Solving a Spatial Evolutionary Game in Prostate Cancer

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Linking structure and evolution

Cancer cell phenotypes vary in space; spatial evolution can explain this

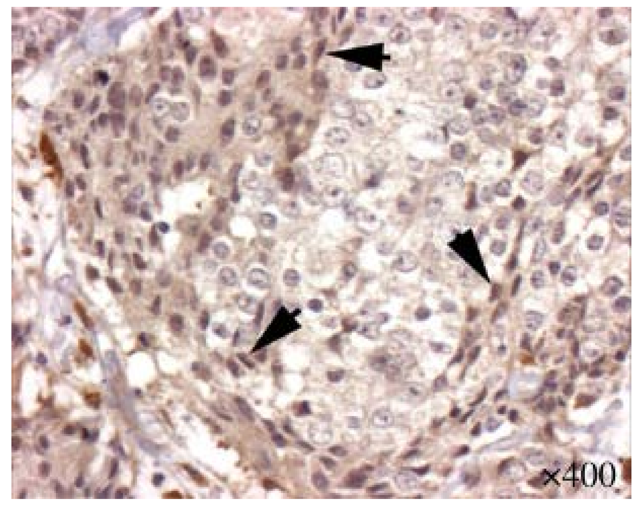


Image from (3)

Our model provides a pithy presentation for a complex relationship between spatial structure and evolution

Evolutionary games model tumor development

Evolutionary game(1) between stromal cells (S), stromal dependent prostate cancer cells (D), and stromal independent prostate cancer cells (I)

$$\begin{array}{c} \\ S \\ D \\ I \end{array}\begin{array}{c} \begin{array}{ccc} S & D & I \end{array} \\ \begin{pmatrix} 0 & \alpha & 0 \\ 1+\alpha-\beta & 1-2\beta & 1-\beta+\rho \\ 1-\gamma & 1-\gamma & 1-\gamma \end{pmatrix} \end{array}$$

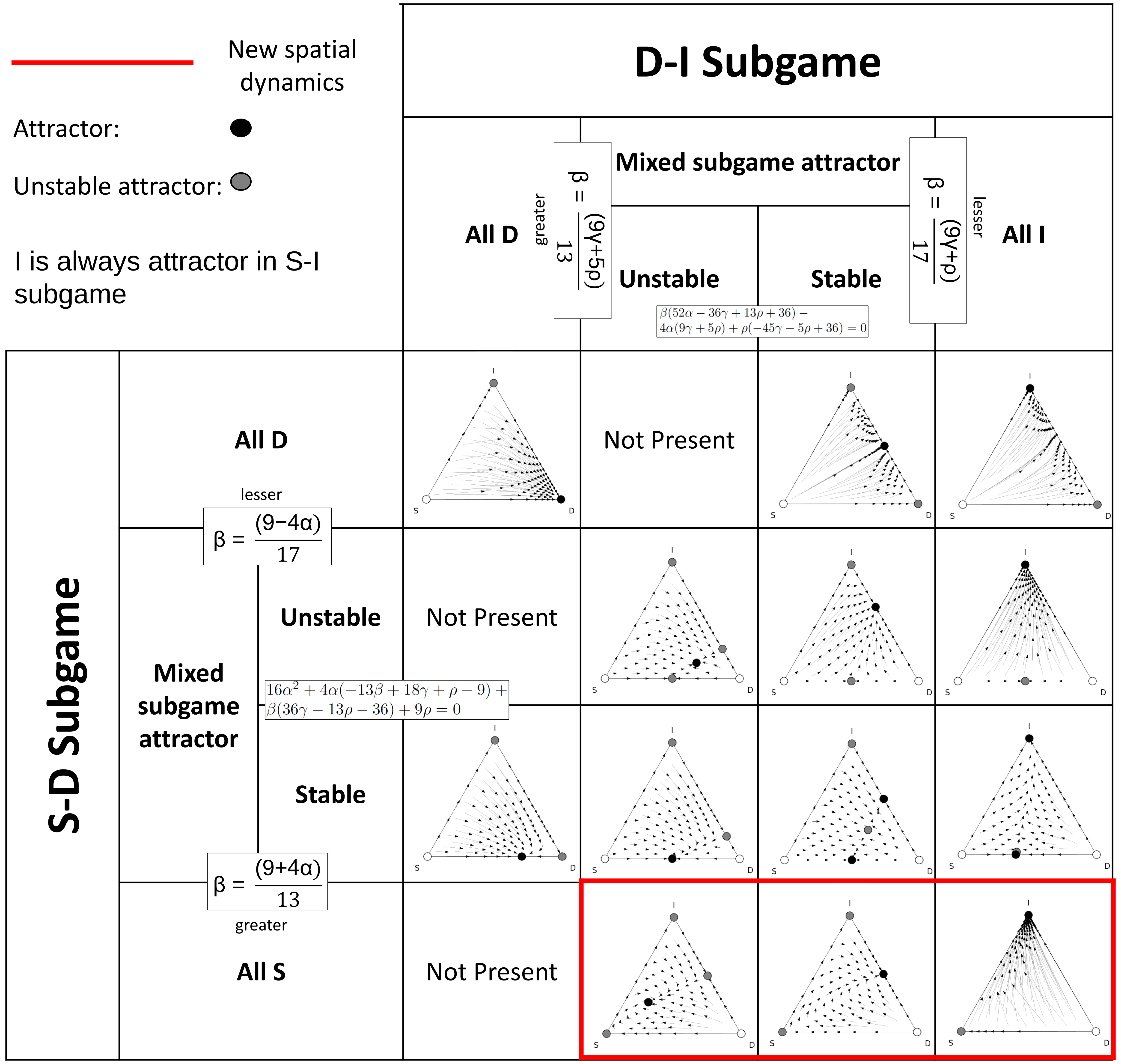
α = benefit derived by stromal cells from dependent cells

