# (Square-)Rooting a spiral: The use of another geometric element in the Rosette Map of the Voynich Manuscript with square roots, golden triangles and spirals

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A recent paper on the geometry of f86v ('Rosette map') of the Voynich manuscript suggested the construction of the basic scaffold of the nine discs via multiple use of the golden ratio with geometric ease and precision (1). However, the Rosette map contains more than just a scaffold and this paper will highlight the use of geometry in a supposedly late stage of the production of this particular folio 86v. Basically, three levels or layers can be distinguished:

1) the *scaffold* (nine discs),
2) the *ornaments* (e.g. links and connections between these discs and the images and other disc content) and
3) the *text and writing* (e.g. on the disc circumference, within the discs and outside).

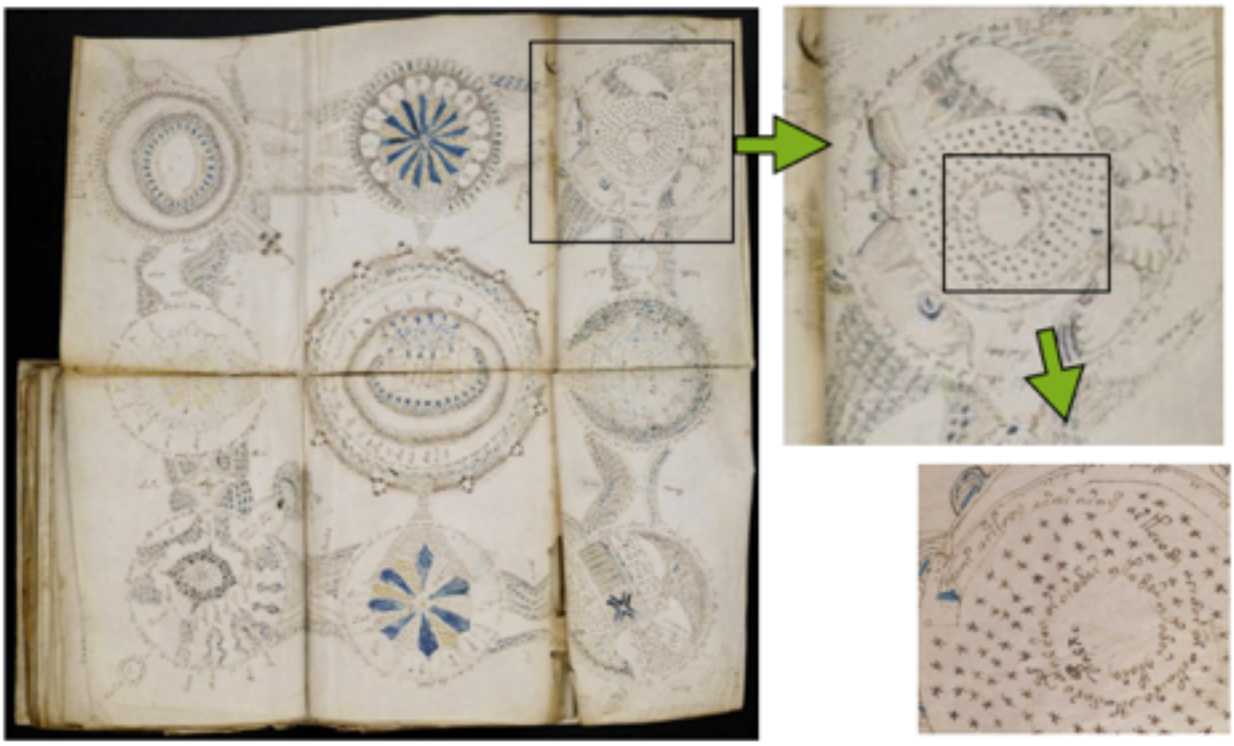
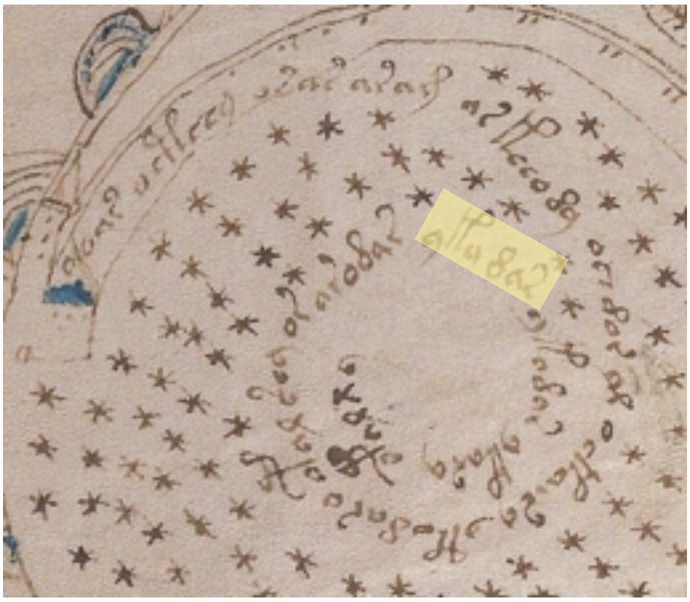


