In Search of Saturation

Probing the Frontier of High-Energy Hadron Scattering

David Zaslavsky with Anna Staśto, Bowen Xiao, Feng Yuan

Penn State University

March 20, 2014 based on PRL **112** 012302 (2014) and work in progress



Parton model of proton structure



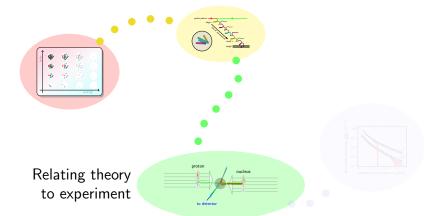




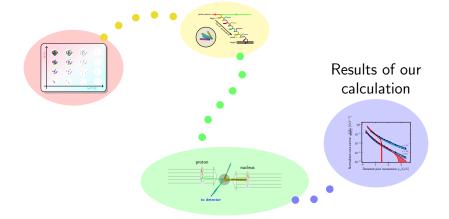


QCD and partons











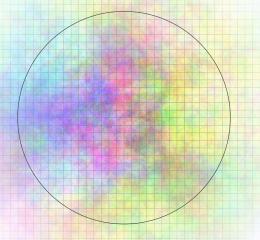




What's in a proton?







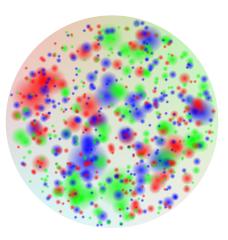
...not directly observable

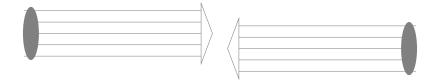


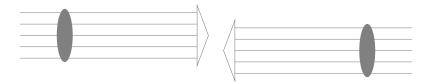
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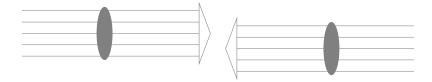
Acts like a collection of particles, or partons

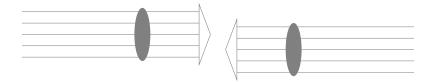
quarks
gluons
photons
electrons
W bosons
Z bosons
Higgs bosons
etc.

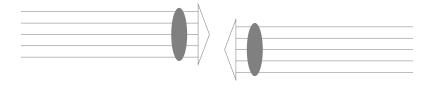


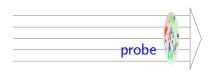








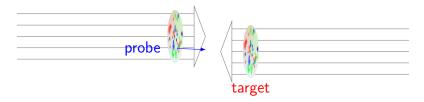






Observing the proton's structure

Information about proton's structure comes from collisions



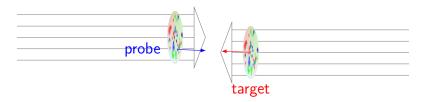
One parton from the probe proton

$$x_p = \frac{\text{forward momentum of parton}}{\text{forward momentum of proton}}$$



Observing the proton's structure

Information about proton's structure comes from collisions



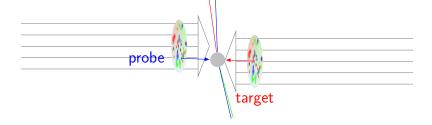
One parton from the target proton

 $x_t = \frac{\text{forward momentum of parton}}{\text{forward momentum of proton}}$



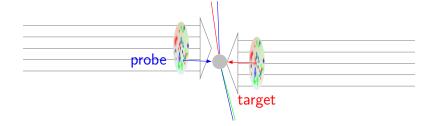
Observing the proton's structure

Information about proton's structure comes from collisions



Particles come out and hit the detector $Q \sim ext{transverse}$ (sideways) momentum transfer between partons





Behavior of target classified by

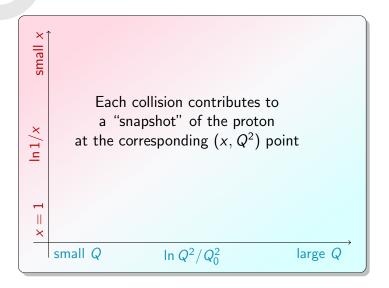
- x_t : forward momentum fraction
- Q: transverse momentum transfer

which are calculated from measurements of outgoing particles



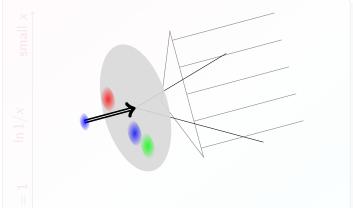
Classifying collisions







Low energy collision:

