Coupled interpolation of 3-component GPS velocities

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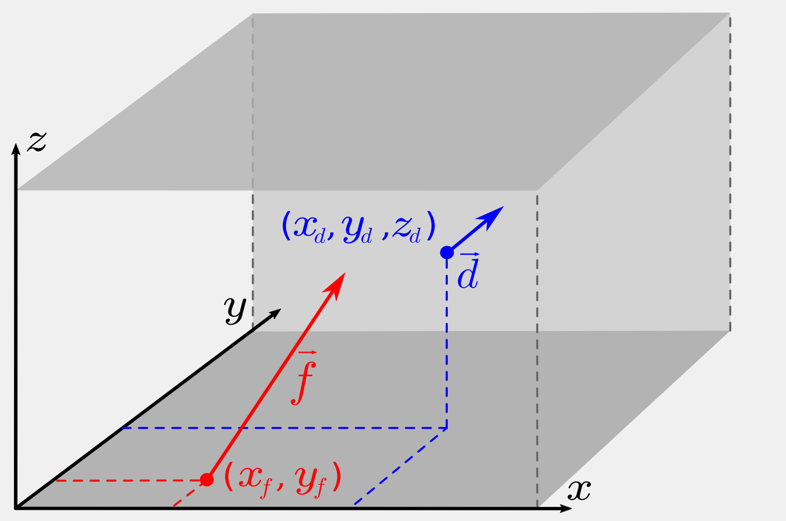
Introduction

GPS/GNSS data have high accuracy and temporal resolution but are spatially sparse. These data often need to be interpolated on regular grids (e.g., to be used as constraints during InSAR processing). Interpolation is done separately for each component of the velocity vector. Sandwell and Wessel (2016) proposed Green's functions based joint interpolation of the horizontal components, which are coupled through elasticity. We propose an extension of this method to include the vertical component, enabling the inclusion of vector data projected in arbitrary directions, such as InSAR line-of-sight velocities.

Interpolation



Elastic half-space with a force acting on it's surface and the corresponding displacement vector.

The Green's functions we adopted for interpolation are a solution to Cerruti's problem for an elastic half-space (Okumura, 1995). Given the x, y, z components of M **forces**, we can calculate the x, y, z components of N **displacements** at arbitrary locations. We introduce a new "coupling parameter" α to control the degree of coupling between the vertical and horizontal components.

$$\begin{bmatrix} G_{xx} & G_{xy} & \alpha\, G_{xz} \\ G_{yx} & G_{yy} & \alpha\, G_{yz} \\ \alpha\, G_{zx} & \alpha\, G_{zy} & G_{zz} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

Assuming that our measured vectors are the displacements, we can place a force beneath each data point, formulate this as a matrix equation, and solve for the forces that best fit our data in a least-squares sense. We use damping regularization to stabilize the solution and avoid over-fitting. Interpolation is done by forward modeling the displacements using our best-fit forces.

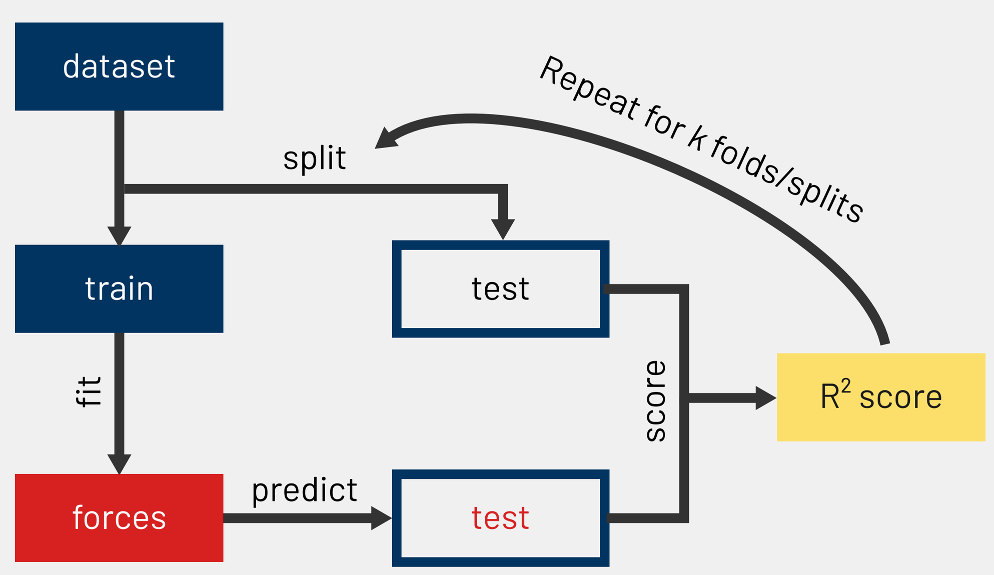
$$\bar{\bar{G}}\bar{f} = \bar{d} \longrightarrow \hat{\bar{f}} = \left(\bar{\bar{G}}^T\bar{\bar{W}}\bar{\bar{G}} + \mu\bar{\bar{I}}\right)^{-1}\bar{\bar{G}}^T\bar{\bar{W}}\bar{d}$$

