The forgotten landscape

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Contrary to common lore, the standard model of particle physics has a "landscape" of physically inequivalent vacua, most of them quite different from ours. I discuss some cosmological consequences and related observational constraints, show how non-perturbative electroweak dynamics selects our special vacuum, and put an old idea to rest.

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I. INTRODUCTION

The standard model (SM) of particle physics is almost disturbingly successful. Textbooks written decades ago, like [1] and [2], remain perfectly good introductions to it. You may want to scribble a note or two in their margins, like "top quark found 1995, mass 173 GeV" and "Higgs found 2012, mass 125 GeV", but other than that, they could have been published last week. As I write this, all convincing signs of physics beyond the SM are related to the one interaction which it never included; gravity.

The SM is a quantum field theory (QFT). Its basic constituents are fields; particles are their excitations. That physics should be described by such a theory may seem like a huge assumption, but if you buy Weinberg's "folk theorem" [3], it boils down to accepting quantum mechanics, Lorentz invariance and locality. All three can of course be questioned, but operationally, there is little room left for doing so within the experimentally explored range of parameter space. In other words, if we live in a simulation running on a classical computer, it is doing an excellent job of faking a universe where physics is local, Lorentz invariant and quantum mechanical.

Another central theme of the SM is gauge invariance. In classical terms, a gauge invariant theory has more degrees of freedom (DoF) than independent equations of motion (EoM). To compute the time evolution of a field from a known set of initial conditions, you must therefore supplement the EoM with extra equations ("fix the gauge"). Gauge invariance is what prevents the result from being completely arbitrary: each set of extra equations which is consistent with the EoM is related to all other such sets by well defined, continuous transformations which leave physical quantities unaffected. Together, those transformations define the gauge group of the theory.

The prototypical gauge invariant theory is classical electromagnetism. In terms of the 4-vector potential A_μ, the electric and magnetic fields are $E_i = \partial_i A_0 - \partial_0 A_i$ and $B_i = \varepsilon_{ijk}\partial_j A_k$, while Maxwell's equations in empty space are $\partial_\mu(\partial^\nu A^\mu - \partial^\mu A^\nu) = 0$. Adding an arbitrary 4-divergence $\partial^\mu f$ to A_μ has no effect on any of them. You are therefore free to reduce the four DoF of A_μ to the two independent polarization states of light by supplementing the EoM with an equation like the Lorenz gauge condition $\partial_\mu A^\mu = 0$.

Classically, the advantage of this formulation may seem debatable (Heaviside apparently wanted A_μ purged from physics [4]). But already non-relativistic QM requires electromagnetic interactions to be written in terms of A_μ rather than of E_i and B_i (a point famously driven home by the Aharonov-Bohm effect), and when you graduate to relativistic QM, Lorentz invariance is violated unless massless vector fields couple to conserved quantities only. By Noether's theorem, every continuous symmetry implies the existence of a conserved quantity, so imposing invariance of the theory under combined gauge transformations of A_μ and of its interaction partners makes Lorentz invariance almost automatic. Gauge invariance as a guiding principle is therefore supported by at least two pillars of Weinberg's "folk theorem".

Yet, there are problems. Since A_μ is not gauge invariant, it does not satisfy our definition of a physical quantity, nor does any field which interacts with it. What, then, is a photon or an electron? This question is eerily reminiscent of those often asked about QM. Could quantum indeterminacy and the incompleteness of gauge invariant theories be two sides of the same coin?

From a more practical point of view, one may question the classical habit of fixing a gauge when doing QFT. Why not just compute expectation values by averaging over all possible field configurations, including ones related by gauge transformations? The implementation of this idea is lattice gauge theory. For QCD it works quite well. But when we turn to electroweak interactions, we run into a complication: the Higgs mechanism.

II. IF IT'S BROKEN, YOU MUST FIX IT

The big difference between QCD and electroweak interactions is that the gauge symmetry of the latter is spontaneously broken. In a way, a gauge theory featuring spontaneous symmetry breaking (SSB) is even more symmetric than the plain vanilla variety. Instead of just one lowest energy state, it has a continuum of them, all related by the gauge symmetry. This continuum of degenerate states is called the vacuum manifold, with the understanding that "vacuum" is not synonymous with "empty"; you tell the vacua apart by the expectation value(s) of some Higgs field(s), so those can not all vanish. Unless you want to break Lorentz invariance, they must be scalars.

In electroweak theory, the gauge symmetry is SU(2)×U(1). The Higgs field is the complex doublet

$$\Phi = \begin{bmatrix} \varphi^+ \\ \varphi^0 \end{bmatrix} = \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_0 - i\phi_3 \end{bmatrix} \tag{1}$$

(the phase factors in front of the real components $\phi_0 \ldots \phi_3$ are chosen to match Eq. (3)). Its vacuum manifold is the 3-sphere ($\mathbb{S}^3$) defined by

$$\Phi^\dagger \Phi = (\phi_0)^2 + (\phi_1)^2 + (\phi_2)^2 + (\phi_3)^2 = \nu^2/2 \tag{2}$$

where the constant $\nu \simeq 246.3$ GeV sets the symmetry breaking scale. When Φ satisfies Eq. (2), it can be parameterized using three angles, $\vec{\theta} = [\theta_1, \theta_2, \theta_3] \in [-2\pi, 2\pi]$:

$$\left[\phi_0, \vec{\phi}\right] \;=\; \frac{\nu}{\sqrt{2}} \left[\cos\left(\frac{\theta}{2}\right), \frac{\vec{\theta}}{\theta} \sin\left(\frac{\theta}{2}\right)\right] \tag{3}$$

This can be viewed as a quaternion encoding a 3D rotation by $\theta = |\vec{\theta}|$ about the axis $\vec{\theta}/\theta$.

Picking a $\langle\Phi\rangle$ somewhere on the vacuum manifold, or equivalently a $\vec{\theta}$, breaks the rotational symmetry of $\mathbb{S}^3$ in the 4-dimensional Euclidean space ($\mathbb{R}^4$) spanned by $\phi_0 \ldots \phi_3$. It also breaks the gauge symmetry, since the Higgs field carries SU(2) and U(1) charges.

Imagine trying to do QCD-style lattice gauge theory with Φ. For every point on $\mathbb{S}^3$ there is an antipodal point where $\phi_0 \ldots \phi_3$ all have the same absolute values but opposite signs. If such antipodal points are all related by gauge transformations, the result of averaging Φ over all possible field configurations, including ones related by gauge transformations, must be $\langle\Phi\rangle = 0$. This is the essence of Elitzur's theorem [5][6]: in gauge theories, SSB requires the gauge to be fixed. What is spontaneously broken is symmetries left over after gauge fixing. If you like the idea of a deep connection between gauge invariance and QM, this could be a problem. Or it might mean that gauge fixing is an ingredient of consistent histories.

Textbooks rarely mention such things. Instead, they note that gauge invariance allows any $\Phi \neq 0$ to be transformed so that only $\phi_0 \neq 0$, or equivalently $\vec{\theta} = 0$, and implicitly apply this transformation to configuration space – the set of all possible field configurations – before quantizing. This is known as imposing the unitary gauge[1]. Its main attraction, besides minimizing the number of DoF, is simple mass terms $\sim \nu^2$ for the weak gauge bosons.

