Relative Illumination and Image Distortion Part 2: how to “sense” the entrance pupil

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ABSTRACT

In a prior journal publication, I showed how to control the relative illumination of a lens system as a function of image distortion and differential distortion, but I did not account for the contribution from the entrance pupil satisfactorily. This technical note tells how to do this by applying a simple geometric method.

In prior work [1 – 3], I showed that, under the condition of low numerical aperture and no vignetting, a lens system’s relative illumination RI may be approximately expressed as

$$RI = \frac{(A/A_o)\cos^4\theta}{(1+D)[1+D+y(dD/dy)]}, \tag{1}$$

where A is the off-axis entrance pupil’s surface area, A_o is the on-axis entrance pupil’s surface area, θ is the chief ray angle in object space, y is the field height (i.e., it is the height of the object), and D is the fractional image distortion defined by

$$D = \frac{y' - y_p'}{y_p'}, \tag{2}$$

where y' is the real image height, and y_p' is the paraxial image height. The quantity dD/dy has been called “differential distortion” [1] and it is the instantaneous rate of change of image distortion with respect to field height. The ray geometry for these quantities is illustrated in Fig. 1, where we assume that the entrance and exit pupils may in general be buried within elements or spaces of the lens system.

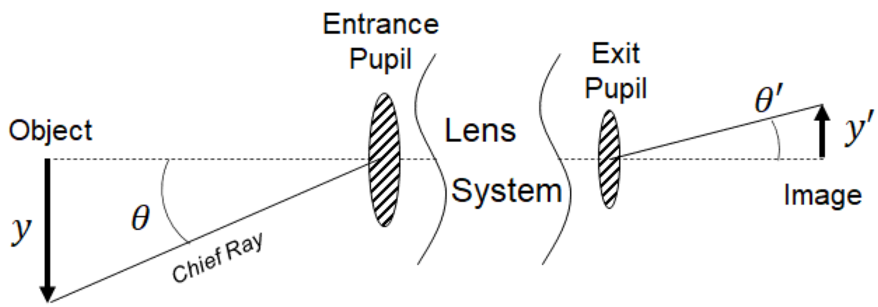

Fig. 1 Geometry of rays for a lens system for the quantities in Eq. (1).

In Eq. (1), it is easy to see that the quantities y, θ, D, and dD/dy may be determined from ray data, but it is not so clear how to determine the ratio A/A_o. To determine this ratio, consider that if a fictitious small thin lens were placed flat at the tip of the object to "look into the lens system", then an image of the off-axis entrance pupil could be made to form at a plane located some distance $-\Delta Z$ behind the small thin lens. Suppose that the full y-dimensional length of this entrance pupil image is ΔY (see Fig. 2). Then, by symmetry, a "dummy surface" placed at $+\Delta Z$ from the object would reveal that the full y-dimension of the ray cone centered at the chief ray also possesses length ΔY. A similar argument can be made for the orthogonal axis at the plane of the dummy surface. If we refer to the orthogonal axis (in and out of the page) as the "+/- x dimension", then at the dummy surface (and assuming a circular aperture stop), one has approximately an ellipse with half dimensions $\Delta Y/2$ and $\Delta X/2$ in orthogonal axes, and the area of this ellipse would be $\pi\Delta X\Delta Y/4$. Similarly, at the dummy surface, there would be a circle of some radius R and area πR^2 for the on-axis cone of rays traveling towards the entrance pupil. Thus, the ratio A/A_o is equal to the ratio $(\pi\Delta X\Delta Y/4)/\pi R^2$. In this way, one may "sense" the ratio A/A_o by way of placing a dummy surface at an arbitrary distance in front of the object, which is easily done in the prescription of an optical design program for a lens system layout.