The parameters that control the behaviour of the interpolator are:

- The regularization parameter μ
- The Poisson's ratio of the elastic medium ν
- The coupling parameter α
- The depth to the forces z_d

Model selection

We employed cross-validation techniques to automatically select optimal values for controlling parameters of the interpolation. Cross-validation works by splitting the dataset into two parts: one for fitting the interpolator (the training set) and one for evaluating the interpolator's performance (the testing set). In particular, we used **k-fold cross-validation**, in which the dataset is randomly shuffled and split *k* times for a more robust measure of performance.



Repeat for all combinations of the controlling parameter values.
The best configuration is the one with the highest R^2 score.

Synthetic

We tested the performance of the 3-component interpolator on data produced by an elastic/viscoelastic model of the San Andreas fault system (Sandwell and Smith-Konter, 2018). We extracted samples for interpolation from the model-produced regular grid of 3-component velocities at the locations of real world stations from the Plate Boundary Observatory (PBO). For comparison, we also interpolated the 3 components separately using a biharmonic spline. Optimal values for the controlling parameters of the 3-component interpolator and the biharmonic spline were determined automatically by systematically testing different combinations and validating against the original model grid.

The figures below show the original model grid (A), extracted samples for interpolation (B), and the differences between the interpolation results and the original grid (C-F). The 3-component interpolator that best recovers the original grid has coupling parameter 0 (uncoupled vertical, C). This configuration outperforms the biharmonic spline (D) and a 3-component interpolation with coupling parameter of 1 (coupled vertical, E).

The depth of the forces is a key parameter that controls the smoothness of the solution. Figure F illustrates the results obtained by placing all forces at a constant depth. The interpolator cannot fit the smooth and sharp portions of the data simultaneously. To overcome this obstacle, we place the forces at depths proportional to the distance to the nearest neighboring force.