**Figure 1: f86v ('Rosette Map', 2) overview, detailed view of sphere containing a text spiral and a detail view of the text spiral (flipped horizontally by 180 degrees). The text spiral begins at the roof of a supposedly towered building.**

Text (and its arrangement) in the Rosette map is often ‘different’ to other folios: This is due to the structure of this folio: Writing (of individual words) occurs in clockwise direction, blocks of texts e.g. in the bottom right disc with imagery indicative of paradise-location (3) or circular inscriptions in all discs but the central one. A particular intriguing writing is found in the top right disc (representing Europe & cold climate according to the suggestion in [3]) where a text is arranged in a spiral (Figure 1). The fact that the text was written before final ornaments were added (star-like features) is visible by the spread of the ‘field’ and flow of stars surrounding the spiral text in the detailed view (Figure 1, Figure 2) where their arrangement of the asterisks follows around the spiral text.



EVA:
o?oas.oetccy.osas.aram.askccody.
ocidor.ol.ockaisy.ytodaro.opalccy.
or.arodar.ykadas.ykodar.ykary.
opvlxy

2a detail text spiral f86v

2b f90r single hit *ykodar*

**Figure 2: 2a) close-up view of the text spiral in the Rosette Map and, based on the EVA transcription, a search at [http://www.voynichese.com](http://www.voynichese.com) reveals a single hit for a transcript with f90r (2b)**

EVA transcription for this text spiral wasn’t available and therefore it is not part of the http://www.voynichese.com search tool (4). Here the transcription for the text spiral is provided via the EVA alphabet (based on [www.voynich.nu/extra/eva.html](www.voynich.nu/extra/eva.html) by Rene Zandbergen and Gabriel Landini (5) :

> EVA: o?oas.oetccy.osas.aram.askccody.ocidor.ol.ockaisy.
> ytodaro.opalccy.or.arodar.ykadas.ykodar.ykary.opvlxy

Analysis of this transcription (word by word) via [http://www.voynichese.com](http://www.voynichese.com) (which does not include the Rosette Map’s EVA transcription) reveals a single hit for the individual transcript of *ykodar* (highlighted in yellow, Figure 2a and 2b), in folio90r in the botanical section. With the exception of one more common word *aram* (54 matches in 33 folios) no direct matches could be retrieved (NB: Two-letter words were not searched or analysed with this tool). The potential content of the text spiral will be discussed later.

This spiral is the single instance of text arranged as such in the entire Voynich manuscript. Now the question arises, if this arrangement of the text as a spiral is random or deliberate. The use of the golden ratio in the establishment of the ***imagery*** of the Rosette map was suggested previously (1). It is unknown, however, if geometric knowledge was used to inform the arrangement of ***text*** in the Rosette map (or the Voynich manuscript in general): In an experiment with overlays using spirals of different origin this paper is going to highlight if, and if so, which geometry was used in the arrangement of the text.

