**Supplemental Information for**

# Evaluating the Transport behavior of $CO_2$ Foam in the Presence of Crude Oil Under High-Temperature and High-Salinity Conditions for Carbonate Reservoirs

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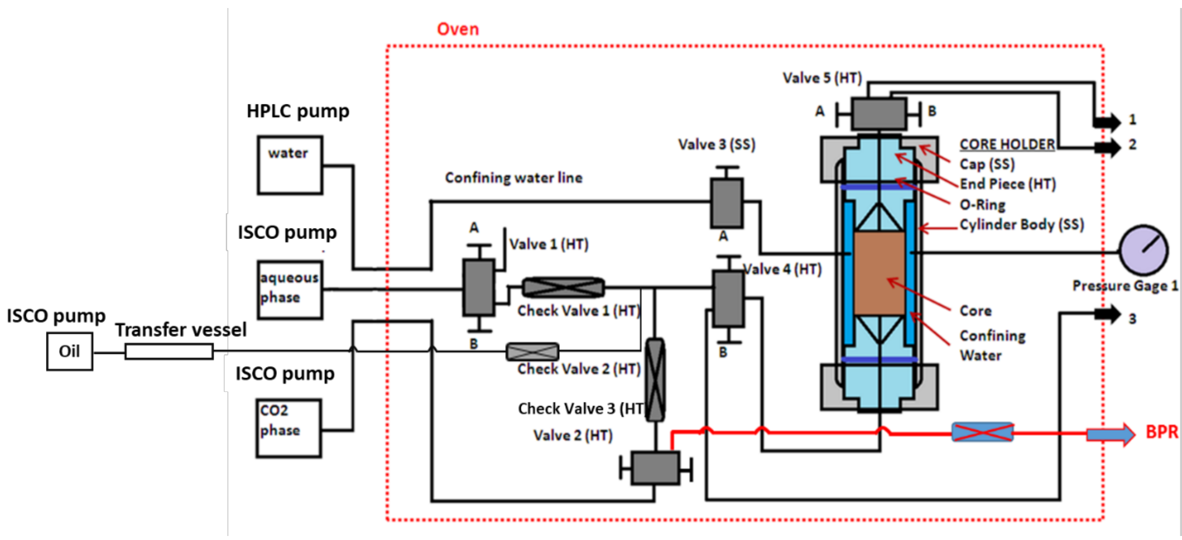


