

# Supporting information

## S2 Appendix. Generalized pivotal quantities and generalized confidences

This appendix details the basic concepts of generalized pivotal quantities and generalized confidence intervals proposed by Weerahandi [11].

A random quantity  $R = R(\mathbf{X}; \mathbf{x}, \zeta)$  is called a generalized pivotal quantity (GPQ) if following two properties hold.

Property D: The distribution of  $R$  is free of unknown parameters.

Property E: The observed value of  $R$ ,  $r_{obs} = R(\mathbf{x}; \mathbf{x}, \zeta)$ , does not depend on nuisance parameters  $\eta$ .

Accordingly, the two-sided equal tailed  $100(1 - \alpha)\%$  generalized confidence interval for  $\theta$  is given by  $(R_{\alpha/2}, R_{1-\alpha/2})$ , where  $R_{\alpha/2}$  and  $R_{1-\alpha/2}$  are the  $100(\alpha/2)$ th and  $100(1 - \alpha/2)$ th percentiles of the distribution of  $R$ . Moreover, the percentiles of  $R$  can be estimated by using Monte Carlo algorithm.