**Another experiment: A geometric exercise with spirals**

Based on the recent findings and methodology (1) a selection of spirals (figure 3) , whose layouts are based on either golden triangle (6) or golden rectangle (7), were first examples for testing if geometry played a role in arranging this text string in form of a spiral. In addition, other layouts, e.g. the tetrahedron based spiral (8) and the ‘Spiral of Theodorus’ (9) were selected for overlay experiments. Other form of spirals were tested, however are not depicted here (amongst these are the Archimedean Spiral and the reciprocal spiral).

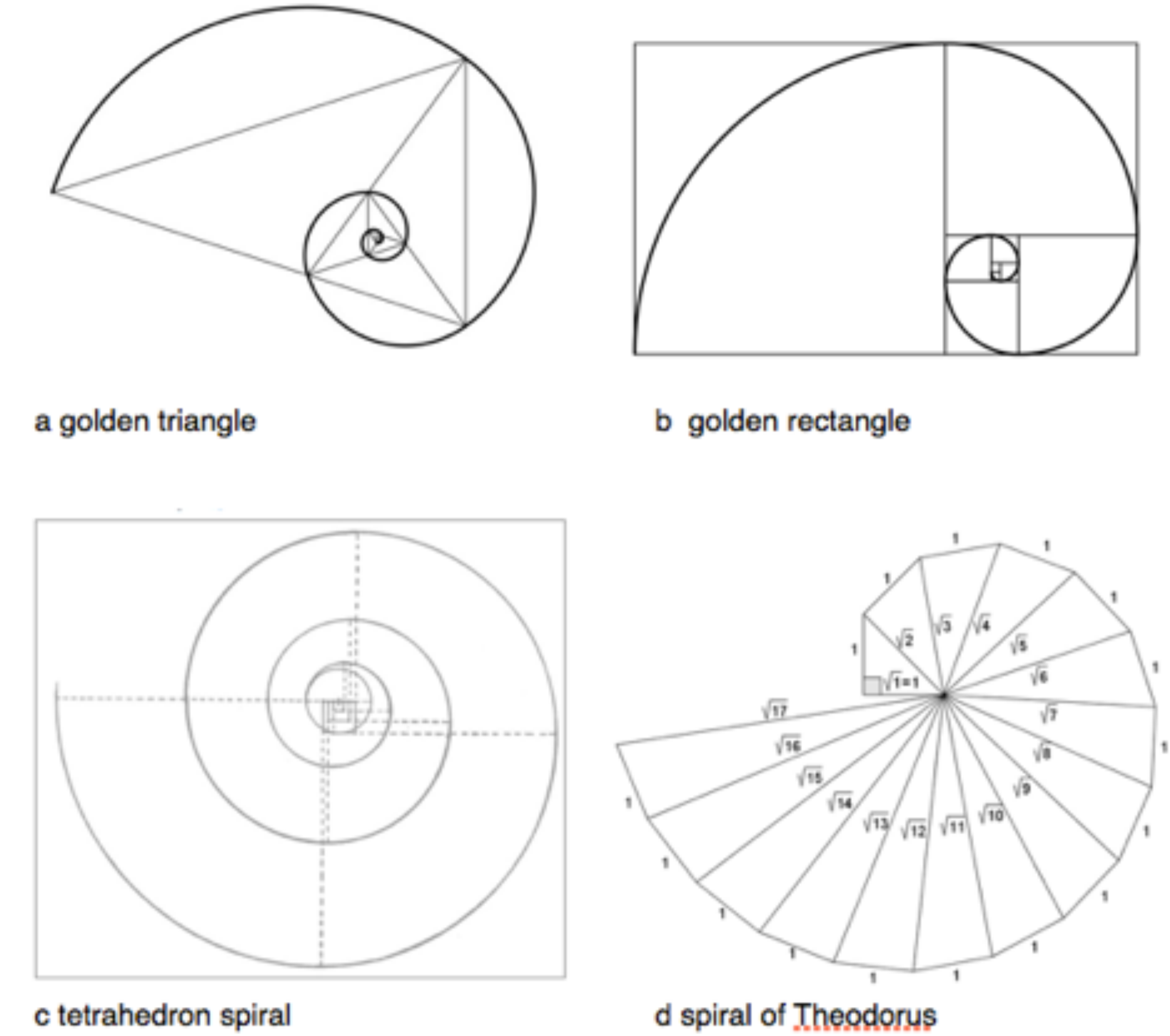
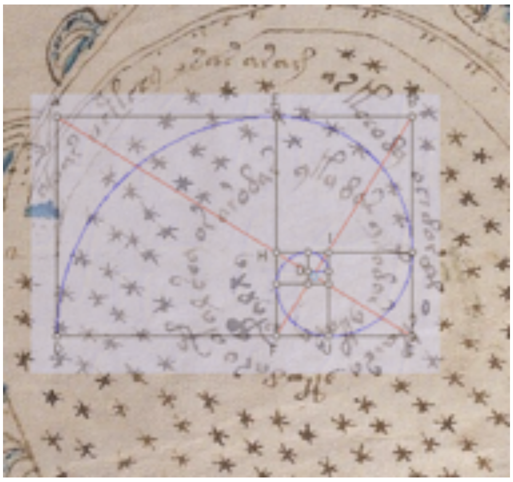