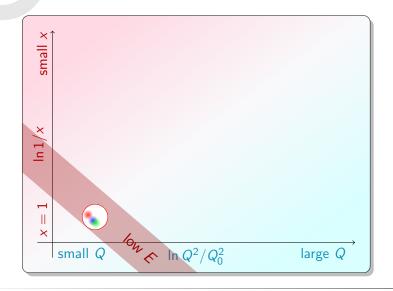


- Target momentum fraction x: large
- Transverse momentum transfer Q: small
- Proton acts like few large partons



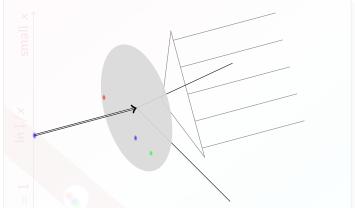
Classifying collisions







High energy collision:

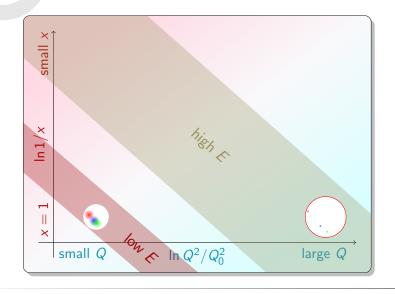


- Target momentum fraction x: large
- Transverse momentum transfer *Q*: large
- Proton acts like few small partons



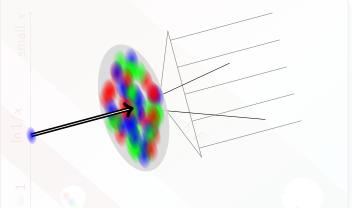
Classifying collisions







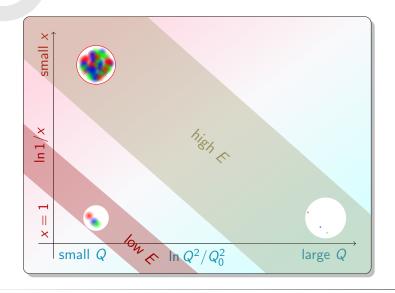
High energy collision:



- Target momentum fraction x: small
- Transverse momentum transfer Q: small
- Proton acts like many large partons

Classifying collisions

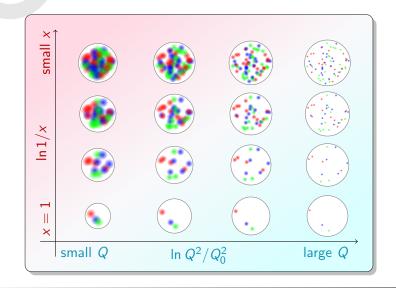




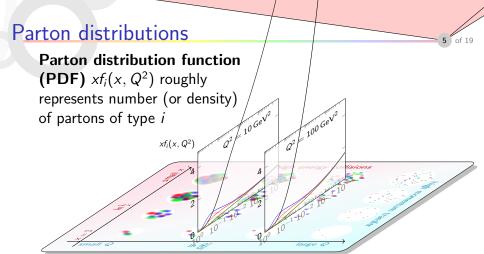


Internal structure of a proton









Shown for some types of partons:

— gluon — up — down — antiup — antidown

This effective description is the parton model







QCD and partons

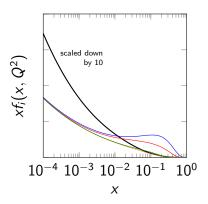


The parton model and QCD



Quantum chromodynamics (QCD):

theory of strong interactions
Can we derive parton distributions from QCD?



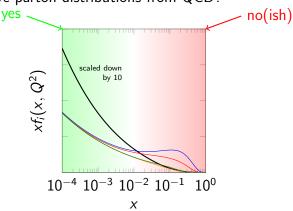


The parton model and QCD



Quantum chromodynamics (QCD):

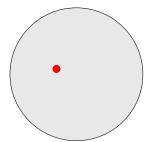
theory of strong interactions
Can we derive parton distributions from QCD?