In principle, this is a legitimate maneuver. If the resulting QFT works, i.e. if it is self-consistent and in agreement with experiment, the procedure used to find it is ultimately irrelevant. But as Weinberg reminisced in [7], it does not work:

> I tried to prove the renormalizability of the electroweak theory using the most convenient gauge that can be introduced in the operator formalism, called unitarity gauge, but I couldn't do it [8]. I suggested the problem to a student [9], but he couldn't do it either, and to this day no one has done it using this gauge.

The problem is that the propagator of a vector boson with a constant mass term tends to a constant value as its momentum goes to infinity, so contributions from Feynman diagrams containing loops are not suppressed at high energy. Perturbation theory becomes an impossible task, requiring the summation of infinitely many, infinitely complicated diagrams.

The now standard electroweak model therefore became generally accepted only after it was proved to be renormalizable in the R_ξ gauges, which retain all components of Φ and give ϕ_1, ϕ_2 and ϕ_3 masses $\propto$ the free parameter $\xi \in [0, \infty]$ [10][11][12]. It is possible to establish a formal equivalence between the unitary gauge and the limit $\xi \to \infty$ of the R_ξ gauges [13], but this does not prove renormalizability in that limit, due to the ambiguities which crop up when multiple quantities are sent to infinity; letting $\xi \to \infty$ before the momenta in loop diagrams produces one result, taking the same limits in the opposite order another one [14].

This has all been known for decades. Yet, bringing it up seems to have become a sure way to win foes and alienate people. Why, I do not know. The experimental success of electroweak theory is based on predictions obtained in the R_ξ gauges, so there is no empirical reason for clinging to the unitary gauge. Maybe it is a cherished childhood belief.

A more serious concern could be that there is no obvious reason why the unitary gauge should fail upon quantization. But there is. Analyzing the ways that values on $\mathbb{S}^3$ can be smoothly mapped to spacetime uncovers configurations which can not be continuously deformed to a constant value [15]. Classically, this is tractable; simply ignore such troublesome cases. The unitary gauge does so implicitly. But in QFT, we are supposed to average over all field configurations. The real surprise would be if properties of a theory which depend crucially on cancellations between contributions from different field configurations, like renormalizability, were to survive arbitrary amputations of configuration space.

III. THE STANDARD MODEL LANDSCAPE

The mass eigenstates of electroweak gauge bosons can be parameterized by the Cartesian coordinates $[\Theta_1, \Theta_2, \Theta_3]$ of an ordinary sphere $\mathbb{S}^2$ in $\mathbb{R}^3$, reflecting the O(3) rotational symmetry left over when the O(4) of $\mathbb{S}^3$ is broken (see Appendix A). Each point on the sphere maps to a different Hopf circle on the Higgs vacuum manifold. They all produce the same masses, but

[1] If you are Weinberg, you get to call it "unitarity" gauge.

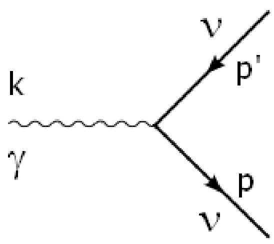

FIG. 1: Feynman diagram for $\gamma \to \nu + \bar{\nu}$. Impossible?

for different linear combinations of gauge bosons. The electromagnetic couplings of left-handed fermion mass eigenstates depend linearly on Θ_3^2. So the SM has its own "landscape" of physically distinct vacua, a continuum of inequivalent classes of Hopf circles.

Why does this happen? Is the SM Lagrangian not supposed to be invariant under SU(2)×U(1) transformations, and is it not true that any choice of vacuum can be reduced to the standard one by such a transformation?

Those are red herrings. The SM Lagrangian is invariant, fermion mass eigenstates are not. So we can not expect invariant expressions in the basis of mass eigenstates. Nor is the mass basis optional if we want to do physics: real particles in real experiments have definite masses, and we can not even begin to compute physical quantities like scattering amplitudes and decay rates without knowing those masses.

For instance, the last term in Eq. (A22) translates to the Feynman diagram of Fig. 1. With the conventions of [1], the corresponding amplitude squared for a photon with momentum k and polarization vector $\varepsilon_{t\mu}(k)$ undergoing the decay $\gamma \to \nu + \bar{\nu}$ is

$$\begin{aligned} |\mathcal{M}|^2 \;=\; & G^2(\vec{\Theta})\,\varepsilon_{t\mu}(k)\varepsilon_t^{\nu}(k)\,\cdot \\ & \mathrm{Tr}\left[\frac{p_\sigma\gamma^\sigma + m_\nu}{2m_\nu}\gamma^\mu(1-\gamma^5)\,\cdot\right. \\ & \left.\frac{p'_\tau\gamma^\tau - m_\nu}{2m_\nu}(1+\gamma^5)\gamma^\nu\right] \end{aligned} \tag{4}$$

You will evidently not get very far without a well defined neutrino mass m_ν.

Note that the unitary gauge breaks the residual O(3), leaving nothing over for SSB. By design, it picks a vacuum where $\Theta_3 = 1 \Rightarrow G(\vec{\Theta}) = 0$. Why? Because that reproduces known physics. But sampling a uniform distribution on $\mathbb{S}^3$, this choice does not seem very likely (see Fig. 2). Maybe you could modify the theory to produce the observed electric charges in a larger region of $\vec{\theta}$ space, or show that $\Theta_3 = 1$ is necessary for the existence of stable atoms or some other prerequisite for life, and invoke the anthropic principle. This should all be starting to sound eerily familiar.

Or you could accept that $\vec{\theta}$ is spacetime-dependent, use a gauge that works, and look to the dynamics of the theory for an explanation.

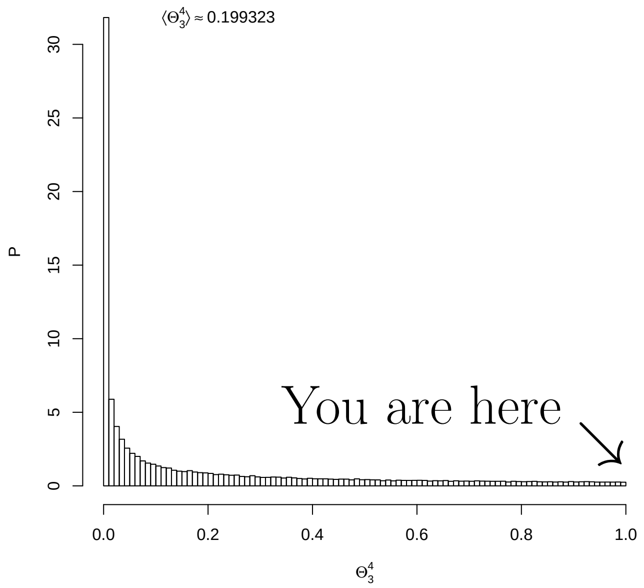

FIG. 2: Distribution of Θ_3^4 (10^5 points sampled uniformly on $\mathbb{S}^3$). Why is our $\Theta_3^4 \simeq 1$?

This drags cosmology into the argument. The standard big bang model starts with a high temperature state where all fields take random values, causing symmetry restoration [17]-[20] (or rather SSB as the universe cools). It also has a finite particle horizon within which information could have been exchanged at any given time. Since random choices could not be communicated beyond the particle horizon, $\vec{\theta}$ started out with gradients on super-horizon scales and then relaxed as the horizon grew.