**Figure 3: Spirals used for overlay with text spiral in f86v; see text for details; for all experiments spirals were kept with constant properties in the overlay process to avoid distortions**

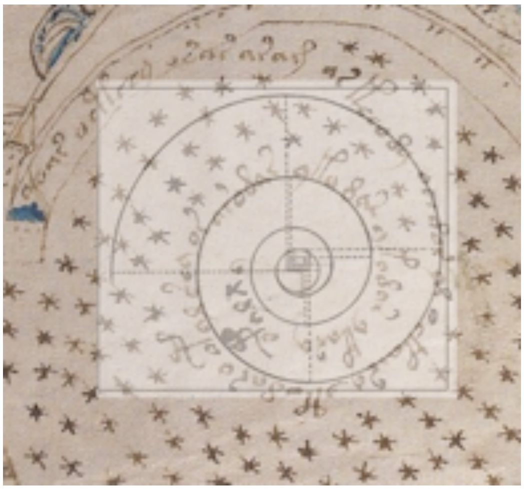
The construction of the spiral of Theodorus is available from multiple sources. However, a description is quoted here that also contains references to golden rectangle and Pythagoras theorem (10):

'*The Wheel of Theodorus begins with a simple right isosceles triangle of base and height 1 unit. Let's call this figure Stage 0. To construct Stage 1, build a second right triangle of height 1 unit whose base is the hypotenuse of the triangle constructed in Stage 0. (consider defining the terms right triangle, base, height, hypotenuse). The design continues in this way, with each newly constructed right triangle having a height of 1 unit and a base of the previous stage's hypotenuse. Over time, a spiral will begin to emerge. This spiral also has significance in mathematics, as it can be enclosed by a rectangle that is known as the "Golden Rectangle". An interesting feature of the Wheel of Theodorus is that although it is a fully unified figure from Stage n to Stage n+1, each new stage reveals a hypotenuse which is either rational or irrational in length. This can be shown using the Pythagorean Theorem.*'