**Figure S1. Schematic of the pump system and core holder module used in the core-flooding setup.**

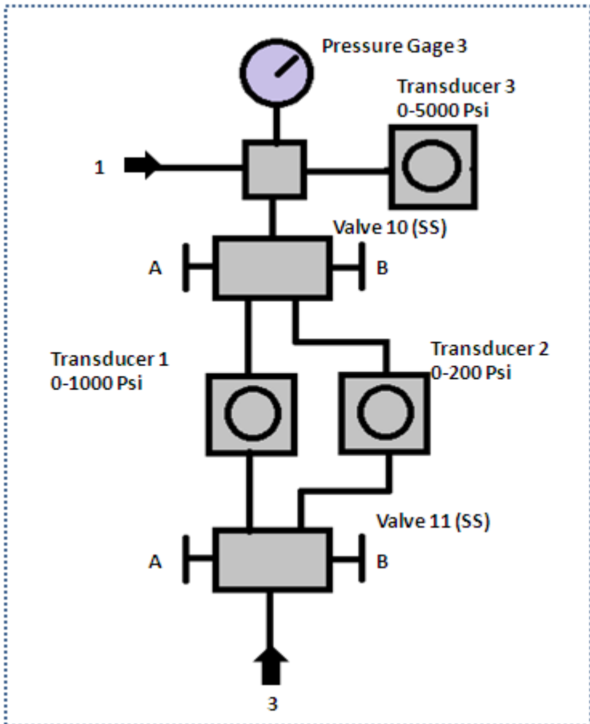


**Figure S2. Detailed schematic of the pressure transducer module.**

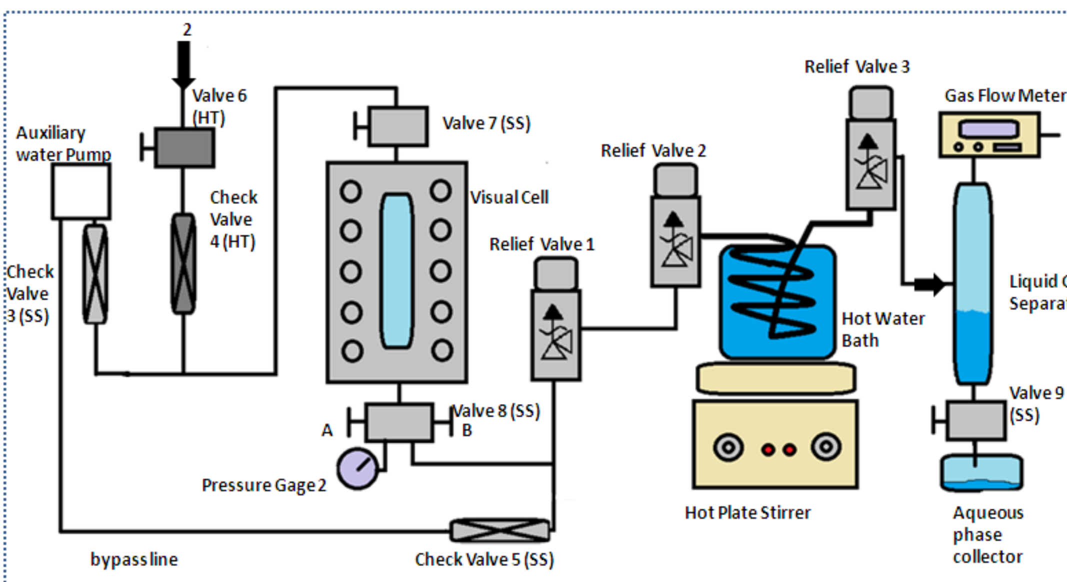


**Figure S3. Detailed schematic of the back pressure regulator (BPR) module.**

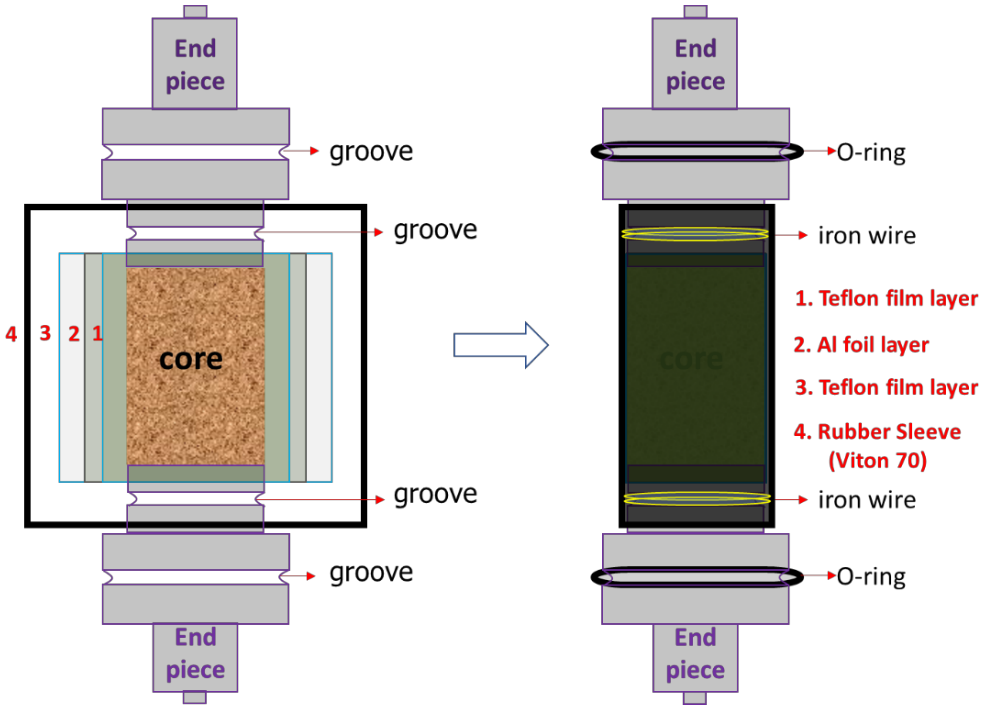


**Figure S4. Schematic of the multi-layer core assembly used to prepare the core for supercritical $CO_2$ flooding.**

| Relative Permeability | Value |
|---|---|
| $k_{rw}^{o}$ | 1 |
| $k_{rg}^{o}$ | 0.1768 |
| $S_{wc}$ | 0.33 |
| $S_{gr}$ | 0 |
| $n_w$ | 2.8 |
| $n_g$ | 1.1 |