Fans of the unitary gauge sometimes do not seem to realize that once the possibility of $\vec{\theta}$ gradients is accepted, the argument is over. Instead, they claim that imposing the unitary gauge turns such gradients into collections of short-lived, massive gauge bosons, which decay on the time scale of weak interactions. This works only if you give up locality – the residual O(3) symmetry must be broken the same way across the whole universe – and forget to quantize.

Remember, the unitary gauge is imposed *before* quantization, while constructing the QFT. Transforming the QFT to a different gauge means going to an inequivalent Hilbert space of physical states. For instance, the $\vec{\theta}$ fields are massless in the R_ξ gauge with $\xi = 0$ (Landau gauge, classically equivalent to the Lorenz gauge), but have the same masses as the gauge bosons with $\xi = 1$ ('t Hooft-Feynman gauge). Above the common ground state level, a massless particle has a continuous energy spectrum which only depends on its 4-momentum; a massive particle has a mass gap, an unphysical energy range below its rest mass. Turning smooth, massless $\vec{\theta}$ fields with low energy density into massive gauge bosons therefore means turning physical ("on shell") configurations into unphysical ones.

We see again the importance of "keeping it real" by working in the physical mass basis. What actually happens is that once the energy density of a massless field falls below that of a pair of its lightest decay products per Compton volume, decays become exponentially suppressed, essentially because they depend on random fluctuations to produce local overdensities.

Suppose we choose a gauge with massless $\vec{\theta}$ (on shell, any working gauge can be used to derive effective EoM for the quantized theory, valid order by order [21]). $\vec{\theta}$ gradients can relax by decaying to neutrinos – see Eq. (A22) – so their energy density must be $\lesssim m_\nu^4$ for (meta)stability. Since the SM Lagrangian is quadratic in derivatives of $\Phi \sim \nu\vec{\theta}$, the energy density of a smooth $\vec{\theta}$ gradient extending over some length scale L is $\sim \nu^2/L^2$. Stability therefore requires $L \gtrsim \nu/m_\nu^2$. If $m_\nu = 0.1$ eV, this translates to $L \gtrsim 10^7$ meters.

That is how tight recently created $\vec{\theta}$ gradients could be. But if they were born shortly after the bang, they may have relaxed until temperature fell below $\sim m_\nu$, and were then stretched by metric expansion. Using the cosmic microwave background (CMB) as a proxy, temperature fell below 0.1 eV at redshift $z \simeq 425$, 1.8 million years after the bang. The particle horizon was then 5.3 million light years (ly). Multiply by the expansion factor $1 + z$, and the current size limit becomes $L \gtrsim 2.3$ Gly.

There is clearly a wide range of possibilities here, depending on initial conditions, the lightest neutrino mass and the dynamics of $\vec{\theta}$. But it can be narrowed down substantially by considering the effect of $\vec{\theta}$ gradients on light reaching us from distant sources.

IV. WHY DO WE SEE THE STARS?

The question about the fate of particles crossing $\vec{\theta}$ gradients was first asked by Weinberg in 1974 [19]. It was quickly answered for light by A. Everett, who tackled the problem as one of classical wave transmission. Using the linearized equations of motion for A_μ, $W^\pm$ and Z, he found that the result depended on the width of the gradient relative to the light's wavelength: total reflection from sharp gradients, total transmission across wide ones, partial transmission in intermediate cases [22]. Absent some reason to expect sharp gradients, there would therefore seem to be no observable consequences.

Presumably because of this conclusion, the question then seems to have been forgotten for three decades, until Penrose brought it up again, asking why we can see distant objects in optical telescopes [23]. In 2005, having read [23] but unaware of Everett's work, I estimated the residual luminosity across a $\vec{\theta}$ gradient, essentially by projecting the photon state at the source onto the photon state at the observer's location, and found

$$\ell(\vec{\omega}) = \sin^2(\theta_W)\frac{(\omega_1^2+\omega_2^2)\cos(|\vec{\omega}|)+\omega_3^2}{|\vec{\omega}|^2} + \cos^2(\theta_W) \quad (5)$$

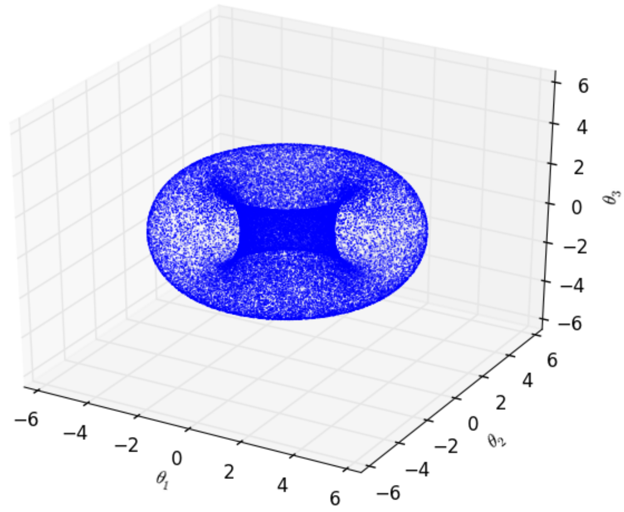

FIG. 3: Randomly sampled $\vec{\theta}$ points where $\Theta_3^4 < 10^{-9}$.

This is just the scalar product of Eq. (A5) for $\vec{\theta} = 0$ and $\vec{\theta} = \vec{\omega}$. The physical picture is of two large regions with constant but different $\vec{\theta}$, separated by an interpolating boundary smooth enough to make reflection negligible. As it crosses the boundary, the photon becomes a linear combination of electroweak gauge bosons, and given enough energy, the massive part of the mix decays to fermions.

There is an implicit assumption here: that electroweak decays dominate over the rotation from source to destination photon state studied by Everett. With Eq. (4) in hand, we can stop assuming and compute the decay rate of A_μ explicitly. For unpolarized photons, a standard calculation (see e.g. Ch. 8 in [1]) yields the center of mass decay rate

$$\Gamma_{CoM} = \frac{G^2 E}{8\pi}\sqrt{(1+2m_\nu/E)(1-2m_\nu/E)}\cdot \left(1 - \frac{(1+2m_\nu/E)(1-2m_\nu/E)}{3}\right) \quad (6)$$

Note the kinematic threshold $E = 2m_\nu$. Photons with lower energy do not decay.

In the limit $m_\nu/E \to 0$, Γ_{CoM} reduces to $G^2E/(12\pi)$. Substituting this into the expression for surviving photon fraction derived in Appendix B yields

$$\ell(\vec{\omega}) \simeq 1 - \frac{G^2}{12\pi}|\vec{\omega}| \quad (7)$$

With the $\langle\Theta_3^4\rangle \simeq 0.2$ sampled from a uniform distribution on $\mathbb{S}^3$ (Fig. 2) this suggests an average luminosity loss in the third decimal, far from the 23% predicted by Eq. (5)[2]. While not much, it compounds. Even a loss of

[2] Drat.