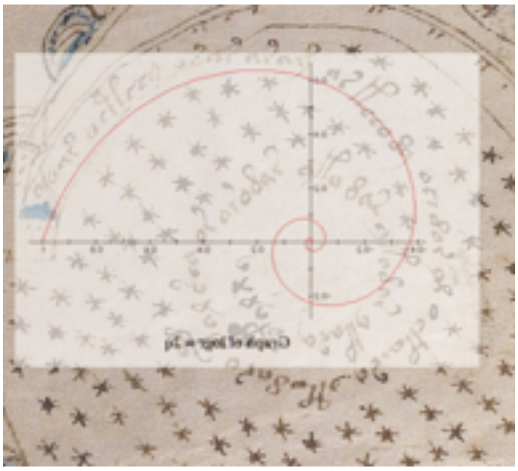
a) overlay based on golden triangle



b) overlay based on golden rectangle



c) overlay tetrahedral spiral



d) logr =2q

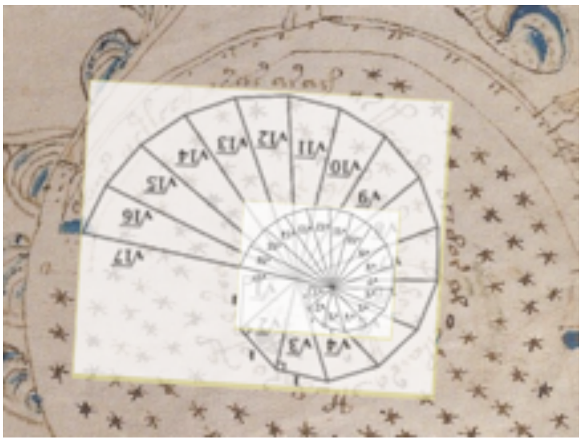
**Figure 4: Various spiral overlays with the text spiral in f86v**

Figure 4 shows the initial results of the overlay experiment with a selection of spirals. Overlay experiments were carried out using spirals, whose proportions were kept constant and constraint throughout the experimental overlay. A first fit was approached using the centre of the adjacent building's 'blue roof' next to the spiral - cf figure 2 and 4 for details - this point marks seemingly the end or starting point of the text spiral. Applying this alignment to have identical starting points (text spiral and overlay), the overlay was adjusted to fit the initial outer bend (see Figure 4a in case of the golden triangle based spiral). Additionally, a fit starting from the inner section was carried out fitting from within the text spiral and analyse the curvature to the opposite end of the spiral. In all 4 cases (Figure 4a - d) no satisfactory approximation beyond an initial fit (where the spiral starts) was noted. Partial fits to the spirals could be observed, however no overall fit seemed possible.

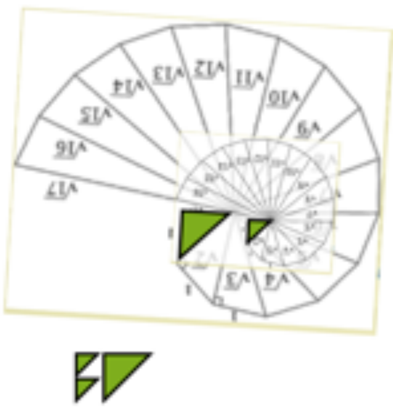
Figure 5 displays the outer and inner fit with the Spiral of Theodorus: In both cases the approximation fits rather well throughout the entire spiral's turn (Figure 5 a, b). Overlaying both approximations reveals an interesting fact: Inner and outer fit results in a parallel arrangement of the spirals (Figure 5c). Next to the parallel layout, an interesting fact was noticed: The core triangle, a simple right isosceles triangle of base and height 1 unit (that starts the spiral construction) of the outer and inner overlay are in ratio 2:1 (Figure 5d).

a) single overlay large spiral (outer bend)

b) single overlay small spiral (inner bend)



c) Overlay combined



d) spirals, overlay and comparative positioning of the core unit triangle of base and height 1 ; ratio 2:1

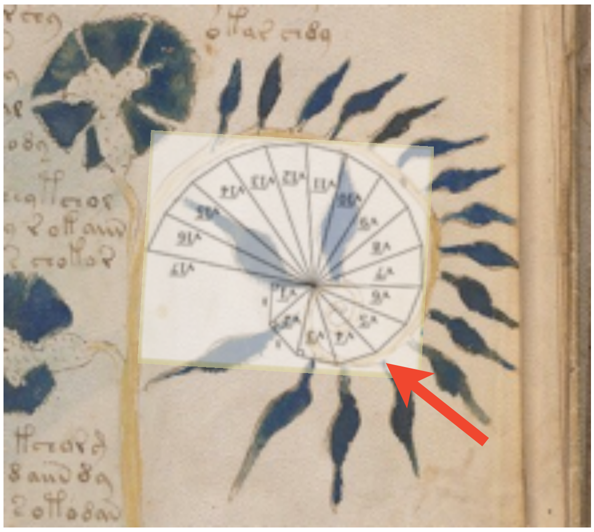
**Figure 5 Spiral of Theodorus, single overlay, double overlay and ratio**

No helplines that could indicate a construction process were identified, although these could have been easily erased after construction. An alternative could have been the use of a stencil that aided the construction of this text spiral. Could such cut pattern be used elsewhere in the manuscript? The discussion below will elaborate on this topic further.