Here's what happens under the hood:



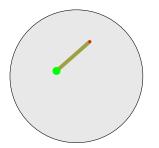
target _____





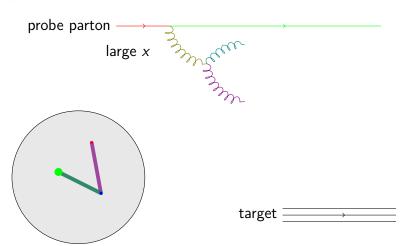
Here's what happens under the hood:

probe parton
$$\longrightarrow$$
 large x



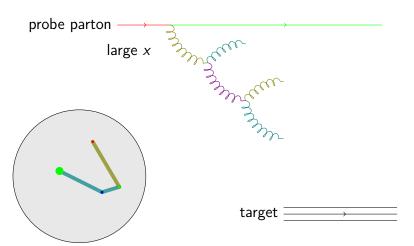
target _____

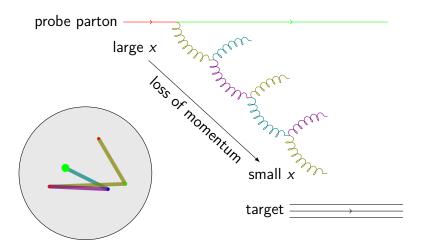


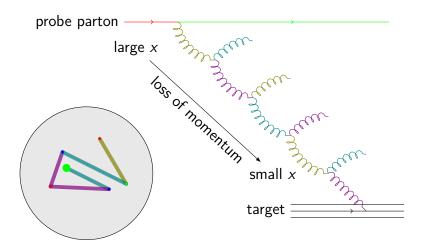










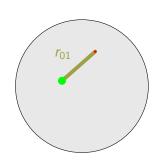


BFKL equation



Translation into math: BFKL equation

$$\frac{\partial S(x, r_{01}^2)}{\partial \ln \frac{1}{x}} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 \mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2} \left[S(x, r_{02}^2) + S(x, r_{12}^2) - S(x, r_{01}^2) - 1 \right]$$



where

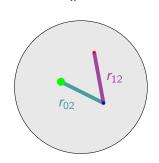
$$F(x, k_{\perp}^2) = \frac{\mathrm{d}}{\mathrm{d}Q^2} \Big[x f_i(x, Q^2) \Big] \Big|_{k_{\perp}^2}$$
 $S(x, r^2) = \int \mathrm{d}^2 \mathbf{k}_{\perp} e^{-i\mathbf{k}_{\perp} \cdot \mathbf{r}} F(x, k_{\perp}^2)$

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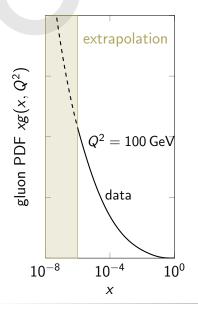


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 $S(x, r^2) = \int \mathrm{d}^2 \mathbf{k}_{\perp} e^{-i\mathbf{k}_{\perp} \cdot \mathbf{r}} F(x, k_{\perp}^2)$

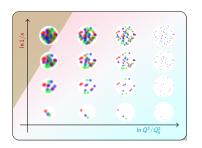
Runaway growth





Problem:

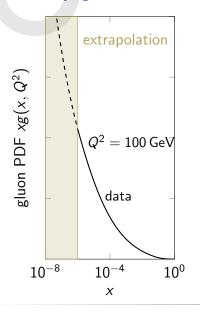
- As $xg(x, Q^2)$ increases, so does gluon interaction probability
- At very small x, trend breaks probability conservation!