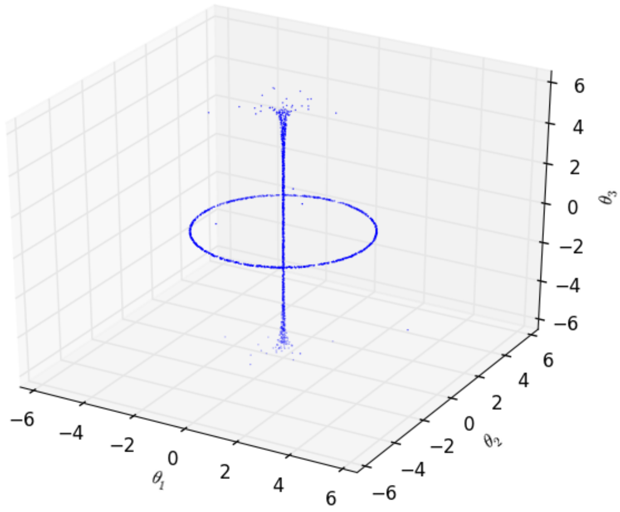

FIG. 4: Randomly sampled $\vec{\theta}$ points where $1 - \Theta_3^4 < 10^{-3}$.

only 10^{-3} per gradient crossing reduces luminosity to 1/3 after 10^3 crossings. Our ability to see the Andromeda galaxy clearly, 2.6 Mly away, therefore implies $L > 10^3$ ly. With objects only visible at longer wavelengths, you can tune m_ν to exploit the kinematic threshold, but the optical visibility of gamma ray burst GRB 080319B over a distance of 7.5 Gly strongly suggests $L > 10^7$ ly.

There is an even stronger constraint from observation. The left-handed electron's coupling to the photon, $e\,\Theta_3^2$, can be viewed as an effective linear dependence of the fine-structure constant α on Θ_3^4. Based on the absorption of quasar light by gas clouds along the line of sight, $\Delta\alpha/\alpha \lesssim 10^{-5}$ [24]. Apparently, the entire visible universe prefers to stay close to the points in Fig. 4.

This calls for an explanation. It also means that a confirmed small variation in α would not necessarily constitute evidence of new physics, as is sometimes claimed. It would be interesting to look for correlations between the α dipole found in [24] and other observed anomalies, like the suspicious alignment between quasar light polarization and low multipole moments in the CMB [25].

V. PRELUDE TO DYNAMICS

There is an obvious way to reconcile cosmology with electroweak symmetry breaking: assume that inflation ended at or below the electroweak scale, so the entire observable universe fits within a patch of constant $\vec{\theta}$. Higgs-mediated inflation [28]-[31] might produce this result. Having the SM Higgs do double duty as the inflaton through a non-minimal coupling to gravity is a particularly attractive idea. But before we jump to conclusions, let's consider the alternatives.

Short of renouncing the Higgs mechanism (in favor of what?) you could take the view, sometimes advocated by Veltman, that the theory is *defined* by its perturbative expansion, while the Lagrangian is just a "bookkeeping device" used to aid the construction of Feynman diagrams [26]. Non-perturbative structures like $\vec{\theta}$ gradients would go away, but at a cost. Empirically, non-perturbative QCD works quite well on the lattice, which raises the question why electroweak interactions should be so different. Theoretically, the perturbation series is not guaranteed to converge (in the simple case of QED it has long been understood to diverge [27]). The closest well-defined object would be a sum over a finite number of terms, which is hard to accept as more than an ad-hoc computational device.

If you accept the possibility of non-perturbative electroweak structures, finite particle horizons make them mandatory: there is no reason to assume perturbatively small differences in $\vec{\theta}$ between regions which have been causally disconnected since before the symmetry was broken (inflation does not change this conclusion, only the spatial scale).

The study of such structures requires other tools than those familiar to most particle physicists. Axial gauges, which simply set a Lorentz component of every gauge boson to zero, stand out in this context [32]. Among them, the time-axial (a.k.a. temporal, Weyl, or Hamiltonian) gauge plays a special role, since it arises naturally in the canonical operator formalism; already in electrodynamics, the conjugate momentum of A_0 vanishes, making it impossible to impose the canonical commutation relation $[A_0(t,\vec{x}), \Pi_0(t,\vec{x}')] = i\delta(\vec{x}-\vec{x}')$. Instead, gauge invariance is invoked to set $A_0 = 0$, and the QFT is written using only the space-like components of A_μ, leaving a residual invariance under time-independent gauge transformations which is removed by imposing Gauss' law. Since the latter commutes with the Hamiltonian, this need only be done at one point in time, e.g. as an initial condition.

The familiar path integral for arbitrary gauges can be derived from this formalism, arguably making it more fundamental [33]. It has proved invaluable for non-perturbative QFT: the instanton solution to the U(1) problem of QCD [34][35], electroweak sphalerons [36]-[40] with related, cosmologically significant violation of baryon number conservation [41]-[43], and extended configurations like Nielsen-Olesen strings [44]-[46] were all found using the time-axial gauge. Its simplicity also makes it ideal for non-perturbative numerical simulations, where it is ubiquitous [47]-[53].

Like the Landau gauge, axial gauges leave $\vec{\theta}$ massless. This sometimes makes people nervous: why don't we see massless $\vec{\theta}$ particles flying around? In the R_ξ gauges, the gauge boson propagators pick up extra poles at the same masses as $\vec{\theta}$, and their contributions to Feynman amplitudes cancel, effectively confining $\vec{\theta}$ particles to within a distance $\sim 1/(g\nu)$ [12]. In axial gauges, the cancellation is against poles in the interaction vertices of massive gauge bosons, with the same result [54].

Absence of $\vec{\theta}$ particles propagating over macroscopic distances does not imply absence of macroscopic and evolving $\vec{\theta}$ configurations, any more than quark confinement implies absence of macroscopic and evolving hadron configurations. You might therefore worry about long-range $\vec{\theta}$ potentials competing with gravity. This too is a non-issue as long as the coupling between $\vec{\theta}$ and fermions $\to 0$ with momentum transfer [55]. It does; see the second row in Eq. (A22).

The convenience of the time-axial gauge becomes apparent when deriving the effective photon mass on a $\vec{\theta}$ gradient, M_O in Eq. (B2). With $A_0 = 0$, the terms of the SM Lagrangian quadratic in A_μ can be rearranged as the quadratic form $\frac{1}{2}\vec{A}^T \mathbb{M} \vec{A}$, where the 3×3 matrix $\mathbb{M}$ has components

$$M_{11} = \left(\partial_0 \vec{\theta}^T \, \mathbb{H} \, \partial_0 \vec{\theta} + \partial_2 \vec{\theta}^T \, \mathbb{H} \, \partial_2 \vec{\theta} + \partial_3 \vec{\theta}^T \, \mathbb{H} \, \partial_3 \vec{\theta}\right) \quad (8)$$

$$M_{12} = \left(-\partial_2 \vec{\theta}^T \, \mathbb{H} \, \partial_1 \vec{\theta}\right) \quad (9)$$

etc. and $\mathbb{H}$ has components

$$H_{ab} = \sin^2(\theta_W) \frac{\partial \Theta_c}{\partial \theta_a} \frac{\partial \Theta_c}{\partial \theta_b} \quad (10)$$

Since $\mathbb{H}$ is real and symmetric, $\mathbb{M}$ is too, and can therefore always be diagonalized. Its three eigenvalues are M_O^2 in the three principal directions defined by its eigenvectors.