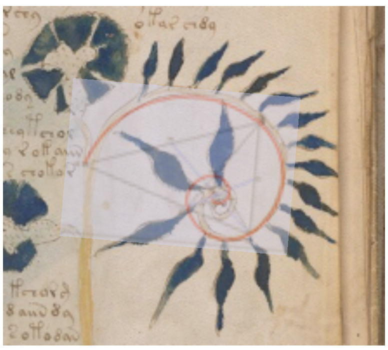
**Other square root spirals in the VM?**

As described above, the investigated text spiral in the Rosette map is the only one where the spiral is based on text and not a line as part of an image or picture. Line-based spirals (particularly in the images of plants in the botanical section) exist. These could be based on a construction process, too. A particular good example is f56r (Figure 6). This image has already been discussed in connection with a spiral: Nick Pelling discussed an inverse hyperbolic spiral (10). Overlay with spirals based on golden triangle, rectangle and Spiral of Theodorus (others not shown) is shown in Figure 6c and 6d, too.

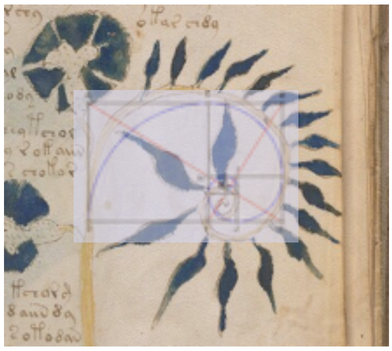
6a: small, inner Theodorus spiral



6b: larger, outer Theodorus spiral



6c: golden triangle



6d: golden rectangle

6e: f56r detail

**Figure 6**

Interesting in this case is the fact that the ‘inner’ (smaller) Theodorus’ spiral seems to have been used ‘completely’ and the outer (larger) one has been followed from sqrt(4) onwards (see Figure 6b, indicated with an arrow). Interestingly, as was the case for the text spiral in the Rosette map, both spirals of Theodorus - once overlayed - are aligned in parallel (see

Figure 5c,d), however the orientation is flipped by 180 degree indicating that here a use of two stencils could be an easy explanation: One for a small spiral, the other for the large spiral of Theodorus.
Spirals based on the golden rectangle and golden triangle seem to fit initially well (better than with the text spiral, see Figure 4), however closer inspection reveals a less accurate fit than with the spiral of Theodorus.

**Discussion**

This paper shows for the first time, that it is possible, and likely, that geometric knowledge has been used in aligning text in f86v in form of a spiral. This may be even a first not just for the Rosette map and the Voynich manuscript, but for medieval manuscripts in general. This use of geometry in artistic display is next to the use of the geometric alignment of the image's scaffold as shown previously (1). This may hint at the fact that the writing and the map construction were carried out by someone very knowledgable in geometry, using the **arrangement of text to display as image** (spiral in this case). The sophisticated use of geometry in the alignment of imagery and text may be an indicator that the all the map's details were made in one process (image, ornaments and writing) and were meant to be 'together'. Assuming this to be true, this could contradict Nick Pelling's idea of a several stage process of the Rosette maps creation (12) although there are other characteristics (to be published) that could highlight a several step process.

The overlay tests with several spirals (Fig. 4-6) provide a valid comparison and alignment with the different types of spirals. Spirals based on golden triangle and golden rectangle don't fit in either setting (starting with the outer [= bigger] or inner [= smaller] bend) and only Theodorus' spiral, in their full extent up to n=sqrt(17), approximates the curvy text near to perfection. In order to approximate the entire text spiral two separate & independent Theodorus' spirals are necessary.
The overlay with two Theodorus spirals makes sense:

- two independent, complete (up to n=sqrt(17)) but differently sized Theodorus spirals in combination provide a perfect fit
- use and/or construction of an extended Theodorus spiral with n> sqrt(17) was not known in medieval ages (i.e. no known depiction is available). The 'natural' border is at the triangle with hypotenuse length sqrt(17). Construction of such an

extended spiral is possible (12) see Figure 7 and such construction with n> sqrt(17) was not proven until 1958 (13).

