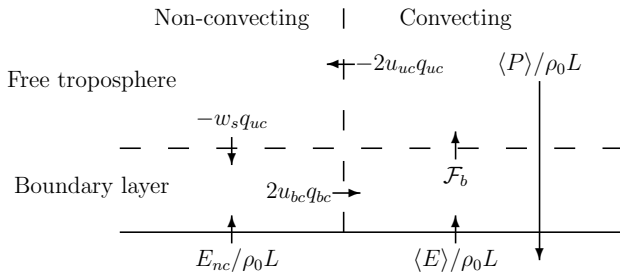
$$-\frac{\tau_b g h^2}{2\theta_0} \frac{d^2\theta_b}{dx^2} = w_b = M_c + w_s.$$





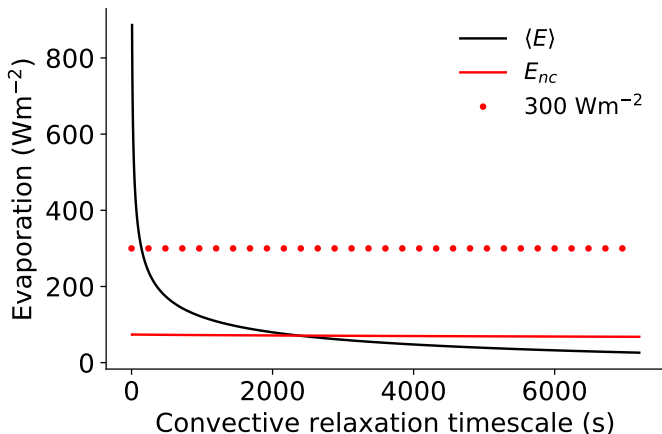
**Figure:** Horizontal winds for : boundary layer ( $u_b$ , black) and upper troposphere ( $u_u$ , red) for control case.

# Moisture fluxes

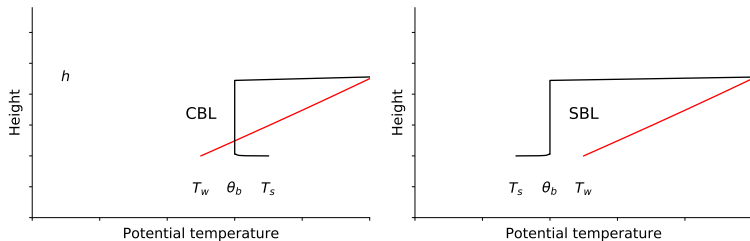


**Figure:** The moisture fluxes assumed between convecting (right), non-convecting (left), boundary layer and free troposphere regions. All fluxes shown are positive.

# Evaporation

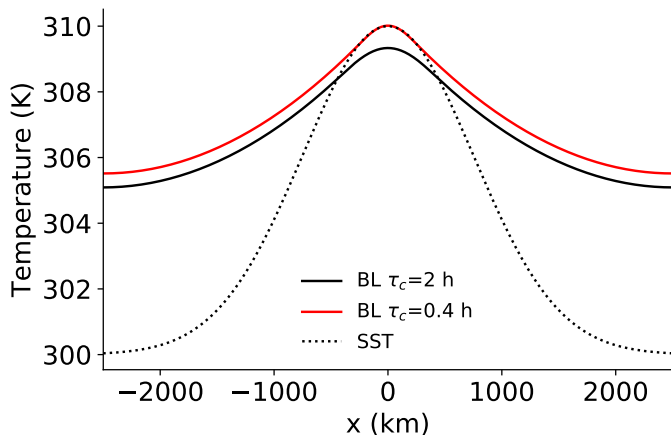


**Figure:** The evaporation averaged over convecting region ( $\langle E \rangle$ ) and from the non-convecting region ( $E_{nc}$ ) plotted against convective relaxation timescale. The red dotted line is the  $300 \text{ Wm}^{-2}$  threshold.



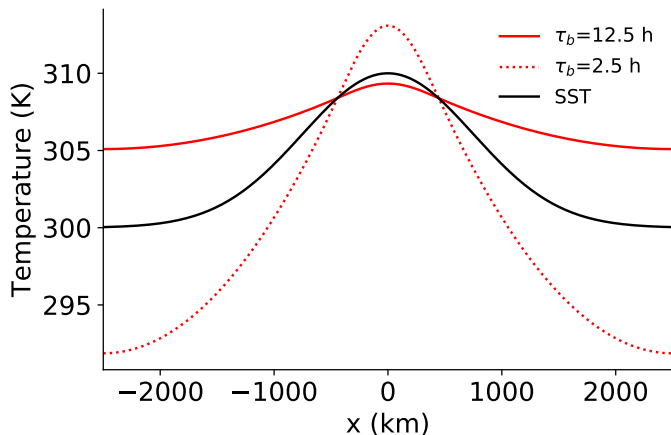
Vertical profiles of potential temperature for the boundary layer (black) with respect to the WTG moist adiabat based on  $T_w$  (red) for the: (left) convective boundary layer or (right) stable boundary layer.