The mapping of $\vec{\Theta}$ to Hopf circles guarantees that $\mathbb{H}$ always has at least one vanishing eigenvalue for any $\vec{\theta}$. The other two are non-negative. The middle one peaks at $\theta = 0$ and falls off for larger θ; the largest one is symmetric about the line $\theta_1 = \theta_2 = 0$ and along it. On that line, the only non-zero elements of $\mathbb{H}$ are

$$H_{11} = H_{22} = 2\sin^2(\theta_W)\,(1 - \cos(\theta_3))\,/\theta_3^2 \quad (11)$$

In $\mathbb{M}$, they only multiply derivatives of θ_1 and θ_2, so M_O vanishes not only for constant $\vec{\theta}$, but also when $\theta_1 = \theta_2 = 0$.

Consider a linear $\vec{\theta}$ gradient along the photon's direction of motion, e.g. the z axis. The two transverse polarization modes then have effective mass

$$M_O = \sqrt{\partial_3 \vec{\theta}^T \, \mathbb{H} \, \partial_3 \vec{\theta}} = \frac{1}{L}\sqrt{\Delta \vec{\theta}^T \, \mathbb{H} \, \Delta \vec{\theta}} \quad (12)$$

Substituting $\sqrt{\Delta \vec{\theta}^T \, \mathbb{H} \, \Delta \vec{\theta}}$ for $|\Delta \vec{\theta}|$ in Eq. (B3), taking the limit $L \to \infty$ and picking random $\vec{\theta}$ pairs uniformly from $\mathbb{S}^3$ produces the residual luminosity distribution of Fig. 5. The average loss is 0.53% per crossing.

VI. DYNAMICAL VACUUM SELECTION

A uniform distribution over static vacuum configurations is just an assumption. How does $\vec{\theta}$ really evolve from the random initial conditions expected at the bang?

It is tempting to derive the classical EoM of the full SM Lagrangian and evolve those, arguing that quantum corrections are small. This works with a plasma hot enough to maintain large occupation numbers for all massive fields [56]-[58]. But at lower energy densities, where mass gaps can no longer be ignored, the infinite divisibility of classical fields ceases to be a good approximation. There are no fractional $W^\pm$ and Z bosons on shell.

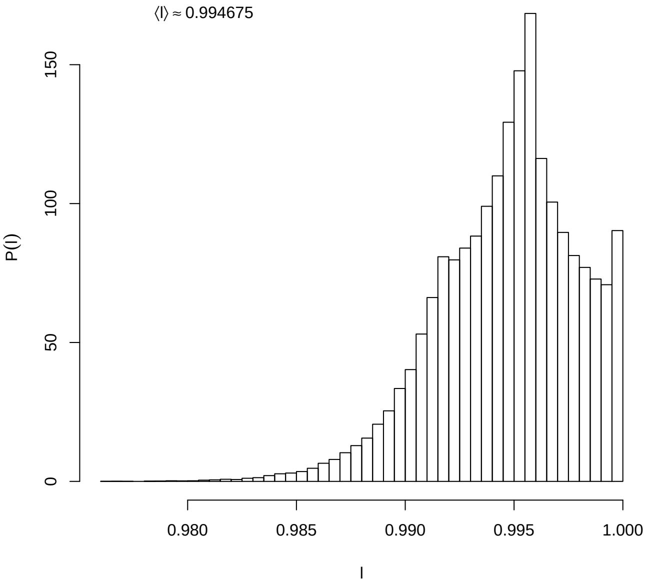

FIG. 5: Residual luminosity distribution for one gradient crossing ($E = 1$ eV, $m_\nu = 0.1$ eV, 10^5 $\vec{\theta}$ pairs sampled uniformly on $\mathbb{S}^3$).

To study the low energy, long distance limit, you must first derive an effective theory. The Appelquist-Carazzone decoupling theorem guarantees that massive fields only affect it through corrections $\propto$ powers of interaction energy over mass [59][60][61]. Below the lowest mass threshold, the EoM for massless fields only are therefore a good approximation. To leading order, they follow from the classical Lagrangian with everything else set $= 0$,

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} G_{ab} \partial_\mu \theta^a \partial^\mu \theta^b \\ &+ \frac{1}{2} \partial_\mu A_\nu \left(\partial^\nu A^\mu - \partial^\mu A^\nu\right) \\ &+ \frac{1}{2} H_{ab} A_\mu \partial_\nu \theta^a (A^\nu \partial^\mu \theta^b - A^\mu \partial^\nu \theta^b) \end{aligned} \quad (13)$$

where the scale parameter ν has been absorbed into A_μ and

$$G_{ab} = \left(\frac{\delta_{ad}\delta_{be}}{2} + \frac{1 - \cos\theta}{\theta^2} \varepsilon_{cda}\varepsilon_{ceb}\right) \frac{\theta_d \theta_e}{\theta^2} \quad (14)$$

is the metric of $\mathbb{S}^3$. Varying $\mathcal{L}$ in A_0 yields the Gauss constraint

$$\nabla \cdot \vec{E} = \partial_0 \vec{\theta}^T \mathbb{H} A_m \partial_m \vec{\theta} \quad (15)$$

With $A_0 = 0$, this is trivially satisfied by any static configuration.

Eq. (13) describes an O(4) non-linear sigma model coupled to electromagnetism by the effective photon mass term. It should hold for energy densities below the pair production threshold of the lightest neutrino, $\sim (0.1\text{ eV})^4$ assuming $m_\nu = 0.1$ eV. As a comparison, the current dark energy density is estimated to be $\simeq (3 \cdot 10^{-3}\text{ eV})^4$.

The decoupling of $\vec{\theta}$ and $\vec{A}$ when $\theta_1 = \theta_2 = 0$ suggests that radiation pressure will drive $\vec{\theta}$ to that line. Mathematically, it should accrete at minima in the speed $s = |\partial_0(\vec{\theta}, \vec{\Pi})|$ of the Hamiltonian phase space flow $\partial_0(\vec{\theta}, \vec{\Pi}) = (\partial H/\partial\vec{\Pi}, -\partial H/\partial\vec{\theta})$, or equivalently in

$$s^2 = \frac{\partial H}{\partial \Pi_a}\frac{\partial H}{\partial \Pi_a} + \frac{\partial H}{\partial \theta_a}\frac{\partial H}{\partial \theta_a} \tag{16}$$

Finding such minima is hard, but if we also require that $\nabla\theta_a = \vec{\Pi} = 0$ (i.e. static vacua), the only non-vanishing second derivatives of the Hamiltonian are

$$\frac{\partial^2 H}{\partial \Pi_a \partial \Pi_b} = \left[\mathbb{K}^{-1}\right]_{ab} \tag{17}$$

where

$$\mathbb{K} = \mathbb{G} + \vec{A}^2\mathbb{H} \tag{18}$$

Since minima of s^2 should have vanishing derivatives in $\vec{\theta}$ and $\vec{\Pi}$, the minima of the largest eigenvalue of $\mathbb{K}^{-1}$ should coincide with the $\vec{\theta}$ least influenced by $\vec{A}$. For $\vec{A}^2 = 0$, the largest eigenvalue at $\vec{\theta} = 0$ is 4; from there, it grows isotropically with θ. For finite $\vec{A}^2$, the minimum is still 4 at $\vec{\theta} = 0$, but larger values form concentric prolate spheroids extending along the θ_3 axis. $\vec{\theta}$ can therefore be expected to settle around that axis.