- Overlay of such an extended Theodorus spiral beyond sqrt(17) does not yield a good match at all (overlay not shown)

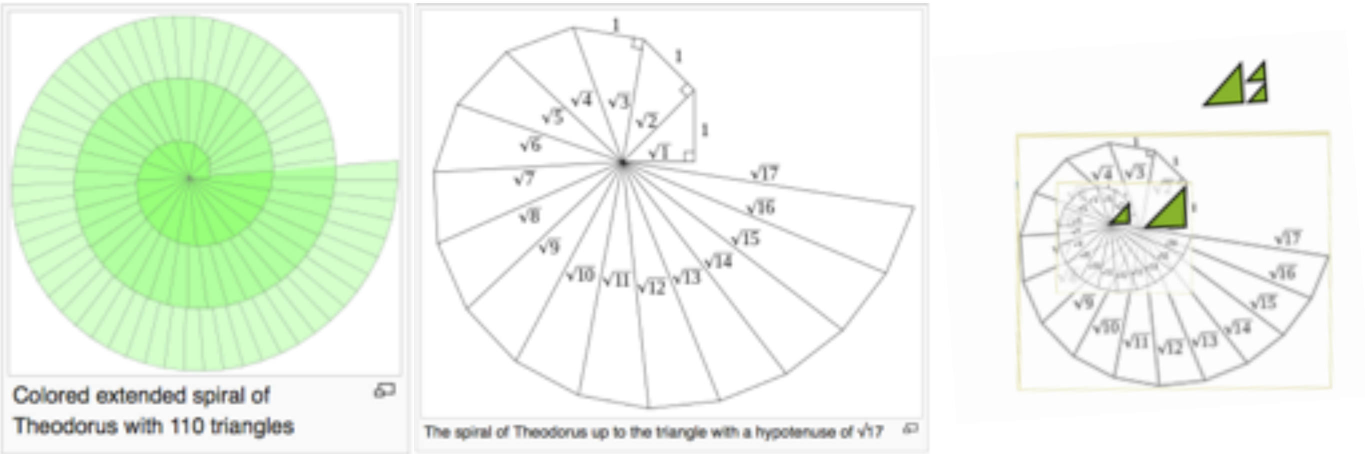



**Figure 7: Extended spiral versus single spiral of Theodorus (12). Suggested use for the Rosette map based on overlay presented in this paper (Fig 5) for comparison.**

A comparison of both these spirals of Theodorus for this text alignment reveals further links: Although no obvious use of a golden ratio (between these two spirals) and different centre points for constructing the spiral, both spirals' baseline (right isosceles triangle in the core of each spiral) are aligned in parallel, which seems deliberately chosen and the triangle 'core' units (to produce the spiral) have the ratio 2:1.

The question remains why two spirals (and not just one or a higher number of spirals) have been chosen: The most obvious reason may lay in the length of text to be written in such manner. The need for two spirals also arose due to the fact that an extended spiral beyond sqrt(17), the natural end, was not known to the map maker.

**Why using a spiral? Hint on location or text?**

The 'end' of the outer part of the text spiral links to what seems like a bent tower and building (Fig. 1 and 2, inset to the right) . The 'bentness' in association with the special sequence of numbers via the spiral led to a first suggestion that the location could have been Pisa: The leaning tower (as *pars-pro-toto* for the Pisa Cathedral Square- cf with 14) was known to be leaning since beginning of its construction late in the 12th century and, after a construction halted for 100 years it then was being resumed and finished after another 100 years with the belltower in 1372) and Fibonacci's *liber abaci* were well known.