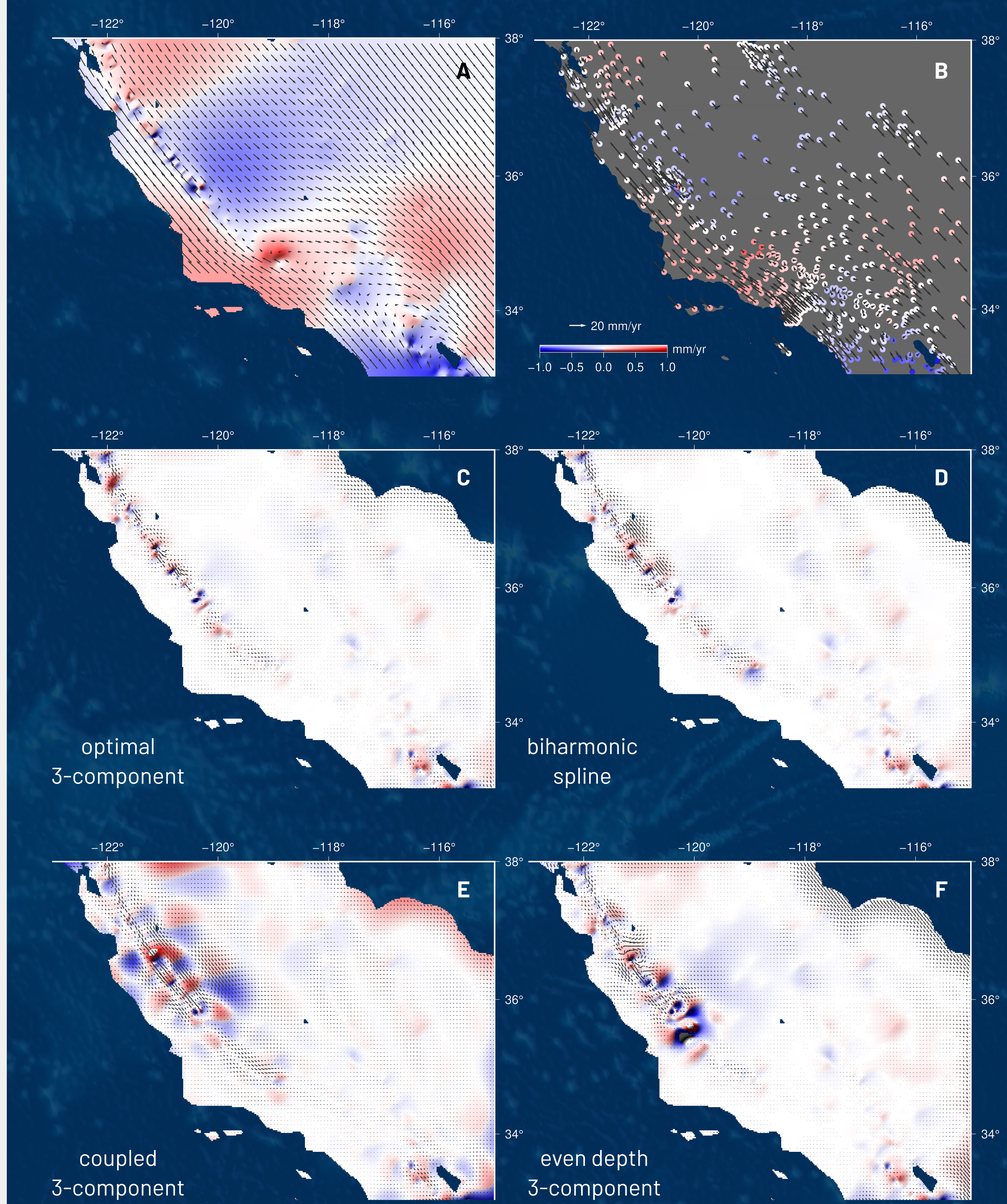
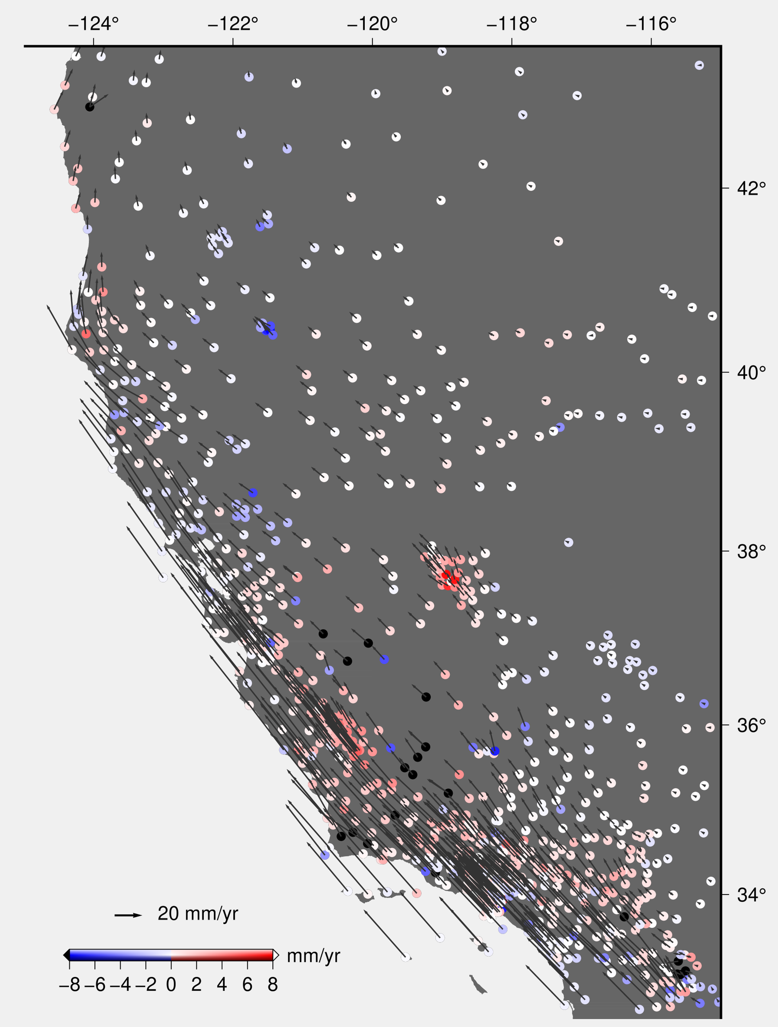
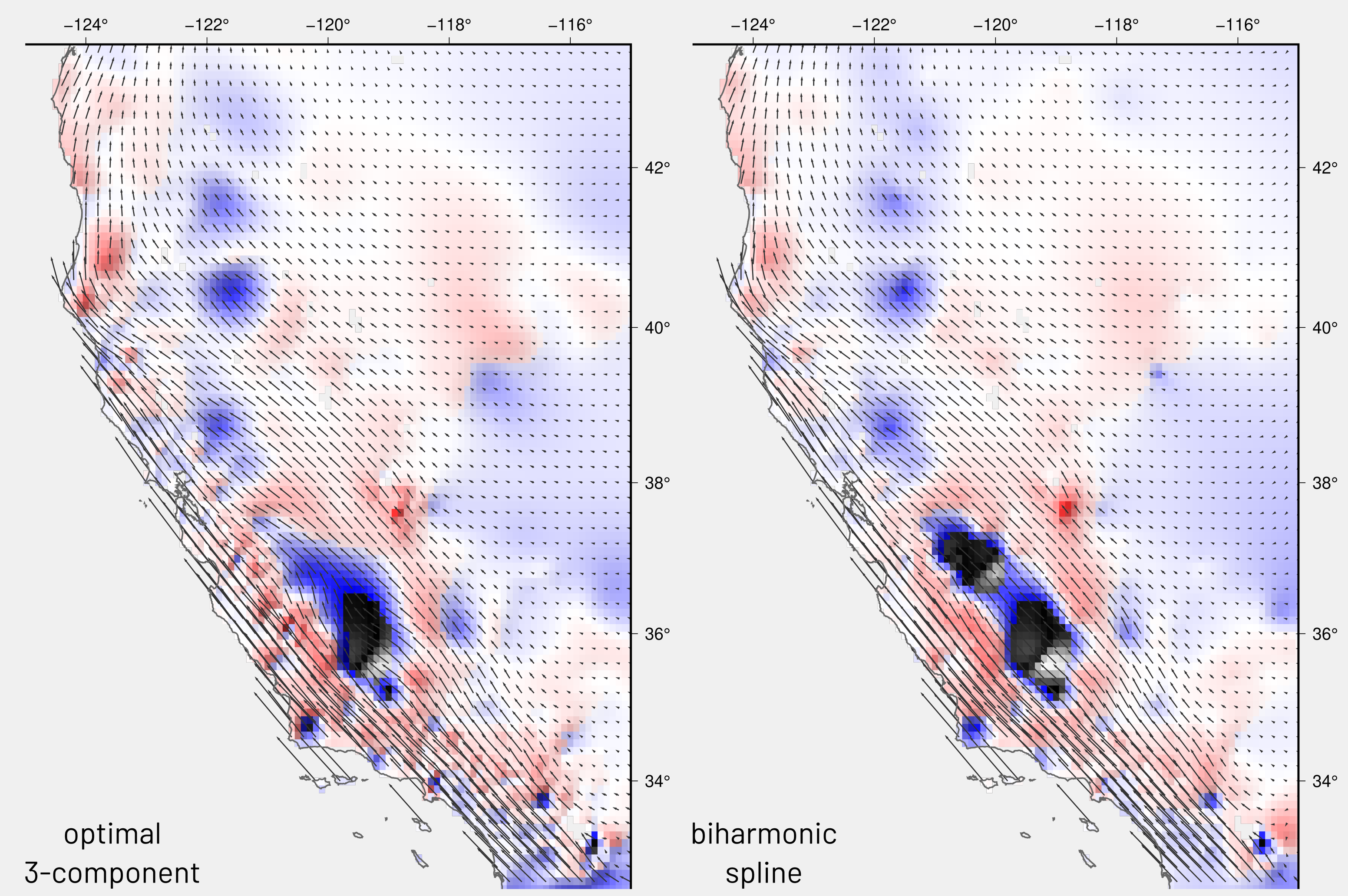