**Table S1. Relative permeability (water/$CO_2$) values[1] used for STARS foam model parameters estimation.**

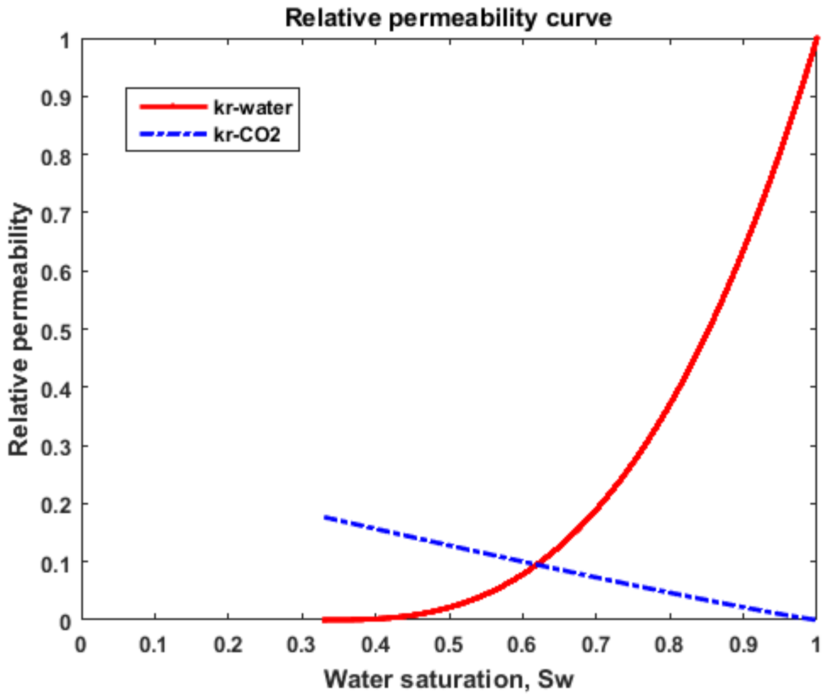


**Figure S5. Relative permeability (water/$CO_2$) curve used for STARS foam model parameters estimation**

Based on Darcy's law, the viscosity of the gas phase and aqueous phase can be expressed by equations shown by **Eq. S1** and **Eq. S2**. The apparent viscosity of foam is expressed calculated using **Eq. S3**. By combining **Eq. S1** and **Eq. S2**, the relative permeability of the aqueous phase can be expressed as shown in **Eq. S4**.

$$\mu_g = -\frac{k_{rock} * k_{rg}^{f}}{u_g} * \nabla P = -\frac{k_{rock} * k_{rg}^{nf} * FM}{u_g} * \nabla P \quad \textbf{(Eq. S1)}$$

**Darcy's law for gas phase**

$$\mu_w = -\frac{k_{rock} * k_{rw}}{u_w} * \nabla \mathrm{P} \quad \textbf{(Eq. S2)}$$

**Darcy's law for the aqueous phase**

$$\mu_{foam} = -\frac{k_{rock}}{(u_w + u_g)} * \nabla \mathrm{P} \quad \textbf{(Eq. S3)}$$

**Darcy's law for foam**

$$k_{rw} = -\frac{\mu_w * u_w}{k_{rock} * \nabla P} = \frac{\mu_w * u_w}{\mu_{foam} * (u_w + u_g)} = \frac{\mu_w}{\mu_{foam}} * \left(1 - f_g\right) \quad \textbf{(Eq. S4)}$$

**Expression of relative permeability of aqueous phase based on Darcy's law**

Based on Corey's model, the relative permeability of the gas phase and the aqueous phase are expressed by **Eq. S5** and **Eq. S6**.

$$k_{rw} = k_{rw}^0 * \left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{gr}}\right)^{n_w} \quad \textbf{(Eq. S5)}$$

**Expression of relative permeability of aqueous phase based on Corey's model**

$$k_{rg} = k_{rg}^0 * \left(\frac{1 - S_w - S_{gr}}{1 - S_{wc} - S_{gr}}\right)^{n_g} \quad \textbf{(Eq. S6)}$$

**Expression of relative permeability of gaseous phase based on Corey's model**

By combining **Eq. S4** and **Eq. S5**, the saturation of water could be expressed as shown in **Eq. S7**.

$$S_w = \left(\frac{\mu_w}{\mu_{foam} * k_{rw}^0} * \left(1 - f_g\right)\right)^{\frac{1}{n_w}} * \left(1 - S_{wc} - S_{gr}\right) + S_{wc} \quad \textbf{(Eq. S7)}$$

**Expression of water saturation**

Once the water saturation is obtained, the value of $FM$ at each fractional flow can be calculated using experiment data $\left(f_g , \mu_{foam}\right)$ from the foam quality scan as shown in **Eq. S8**.