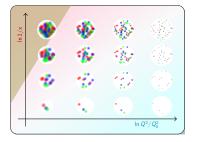


Runaway growth



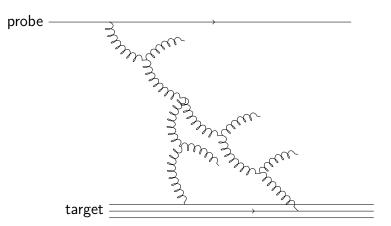


AHHHHHHHH PARTICLE PHYSICS IS BROKEN



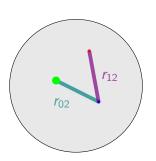


Multiple interactions occur with lots of partons in the target



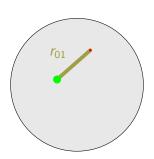
Translation into math: BK equation

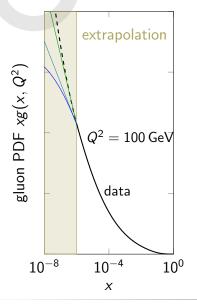
$$\frac{\partial S(x, r_{01}^2)}{\partial \ln \frac{1}{x}} = -\frac{\alpha_s N_c}{2\pi^2} \int d^2 \mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2} \left[S(x, r_{01}^2) - S(x, r_{02}^2) S(x, r_{12}^2) \right]$$



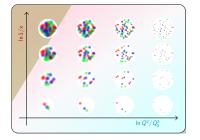
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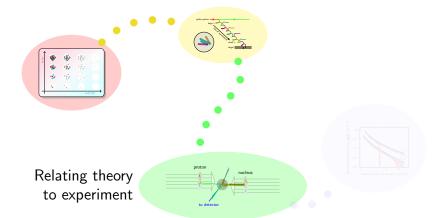


BK predicts slower growth
This is called **saturation**Details depend on parameters in the equation

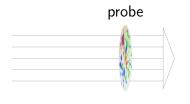


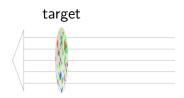


Outline



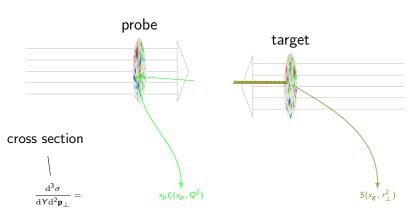


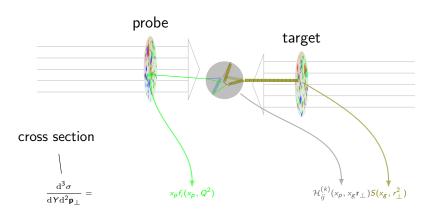


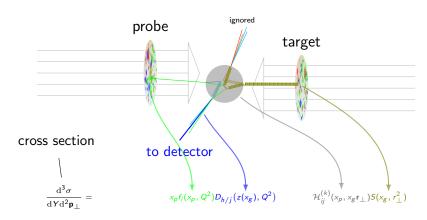


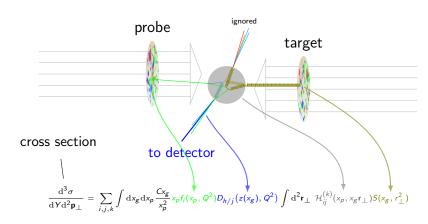
cross section

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Y\mathrm{d}^2\mathbf{p}_{\perp}} =$$









$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Y\mathrm{d}^2p_\perp} = \sum_{i,i,k} \int \mathrm{d}x_g \, \mathrm{d}x_p \frac{\mathrm{C}x_g}{x_p^2} \, x_p f_i(x_p,\,Q^2) D_{h/j} \big(z(x_g),\,Q^2 \big) \int \mathrm{d}^2r_\perp \,\, \mathcal{H}^{(k)}_{ij}(x_p,x_gr_\perp) S(x_g\,,r_\perp^2) \, d^2r_\perp \, \mathcal{H}^{(k)}_{ij}(x_p,x_gr_\perp) S(x_g\,,r_\perp^2) \, d^2r_\perp \,\, \mathcal{H}^{(k)}_{ij}(x_p,x_gr_\perp) S(x_g\,,r_\perp^2) \, d^2r_\perp \, \mathcal{H}^{(k)}_{ij}(x_p,x_gr_\perp) S(x_g\,,r_\perp^2) \, d^2r_\perp \,\, \mathcal{H}^{(k)}_{ij}(x_p,x_gr_\perp) \, d^2r_\perp \,\, \mathcal{H}^{(k)}_{ij}(x_p,x_gr_\perp) \, d^$$

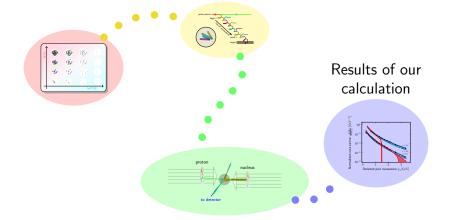
Leading order:

Next to leading order:

$$\mathcal{H}_{qq}^{(0)} \leftrightarrow \qquad \qquad \mathcal{H}_{qq}^{(1)} \leftrightarrow \qquad + \qquad \qquad + \qquad$$

But are these contributions enough?