This is confirmed by numerical simulations [62]. The θ_3 axis acts as an attractor, growing stronger with electromagnetic energy density. The larger the latter, the more $\vec{\theta}$ spreads out along θ_3, reflecting the cigar-like shape of the surfaces in Fig. 6. At comparable energy densities, $\vec{\theta}$ settles around $\theta_1 = \theta_2 = 0$ after a few collisions between wavefronts. In Minkowski space, M_O then approaches some finite value asymptotically as radiation pressure finds a balance with the thermal motion of $\vec{\theta}$.

An expanding (FRW) metric adds two notable effects: it cools $\vec{\theta}$, causing the "cigar" to keep shrinking about some point on the θ_3 axis, and it aids the formation of extended structures, apparently by pumping up plasma oscillations in $\vec{\theta}$ (max extension gets an extra push, increasing $\vec{E}$, while max velocity is reduced, damping $\vec{B}$). The result is a foam-like structure of large "bubbles" with low M_O, separated by boundaries where it's larger (but still unobservable in the parameter space accessible with a PC).

Neutral domains with charged boundaries have been proposed in the past to explain baryogenesis, dark matter and even dark energy. But the main point is that the dynamics always drives $\Theta_3 \to 1$, without extra assumptions.

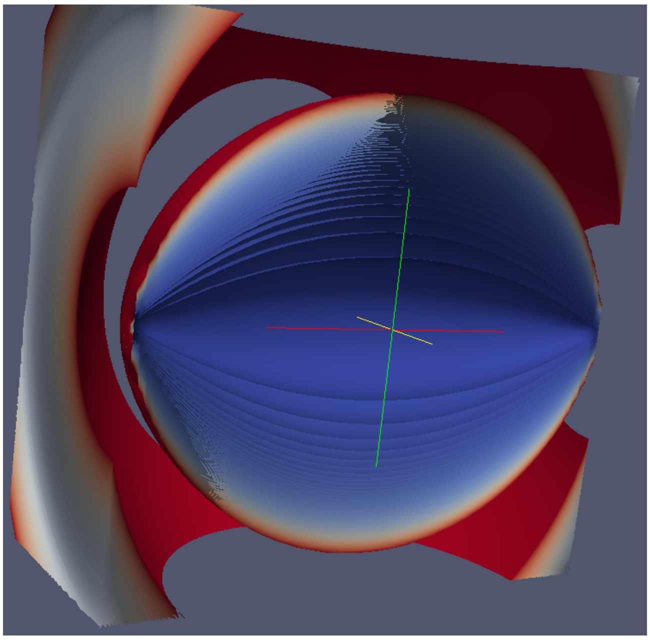

FIG. 6: Max eigenvalues of $\mathbb{K}^{-1}$ with $\vec{A}^2 = 100^2$ (cut through isosurfaces, θ_3 axis in red).

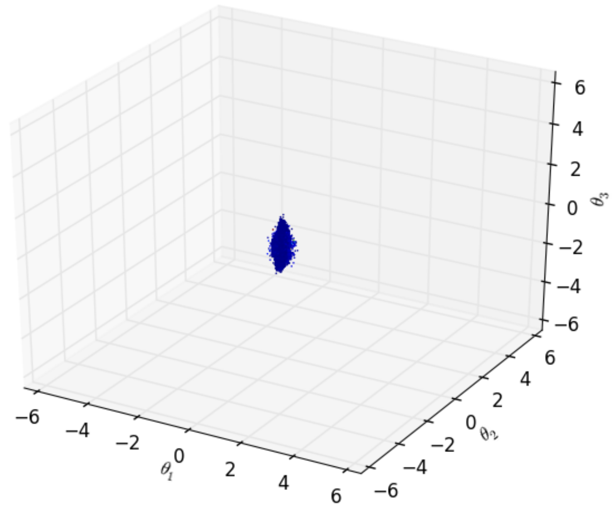

FIG. 7: Typical asymptotic "cigar" distribution of $\vec{\theta}$.

VII. CONCLUSION

The standard model of particle physics has a landscape of physically inequivalent vacua. Naively assigning equal probability to each makes our universe look very unlikely. At first, rejecting the "just so" unitary gauge and anthropic principle seems to make things worse. But by forcing us to consider the dynamics, this intransigence eventually leads to a natural explanation.

Any resemblance to other theories, living or dead, may be purely coincidental.

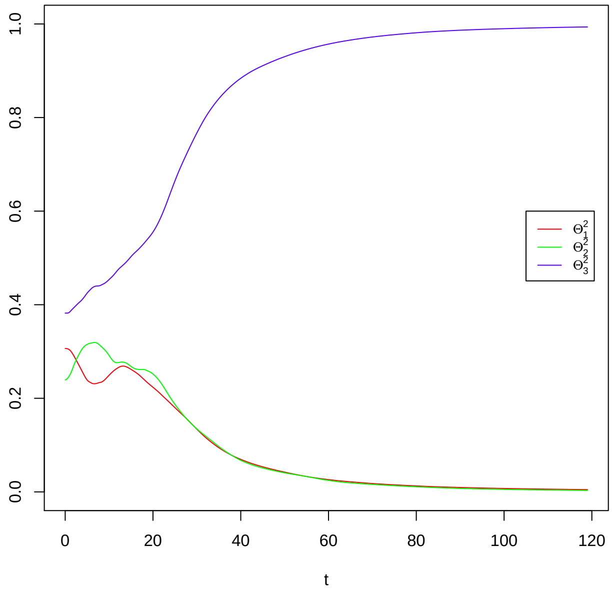

FIG. 8: Evolution of $\langle \Theta_a^2 \rangle$ in expanding universe.

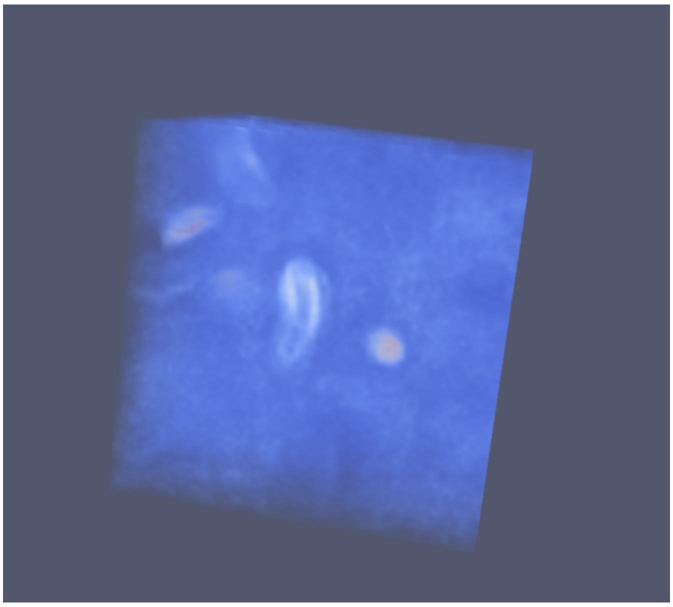

FIG. 9: Max M_O in expanding universe.