β = cost of being dependent

ρ = benefit derived by dependent cells from independent cells

γ = cost of being independent

Space allows new population dynamics



Space reduces interactions between individual cells

ON(2,3) transform with the largest effect of space: 3 interactions (k = 3) with death-birth dynamics.

$$ON_k = A + \frac{1}{k-2}(\Delta 1^T - 1\Delta^T) + \frac{1}{(k+1)(k-2)}(A - A^T)$$

In original game every cell interacts with every other cell in the total population; spatial game each cell interacts with exactly 3 other cells

$$\begin{pmatrix} 0 & -\frac{5}{4}+\alpha+\frac{9\beta}{4} & \frac{5}{4}(1-\gamma) \\ \frac{9}{4}+\alpha-\frac{13\beta}{4} & 1-2\beta & \frac{1}{4}(4-13\beta+5\gamma+5\rho) \\ -\frac{9}{4}(1-\gamma) & \frac{1}{4}(4+9\beta-9\gamma-\rho) & 1-\gamma \end{pmatrix}$$

Subgames give intuitive solutions

$$\begin{pmatrix} \boxed{\begin{matrix} 0 & \alpha \\ 1+\alpha-\beta & 1-2\beta \end{matrix}} & \begin{matrix} 0 \\ 1-\beta+\rho \end{matrix} \\ \begin{matrix} 1-\gamma & 1-\gamma \end{matrix} & 1-\gamma \end{pmatrix}$$

Subgame

$$\begin{array}{c} \\ S \\ D \end{array}\begin{array}{c} \begin{array}{cc} S & D \end{array} \\ \begin{pmatrix} 0 & \alpha \\ 1+\alpha-\beta & 1-2\beta \end{pmatrix} \end{array}$$

$(1+\alpha-\beta)-(0) > 0$ D invades S

$(\alpha) - (1-2\beta) > 0$ S invades D

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References

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2. Ohtsuki H, Nowak MA. The replicator equation on graphs. J Theor Biol 2006; 243(1): 86-97.
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