Plate Boundary Observatory data

We applied our interpolator to data from the EarthScope Plate Boundary Observatory (PBO) for the US West coast (right). The dataset includes uncertainty estimates which we used as weights in the interpolation. The vertical signal in the California Central Valley is dominated by strong subsidence associated with groundwater extraction.



k-fold cross-validation results indicate an optimal coupling parameter of 1 (coupled vertical), Poisson's ratio of 0.5, and minimum depth of the forces 500 meters (bottom left). This configuration resulted in an R^2 score of 0.94 (the best score possible is 1). The 3 independent biharmonic splines also achieved a score of 0.94, indicating that both methods are equally good at predicting data which was not included in the fitting. Vertical velocities are compatible with Hammond et al. (2016), though the bimodal subsidence pattern in the Central Valley is not as pronounced. This difference is likely due to the lack of observations in that region.



Conclusions

- The location of the forces is critical.
- Coupling through elasticity doesn't always work.
- Uncoupled results on synthetic are better than biharmonic spline.
- Coupled and biharmonic spline results on PBO data are comparable.
- Cross-validation allows automatic optimal configuration.
- Computational load is high but can be overcome.

References

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All figures were generated using the Generic Mapping Tools.
The Python implementation of this method is based on the Verde library (fatiando.org/verde).
The poster, data, and code can be found at github.com/leouieda/agu2018

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