APPENDIX A: EVERY UNITARY GAUGE

In this appendix, I follow the introductory textbook approach of taking the classical SM Lagrangian and picking a point on the Higgs vacuum manifold, Eq. (2), but with a twist. Instead of choosing the standard vacuum and merging it into the math as an anonymous number, I carry it along explicitly as a variable. The payoff at the end of the calculation is be the ability to easily read off the effect of choosing different vacua. If they are all physically equivalent, physical quantities will be unaffected.

Substituting Eq. (3) into the SM Lagrangian and reading off terms quadratic in the electroweak gauge fields $\left[B_\mu, W_\mu^1, W_\mu^2, W_\mu^3\right]$ yields the mass matrix

$$\frac{\nu^2}{2}\begin{bmatrix} g_B^2 & g_B g_W \Theta_1 & g_B g_W \Theta_2 & -g_B g_W \Theta_3 \\ g_B g_W \Theta_1 & g_W^2 & 0 & 0 \\ g_B g_W \Theta_2 & 0 & g_W^2 & 0 \\ -g_B g_W \Theta_3 & 0 & 0 & g_W^2 \end{bmatrix} \quad \text{(A1)}$$

where g_B and g_W are the gauge coupling constants. You may recognize the auxiliary quantities

$$\begin{aligned} \Theta_1 &= \left[\theta_1\theta_3(\cos(\theta)-1)+\theta\theta_2\sin(\theta)\right]/\theta^2 & \text{(A2)} \\ \Theta_2 &= \left[\theta_2\theta_3(\cos(\theta)-1)-\theta\theta_1\sin(\theta)\right]/\theta^2 & \text{(A3)} \\ \Theta_3 &= \left[(\theta_1^2+\theta_2^2)\cos(\theta)+\theta_3^2\right]/\theta^2 & \text{(A4)} \end{aligned}$$

as the Hopf map [16], which takes disjoint circles on $\mathbb{S}^3$ to different points on $\mathbb{S}^2$: $(\Theta_1)^2+(\Theta_2)^2+(\Theta_3)^2=1$.

The eigenvalues of Eq. (A1) are independent of $\vec{\theta}$, and easily recognized as the tree level masses squared of photon, $W^\pm$ and Z^0: 0, $g_W^2\nu^2/4$ and $(g_W^2+g_B^2)\nu^2/4$. As you may have expected, all vacua are equivalent as far as the gauge boson masses are concerned.

The two degenerate eigenstates can be orthogonalized to obtain

$$\begin{aligned} A_\mu &= [g_W/g_B, -\Theta_1, -\Theta_2,\ \Theta_3]/\sqrt{1+g_W^2/g_B^2} & \text{(A5)} \\ \acute{W}_\mu^1 &= [0, \quad -\Theta_2, \quad \Theta_1, \quad 0]/\sqrt{\Theta_1^2+\Theta_2^2} & \text{(A6)} \\ \acute{W}_\mu^2 &= [0, \Theta_1\Theta_3, \Theta_2\Theta_3, \Theta_1^2+\Theta_2^2]/\sqrt{\Theta_1^2+\Theta_2^2} & \text{(A7)} \\ Z_\mu &= [-g_B/g_W, -\Theta_1, -\Theta_2, \Theta_3]/\sqrt{1+g_B^2/g_W^2} & \text{(A8)} \end{aligned}$$

all in terms of the basis $\left[B_\mu, W_\mu^1, W_\mu^2, W_\mu^3\right]$. Eq. (A5) is the massless combination, i.e. the photon, in an arbitrary vacuum.

Eqs. (A5)-(A8) are orthonormal eigenstates, so they are trivially inverted. In particular, setting all massive combinations to zero and keeping only the photon yields

$$\begin{aligned} B_\mu &= A_\mu\cos(\theta_W) & \text{(A9)} \\ W_\mu^1 &= -A_\mu\Theta_1\sin(\theta_W) & \text{(A10)} \\ W_\mu^2 &= -A_\mu\Theta_2\sin(\theta_W) & \text{(A11)} \\ W_\mu^3 &= A_\mu\Theta_3\sin(\theta_W) & \text{(A12)} \end{aligned}$$

where θ_W is the Weinberg angle, $\sin^2(\theta_W)=g_B^2/(g_B^2+g_W^2)\simeq 0.232$.

Note that each set of mass eigenstates applies to an entire Hopf circle of vacua. For every $\vec{\theta}$, there is therefore a direction on the Higgs vacuum manifold along which the mass eigenstates remain unchanged.

The SM Higgs field also produces fermion masses through Yukawa couplings on the form

$$f_u\bar{q}_L\tilde{\Phi}u_R + f_d\bar{q}_L\Phi d_R + H.c. \quad \text{(A13)}$$

where f_u and f_d are the coupling constants, $q_L=[u_L,d_L]^T$ is an SU(2) doublet of left-handed fermions (e.g. up and down quarks), u_R and d_R are the corresponding right-handed fermions, H.c. is short for Hermitian conjugate, and $\tilde{\Phi}=[\phi^{0*},-\phi^{+*}]^T$. For the cognescenti, I use 4-component Dirac spinors with the opposite chirality projected out: $u_L=\frac{1}{2}(1-\gamma^5)u_L$, $u_R=\frac{1}{2}(1+\gamma^5)u_R$, $\bar{q}_L=q_L^\dagger\gamma^0$. The γ^μ are Dirac's gamma matrices.

This can equivalently be written in matrix form by assembling u_R and d_R in a right-handed doublet q_R, the Higgs components in the SU(2) matrix

$$\begin{bmatrix} \varphi^{0*} & \varphi^+ \\ -\varphi^{+*} & \varphi^0 \end{bmatrix} = \frac{\nu}{\sqrt{2}}\mathbf{\Sigma} \quad \text{(A14)}$$

and the Yukawa couplings in a diagonal matrix

$$\mathbf{M}_L = \frac{\nu}{\sqrt{2}}\begin{bmatrix} f_u & 0 \\ 0 & f_d \end{bmatrix} \quad \text{(A15)}$$

Eq. (A13) then becomes

$$\bar{q}_L\mathbf{\Sigma}\mathbf{M}_L q_R + \bar{q}_R\mathbf{M}_L^\dagger\mathbf{\Sigma}^\dagger q_L \quad \text{(A16)}$$

In the standard vacuum, $\mathbf{\Sigma}$ is diagonal, so the fermions are mass eigenstates. In other vacua, it is evident by inspection that the mass matrix is diagonalized by the rotation $q_L \to q'_L = \mathbf{\Sigma}^\dagger q_L$, i.e. by

$$q_L = \mathbf{\Sigma}q'_L \quad \text{(A17)}$$

The SM Lagrangian can therefore be expressed using fermion mass eigenstates q'_L by substituting Eq. (A17) into it. The task is simplified by using Eq. (3), Eq. (A14) and the Pauli matrices

$$\tau^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \tau^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \tau^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{(A18)}$$

to write

$$\begin{aligned} \mathbf{\Sigma} &= \cos\left(\frac{\theta}{2}\right) + i\,\frac{\theta_a\tau_a}{\theta}\sin\left(\frac{\theta}{2}\right) \\ &= \exp\left(\frac{i}{2}\,\theta_a\tau_a\right) & \text{(A19)} \end{aligned}$$