## Boundary-layer potential temperature



**Figure:** The distribution of SST (dotted) and boundary-layer potential temperature for  $\tau_c = 2$  h (black) and  $\tau_c = 0.4$  h (red).

## Sensitivity to drag



**Figure:** The sensitivity of the boundary-layer potential temperature to decreasing  $\tau_b$  (increasing drag) from 12.5 h (red) to 2.5 h (red dotted).

## Summary

- ▶ Simple model coupling boundary layer, deep convection and large scale.

## Summary

- ▶ Simple model coupling boundary layer, deep convection and large scale.
- ▶ Model predicts a  $\tau_c^{1/3}$  variation of convective width.



# Summary

- ▶ Simple model coupling boundary layer, deep convection and large scale.
- ▶ Model predicts a  $\tau_c^{1/3}$  variation of convective width.
- ▶ Boundary layer constrains the flow:
  - ▶ The evaporation in the non-convecting region sets the horizontal advection of moisture.

# Summary

- ▶ Simple model coupling boundary layer, deep convection and large scale.
- ▶ Model predicts a  $\tau_c^{1/3}$  variation of convective width.
- ▶ Boundary layer constrains the flow:
  - ▶ The evaporation in the non-convecting region sets the horizontal advection of moisture.
  - ▶ Maintaining a convective boundary layer- lower limit on relaxation timescale, upper limit on drag.

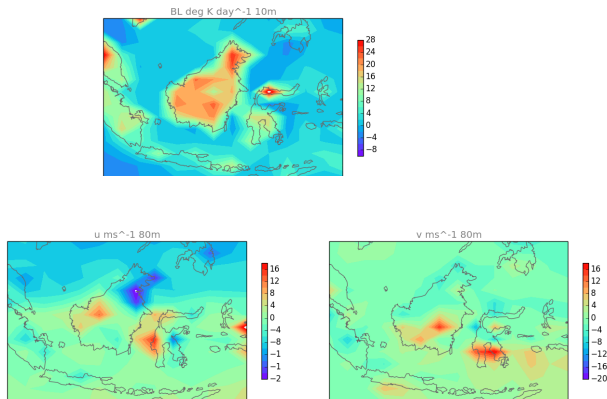
# Summary

- ▶ Simple model coupling boundary layer, deep convection and large scale.
- ▶ Model predicts a  $\tau_c^{1/3}$  variation of convective width.
- ▶ Boundary layer constrains the flow:
  - ▶ The evaporation in the non-convecting region sets the horizontal advection of moisture.
  - ▶ Maintaining a convective boundary layer- lower limit on relaxation timescale, upper limit on drag.
- ▶ Motivates tests of weather and climate models:
  - ▶ How well does convection scheme maintain WTG?
  - ▶ Does the convective width decrease with increased efficiency of convection scheme?
  - ▶ How close to Ekman balance is tropical convergence on weekly-monthly timescales?
  - ▶ See Susannah Hearn's poster - defining balanced regimes in MetUM.

# Summary

- ▶ Simple model coupling boundary layer, deep convection and large scale.
- ▶ Model predicts a  $\tau_c^{1/3}$  variation of convective width.
- ▶ Boundary layer constrains the flow:
  - ▶ The evaporation in the non-convecting region sets the horizontal advection of moisture.
  - ▶ Maintaining a convective boundary layer- lower limit on relaxation timescale, upper limit on drag.
- ▶ Motivates tests of weather and climate models:
  - ▶ How well does convection scheme maintain WTG?
  - ▶ Does the convective width decrease with increased efficiency of convection scheme?
  - ▶ How close to Ekman balance is tropical convergence on weekly-monthly timescales?
  - ▶ See Susannah Hearn's poster - defining balanced regimes in MetUM.
- ▶ Beare and Cullen (2018), *submitted to JAS*.
- ▶ See also Beare and Cullen (2013 PhilTransA, 2016 QJRMS) for balanced diagnostics applied to mid-latitudes.

# Cullen 2018, in preparation



**Figure 4.** Diagnostics calculated over a region 10°S to 10°N and 100°E to 130°E at 80m height above the surface. (a) Boundary layer heating, units  $^{\circ}\text{K day}^{-1}$ : (b) total zonal wind calculated from (19), (c) total meridional wind calculated from (19).

Diagnosed convergence from MetUM over Borneo, based on Ekman balance (semi-geotriptic theory).