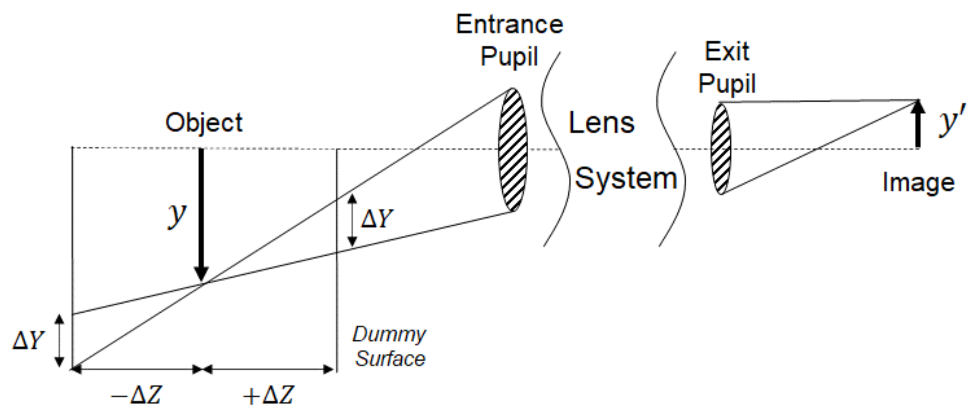

Fig. 2 "Sensing" the entrance pupil area using a dummy surface in front of the object.

As an example, consider the lens prescription given by "Figure 1" in U.S. Patent 2,031,792, which is a symmetrical lens of 66 mm effective focal length (EFL) at wavelength 588 nm [4]. Using Zemax® OpticStudio® [5], the lens layout is shown in Fig. 3 without vignetting at an aperture of f/6.3, semi-field angle 35 degrees, object at infinity, and image plane set at 51.9421 mm from the back vertex of the last element. The relative illumination given by OpticStudio (OS) is shown in Fig. 4. For an object at infinity, it can be shown (see Appendix A) that Eq. (1) may be expressed as

$$RI = \frac{(A/A_o)\cos^4\theta}{(1+D)[1+D+\sin\theta\cos\theta\,(dD/d\theta)]}\,. \tag{3}$$

We now show that Eq. (3) may be used to estimate the relative illumination of this lens. In particular, we shall calculate it at full field (i.e., at the field half angle of 35 degrees) and compare the computed value with the value given by OS in Fig. 4, which is roughly 0.35.

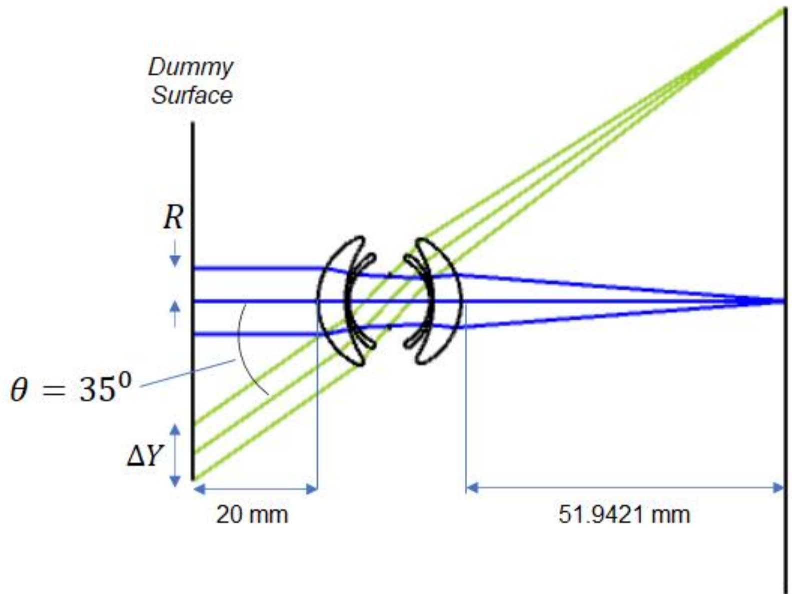

Fig. 3 Layout in Zemax® OpticStudio® for the lens given by “Figure 1” in U.S. Patent 2,031,792.

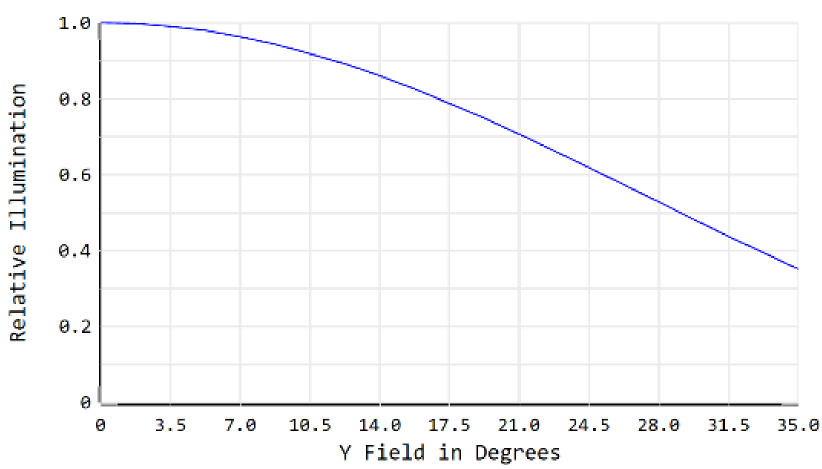

Fig. 4 Relative illumination plotted in Zemax OpticStudio for the lens in Fig. 3.