We now have all we need to rewrite the standard kinetic term $i\bar{q}_L D_\mu\gamma^\mu q_L$, where

$$D_\mu = \partial_\mu + \frac{ieB_\mu}{\cos(\theta_W)}\left(q-\frac{\tau^3}{2}\right) + \frac{ieW_\mu^a\tau^a}{2\sin(\theta_W)} \quad \text{(A20)}$$

is the covariant derivative, e is the electric charge of the proton (related to the fine-structure constant $\alpha \simeq 1/137.04$ by $e^2 = 4\pi\alpha$) and q is the electric charge operator. For brevity, let's focus on electromagnetic interactions by using Eqs. (A9)-(A12) and a neutrino-electron doublet $L_L = [\nu_L, e_L]^T$ for q_L. Applying Eqs. (A17), (A19) and

$$\mathbf{\Sigma}\partial_\mu\mathbf{\Sigma}^\dagger = \frac{i}{2}\,\partial_\mu\theta^a\mathbf{\Sigma}^\dagger\tau^a\mathbf{\Sigma} \tag{A21}$$

this becomes

$$\begin{aligned} i\bar{L}_L D_\mu\gamma^\mu L_L &= i\,\bar{\nu}'_L\partial_\mu\gamma^\mu\nu'_L \\ &+ \frac{1}{2}\left[\Theta^1\partial_\mu\theta^1 + \Theta^2\partial_\mu\theta^2 - \Theta^3\partial_\mu\theta^3\right]\bar{\nu}'_L\gamma^\mu\nu'_L \\ &+ GA_\mu\bar{\nu}'_L\gamma^\mu\nu'_L + \ldots \end{aligned} \tag{A22}$$

where

$$G = e\left(\Theta_1^2 + \Theta_2^2\right) = e\left(1 - \Theta_3^2\right) \tag{A23}$$

and "…" contains all terms involving e_L and massive gauge bosons. The last row in Eq. (A22) says that even at interaction energies too low to involve those, ν'_L and photons interact with effective coupling strength G. When $\vec{\theta} \to 0$, $\Theta_3 \to 1$ and $G \to 0$. The equivalent expressions for e'_L are similar; the $\partial_\mu\vec{\theta}$ terms change sign, the effective photon coupling becomes $e\,\Theta_3^2$.

So, as $\vec{\theta}$ moves away from 0, ν'_L picks up an effective electric charge $G = e(1-\Theta_3^2)$, while that of e'_L is reduced by the same amount. $G = 0$ only on the axis $\theta_1 = \theta_2 = 0$ and on a ring in the $\theta_3 = 0$ plane with radius squared $\theta_1^2 + \theta_2^2 = \pi^2$. Centered on that ring, there is a torus where $G = e$, with inner radius squared $\pi^2/4$ and outer radius squared $9\pi^2/4$ extending along $\theta_3^2 \simeq [0, 2\pi]$. On that torus, ν'_L has unit electric charge, while e'_L has none.

There are also mixed interaction terms

$$A_\mu\left(F\bar{\nu}'_L\gamma^\mu e'_L + F^*\bar{e}'_L\gamma^\mu\nu'_L\right) \tag{A24}$$

where

$$F = e\,\Theta_3\left[\Theta_1 - i\,\Theta_2 - 2\,(i\,\theta_1 + \theta_2)\,\frac{\sin(\theta)}{\theta}\right] \tag{A25}$$

This may look alarmingly messy, but its contribution to the amplitude squared of such interactions (i.e. to their probability) is

$$|F|^2 = (e\Theta_3)^2(\Theta_1^2 + \Theta_2^2) = e\,G\,\Theta_3^2 \tag{A26}$$

For quarks, it is sufficient to note that the kinetic term $i\,\bar{Q}_L D_\mu\gamma^\mu Q_L$ with

$$Q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix} \tag{A27}$$

can be obtained from its lepton counterpart by the substitutions $\nu_L \to u_L$, $e_L \to d_L$, $B_\mu \to -B_\mu/3$. The form of the mixed interaction terms does not change, while the diagonal terms become

$$e\left(\frac{1}{3} - \Theta_3^2\right)A_\mu\bar{u}'_L\gamma^\mu u'_L \tag{A28}$$

and

$$e\left(\Theta_3^2 - \frac{2}{3}\right)A_\mu\bar{d}'_L\gamma^\mu d'_L \tag{A29}$$

Like ν'_L and e'_L, u'_L and d'_L swap electric charges as Θ_3 goes from 1 to 0.

Right-handed fermions do not partake in weak interactions and work as usual, independently of $\vec{\theta}$. For instance,

$$\begin{aligned} i\bar{L}_R D_\mu\gamma^\mu L_R &= i\,\bar{\nu}_R\gamma^\mu\partial_\mu\nu_R + i\,\bar{e}_R\gamma^\mu\partial_\mu e_R \\ &+ eA_\mu\bar{e}_R\gamma^\mu e_R \end{aligned} \tag{A30}$$

APPENDIX B: PHOTON DECAY

Particles with decay rate Γ have mean life $\tau = 1/\Gamma$; the surviving fraction after a time t is $\exp(-t/\tau) = \exp(-t\Gamma)$. If they are moving at relativistic speed β in the observer's reference frame and Γ is known in their center of mass (CoM) frame, the observer will see a dilated lifetime, or equivalently a reduced decay rate

$$\Gamma_O = \Gamma_{CoM}\sqrt{1-\beta^2} \tag{B1}$$

where $\beta = M_O/E$, M_O is particle mass in the observer frame and E is total particle energy. The time t needed to traverse a gradient of spatial width L is $t = L/\beta$, so the fraction of particles which survive the crossing is

$$\exp(-t/\tau) = \exp\left(\frac{-L\Gamma_{CoM}}{\sqrt{E^2/M_O^2 - 1}}\right) \tag{B2}$$

In our case, M_O is the effective photon mass due to the $\vec{\theta}$ gradient. To leading order, it can be obtained by substituting Eqs. (A9)-(A12) into the SM Langrangian and reading off all terms quadratic in A_μ. Since A_μ is defined by its lack of an ordinary mass $\propto \nu$, such terms can only come from the quartic interactions of W_μ^a. Even without a full derivation, we can therefore tell that $M_O \sim |\Delta\vec{\theta}|/L$, where $\Delta\vec{\theta}$ is the difference in $\vec{\theta}$ across the gradient and L its width. Substituting M_O into $\exp(-t/\tau)$ and expanding in $1/L$ yields

$$\exp(-t/\tau) \simeq \exp\left(-\frac{\Gamma_{CoM}|\Delta\vec{\theta}|}{E}\right)\left[1 - \mathcal{O}\left(\frac{\Gamma_{CoM}}{L^2E^3}\right)\right] \tag{B3}$$

Visible light has E $\simeq$ 1 eV $\simeq 1/(10^{-6}$ m). One ly is $\simeq 10^{16}$ m, so the first term is quite sufficient for astronomic L. Eq. (B3) essentially says that the longer mean life of a photon crossing a wider gradient is compensated by the longer flight time, so the surviving fraction stays the same.

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