However, the geometric implementation (and near perfect execution) of the spiral may rather hint at a link to Theodorus (of Cyrene), a mathematician, who lived and worked most of his life in Athens. Could either of these cities be presented by the building works at the beginning of the text spiral? The latter location would coincide with the suggested continental location by Wastl & Feger (3), whereas the former (Cyrene, North Africa) would be incompatible with an "European' disc identification. If it fits another geographic map as identified by Diane O'Donavan can't be fully excluded or confirmed (a full overview of her work on this particular building was not retrieved or available while writing this manuscript - although based on one of her summaries on her blog (15), this tower/building hasn't been discussed yet and both options would look unlikely: D. O'Donavan has an Asian/Turkish location (Cappadocia) close to the start of the spiral.
Understanding the text will reveal if a potential location is depicted. The findings in this manuscript - i.e. the link of the text spiral to either names of mathematicians or famous places, where these resided in - could possibly aid the deciphering of the text in the spiral alignment- a link between the meaning of the text and its visual depiction as spiral seems likely hence names of places and/or people (in this or the next paragraph) could possibly be used to identify the text sequence in the spiral.

Theodorus' works didn't survive (only indirectly and partially - the few surviving quotes of his works are by his pupil Plato), however, the ancient historian Proclus strongly linked him to his teacher Protagoras, and even linked one of Euclid's Elements to descend from Theodorus. The Element that Proclus links to Theodurus is **the golden ratio** (16).
The spiral of Theodorus is one of many, possibly one of the most famous, visualisation of the Phytagorean theorem (17) allowing for the construction of the square root of any integer starting with a right isosceles triangle of base and height of 1 unit (and hypotenuse sqrt(2)). Constructing geometrically the square root in sequence (sqrt(2), sqrt(3), sqrt(4), sqrt(5), etc. including irrational numbers), is a consequence of using the rectangular triangle. In this manner, the spiral of Theodorus may be seen as an early version and representation of irrational numbers. Therefore, it can be considered to be a kind of visual representation of number theory (18).
Although this square root spiral by Theodorus shows the interdependencies between integers, natural numbers and irrational numbers in a geometric and visual way, it is rather unlikely that this was the intention of the mapmaker. It is more likely that the (known) use of geometric tools (compare with the use of the golden ratio [1]) led to an artistic impression to please the medieval mind and eye rather than provide an (early) highlight of

a mathematical proof -via geometry- in number theory. This may not have been the intention while producing the Rosette map in the Voynich manuscript.

**The construction process of the spiral of Theodorus: Use and re-use?**

Would the mapmaker have used the full construction process of Theodorus' spiral (10) *in-situ* to produce the text spiral? This seems rather unlikely: No helplines have been identified (although these could have easily been erased after construction). In addition, it seems that this construction process would have outweighed its purpose. I would like to suggest the use of a stencil or a cut pattern instead that would have been available for not just one but several instances of use in application of a spiral pattern. Re-use of such stencils and, as shown in case of the flower spiral in f56r showcases one example where this could have bee re-used (Figure 6). A direct overlay (to the best of folio sizes) reveals that the Spiral of Theodorus of the exact same dimensions could have been used to not only align the text in f86v, but also the plants spiral in f56r (Figure 8).

**Figure 8: Overlay with large spiral of Theodorus (Fig5, and Fig6) in aligned folios 56r (left) and f86v (right).**

This also would be another hint - albeit from a technical perspective - that, across different sections in the Voynich manuscript, identical tools and techniques were used. This can be shown in the case of the Spiral of Theodorus as stencil for imagery and the text spiral in the Rosette Map in the Voynich manuscript.

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11) Pelling, N. (2014) www.ciphermysteries.com/the-voynich-manuscript/voynich-theories#comment-313466 : (comment written December 14, 2014 10:16 pm) ...[..]. '*My 'takeaway' is rather different, though: if the shape is a representation of Paradise in the manner of the Beatus map, then it is one to which extra (curved internal) lines have been added apparently to obscure the overall visual meaning. Which points to at least two stages of construction (i.e. writing, and then obfuscation), which matches what we see elsewhere (e.g. on the reverse side of the same hexfolio). But this also suggests that we should be suspicious of everything we see on the page: that the original alpha state of the rosettes may well have been far simpler, but that the way they all ended up was far more distracting to the eye.'* accessed 01/05/2015

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