If one were to enter the prescription for the lens in Fig. 3, one would find that the image at full field has pincushion distortion at 1.22%, which means that $D = 0.0122$. At full field, the differential distortion is approximately $dD/d\theta \approx 0.0416$. Applying these values and $\theta = 35^0$ into Eq. (3), we have:

$$RI = \frac{(A/A_o)\cos^4(35^0)}{(1 + 0.0122)[1 + 0.0122 + \sin(35^0)\cos(35^0)\,(0.0416)]}$$

$$\approx (A/A_o)0.4311. \tag{4}$$

It remains to determine the quantity A/A_o. To do this, consider the spot diagram shown in Fig. 5 for the on and off-axis ray bundles at the dummy surface of the layout in Fig. 3. One can obtain values for ΔX, ΔY, and R from this spot diagram or from special operands in the merit function. In fact, the values shown in Fig. 5 were obtained from OS's merit function operands for ray heights. Applying the values for ΔX, ΔY, and R shown in Fig. 5 into the relation $A/A_o = (\pi \Delta X \Delta Y/4)/\pi R^2$, we have $A/A_o = 0.8174$. Applying this value into Eq. (4), we have

$$RI \approx (0.8174)0.4311 \approx 0.3524. \tag{5}$$

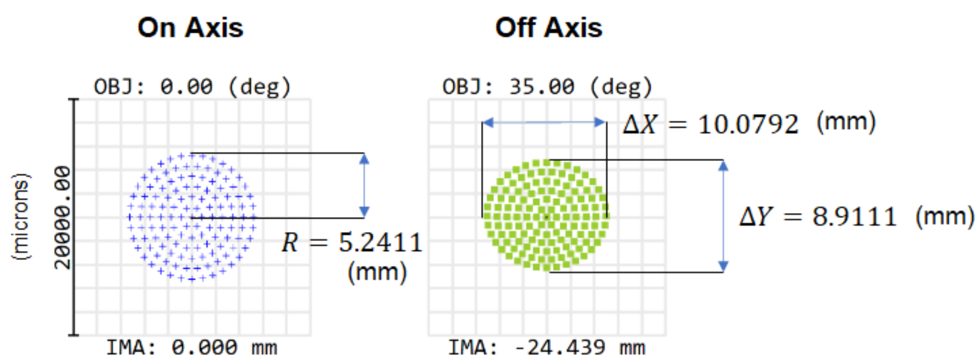

Fig. 5 Spot diagram for the two fields at the dummy surface shown in Fig. 3.

Note that the value for the full field relative illumination computed in Eq. (5) is quite close to the full field relative illumination given by the OS plot in Fig. 4, proving that the method for "sensing" the entrance pupil area ratio A/A_o in Eq. (1) [or, equivalently, in Eq. (3) for an object at infinity] works. The method could also potentially be applied to lens systems with non-circular apertures. In such cases, one would not assume that the on and off-axis entrance pupils have circular and elliptical shapes, respectively.

Why would one apply the method described here (and in Refs. 1 – 3) to estimate relative illumination when modern optical design programs can provide this data automatically? In fact, there are even more scholarly ways to compute and analyze relative illumination and aberrations [6 – 8]. The simple answer is that the method in this paper is just the way I do it, and it gives me quick information about what I'm controlling and constraining when designing a lens. Eqs. (1) and (3) also tell me how to tailor the relative illumination. I could — if I wanted to — make relative illumination uniform throughout the field by controlling image distortion and differential distortion [2], and also the entrance pupil area. We know a lot about image distortion, but not so much about differential distortion. The latter has a significant impact on relative illumination even when image distortion is zero at one or more field points.