Outline





Determining the gluon distribution

- Postulate a form for gluon distribution $S(x, r^2)$ using e.g. BK equation (section 2)
- Plug in to cross section formula (section 3)
- Compare results to data

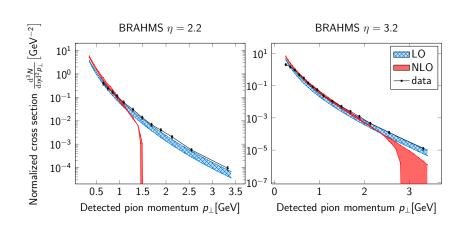
Two reasons it might not match:

- Proposed model of gluon distribution is not accurate; or
- Cross section formula is not precise enough



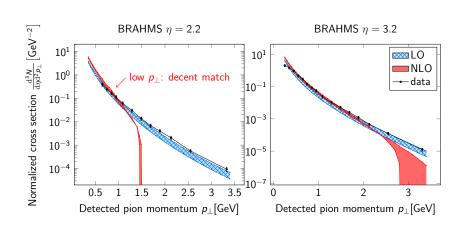
Comparison with data

Best available: from BRAHMS @ RHIC (Brookhaven)



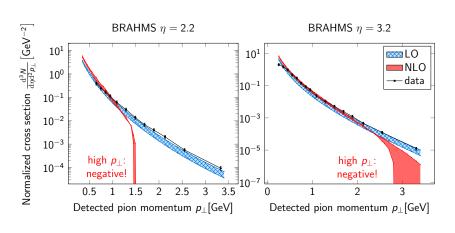
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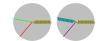
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Strongly suggests the LO+NLO terms are insufficient!

What next?

If the leading order



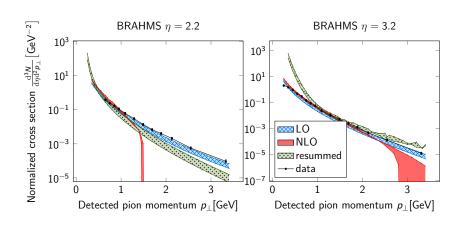
and next to leading order terms



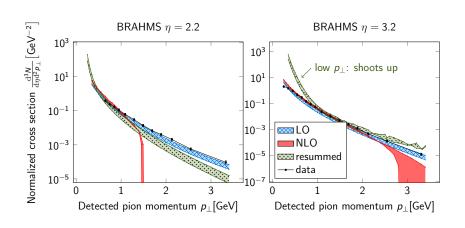
don't work, look for largest terms at higher orders.



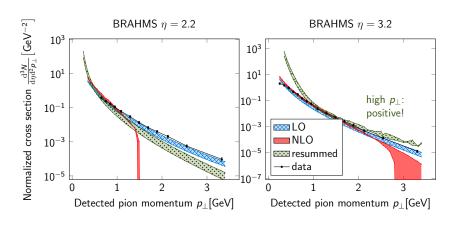
Simplest resummation scheme:



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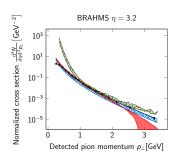


This is promising! But needs better tuning.

What does this tell us?

Three possible conclusions:

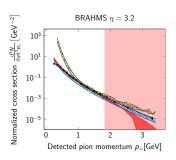
- Cross section formula is not precise enough
- Model of gluon distribution is not accurate, or
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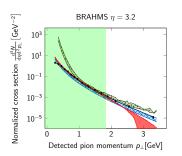
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NLO formula is useful for looking for saturation

Acknowledgments

My collaborators:

- Anna Sta
 śto
- Bowen Xiao
- Feng Yuan

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- Milton Cole
- Andrew Magyar
- Jason Wittenbach
- Kathryne Sparks Woodle

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- Milton Cole
- Stéphane Coutu
- Jorge Sofo
- Anna Sta
 śto
- Mauricio Terrones

And YOU

for staying awake