$$FM_{exp} = \frac{f_g * \mu_g}{\mu_{foam} * k_{rg}^0 * \left(\frac{1 - S_w - S_{gr}}{1 - S_{wc} - S_{gr}}\right)^{n_g}} \quad \textbf{(Eq. S8)}$$

**Expression of FM based on experimental data**

$$FM_{model} = \frac{1}{1 + fmmob * F_{water}} = \frac{1}{1 + fmmob * \{0.5 + \frac{arctan[epdry * (S_w - fmdry)]}{\pi}\}} \quad \textbf{(Eq. S9)}$$

**Expression of FM in the STARS foam model**

The dry out function parameters, $epdry$ and $fmdry$ shown in **Eq. S9**, can be uniquely determined by performing a linear regression for different values of $fmmob$ as shown by

**Eq. S10**[2]. Then, by performing a value scan of $fmmob$, the final $fmmob$ can be determined through the minimization of the objective function as shown in **Eq. S11**.

$$y(FM_{exp}, fmmob) = tan\left(\left(\frac{\left(1/FM_{exp}\right)-1}{fmmob} - 0.5\right)\pi\right) = epdry(S_w - fmdry) \quad \textbf{(Eq. S10)}$$

**Formula for determining $epdry$ and $fmdry$ at each $fmmob$**

$$\min f(fmmob) = \left\{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\mu_{app,i}^{exp}-\mu_{app,i}^{STARS}}{\mu_{app,imax}^{exp}}\right)^2 + P * \left(\frac{f_{g,imax}^{exp}-f_{g,transition}^{STARS}}{f_{g,imax}^{exp}}\right)^2\right\} \quad \textbf{(Eq. S11)}$$

$$P = \begin{cases} 0 & f_{g,imax-1}^{exp} < f_{g,transition}^{STARS} < f_{g,imax+1}^{exp} \\ 1 & Otherwise \end{cases}$$

**Objective functions for $fmmob$ obtimization**

For estimating the shear thinning parameters, $fmcap$ and $epcap$, a linear regression using $fmmob$ and the $Fwater$ values at a specific flow rate can be performed, as shown in **Eq. S12** and **Eq. S13**. The same concept can be used to determine the surfactant concentration dependent function, as shown in **Eq. S14** and **Eq. S15**.

$$FM = \frac{1}{1+fmmob*F_{water}*\left(\frac{fmcap}{N_{Ca}}\right)^{epcap}} \quad \textbf{(Eq. S12)}$$

**Expression of FM regarding dry out and shear thinning functions**

$$y(FM_{exp}) = log_{10}\left(\frac{\left(1/FM_{exp}\right)-1}{fmmob*F_{water}\left(N_{Ca}(S_w)\right)}\right) = -epcap * \left(log_{10}(N_{Ca}) - log_{10}(fmcap)\right) \quad \textbf{(Eq. S13)}$$

**Expression of linear regression form for determining the $fmcap$ and $epcap$**

$$FM = \frac{1}{1+fmmob*F_{water}*\left(\frac{C_{SW}}{fmsurf}\right)^{epsurf}} \quad \textbf{(Eq. S14)}$$

**Expression of FM regarding dry out and shear thinning functions**

$$y(FM_{exp}) = log_{10}\left(\frac{\left(1/_{FM_{exp}}\right)-1}{fmmob*F_{water}(C_{sw}(S_w))}\right) = epsurf * \left(log_{10}(C_{sw}) - log_{10}(fmsurf)\right) \quad \textbf{(Eq. S15)}$$

**Expression of linear regression form for determining the $fmsurf$ and $epsurf$**

**References**


(1) Bennion, B.; Bachu, S. Drainage and Imbibition Relative Permeability Relationships for Supercritical CO2/Brine and H2S/Brine Systems in Intergranular Sandstone, Carbonate, Shale, and Anhydrite Rocks. *SPE Reservoir Evaluation & Engineering* **2008**, *11* (03), 487–496. https://doi.org/10.2118/99326-PA.

(2) Zeng, Y.; Muthuswamy, A.; Ma, K.; Wang, L.; Farajzadeh, R.; Puerto, M.; Vincent-Bonnieu, S.; Eftekhari, A. A.; Wang, Y.; Da, C.; et al. Insights on Foam Transport from a Texture-Implicit Local-Equilibrium Model with an Improved Parameter Estimation Algorithm. *Ind. Eng. Chem. Res.* **2016**, *55* (28), 7819–7829. https://doi.org/10.1021/acs.iecr.6b01424.