APPENDIX A: RELATIVE ILLUMINATION WHEN THE OBJECT IS AT INFINITY

To get from Eq. (1) to Eq. (3), one simply notes that the quantity $y(dD/dy)$ is precisely equal to $y_p'(dD/dy_p')$, where y_p' is the paraxial image height as defined in Eq. (2). Substituting $y_p'(dD/dy_p')$ in place of $y(dD/dy)$ in Eq. (1) yields

$$RI = \frac{(A/A_o)\cos^4\theta}{(1+D)\left[1+D+y_p{}'(dD/dy_p{}')\right]}\,. \tag{A1}$$

Eq. (A1) may be applied to compute relative illumination (at low numerical aperture and no vignetting) for an object at infinity. However, for objects at infinity, since optical design programs usually provide the image distortion and relative illumination plot in terms of θ, it would be convenient if Eq. (A1) is expressed completely in terms of θ. To do this, first substitute Eq. (2) into Eq. (A1) to get

$$RI = \frac{(A/A_o)\cos^4\theta}{\dfrac{y'}{y_p{}'}\left[\dfrac{y'}{y_p{}'}+y_p{}'\dfrac{dD}{dy_p{}'}\right]}\,. \tag{A2}$$

Now, by applying Eq. (2), we note that

$$\frac{dD}{dy_p{}'} = \frac{d}{dy_p{}'}\left(\frac{y'-y_p{}'}{y_p{}'}\right) = \frac{y_p'(dy'/dy_p{}')-y'}{y_p{}'^2}\,. \tag{A3}$$

Substituting the result on the right-side of Eq. (A3) into Eq. (A2) we have

$$RI = \frac{(A/A_o)\cos^4\theta}{\dfrac{y'}{y_p{}'}\left[\dfrac{y'}{y_p{}'}+y_p{}'\left(\dfrac{y_p'(dy'/dy_p{}')-y'}{y_p{}'^2}\right)\right]}$$

$$= \frac{(A/A_o)y_p{}'\cos^4\theta}{y'(dy'/dy_p{}')}\,. \tag{A4}$$

For an object at infinity, there is no distinction between θ measured at the entrance pupil and at the object space principal plane of a lens system, hence, $\tan\theta = y_p{}'/f$, where f is the EFL of the lens system. Therefore, $dy_p' = f\sec^2\theta\, d\theta = f d\theta/\cos^2\theta$. Substituting this and $\tan\theta = y_p{}'/f$ into Eq. (A4) we have

$$RI = \frac{(A/A_o)f^2\tan\theta\cos^4\theta}{y'(dy'/d\theta)\cos^2\theta} = \frac{(A/A_o)f^2\sin\theta\cos\theta}{y'(dy'/d\theta)}. \tag{A5}$$

Eq. (A5) is actually the form described by Rudolf Kingslake [9] when he applied Max Reiss's [10] technique for deriving the relative illumination of a lens system from the point of view of the object (for an object at infinity). It is rather interesting to note that neither Kingslake nor Reiss performed the full differentiation $dy'/d\theta$. Had they done it — and had they expressed image distortion in the form given by Eq. (2) — perhaps they would have "discovered" differential distortion. Interestingly, Welford [11] refers to dy'/dy as "differential distortion", but not in the context of its role in relative illumination. Anyway, Eq. (A5) may now be recast into the form given by Eq. (3) if we now look to Eq. (2) and re-arrange it as $y' = y_p{}'(1+D)$ and note that for an object at infinity, $\tan\theta = y_p'/f$, so $y_p' = f\tan\theta$. Therefore, $y' = f\tan\theta\,(1+D)$. Differentiating this, we have

$$\frac{dy'}{d\theta} = f(1+D)\sec^2\theta + f\tan\theta\frac{dD}{d\theta}. \tag{A6}$$

Substituting Eq. (A6) into Eq. (A5) and using the fact that $y' = f\tan\theta\,(1+D)$, we have

$$RI = \frac{(A/A_o)f^2\sin\theta\cos\theta}{f\tan\theta\,(1+D)\left[f(1+D)\sec^2\theta + f\tan\theta\frac{dD}{d\theta}\right]}. \tag{A7}$$

Finally, by performing a little bit of algebraic manipulation, Eq. (A7) becomes Eq. (3). We could have arrived at Eq. (A7) from Eq. (A2) without first deriving Eq. (A5), but the reason I did not do it was because I wanted to show how to arrive at Max Reiss's formula [i.e., Eq. (A5)].

APPENDIX B: CONCEPT OF COMPUTING RELATIVE ILLUMINATION IN MODERN OPTICAL DESIGN PROGRAMS

According to the Zemax OS manual, Matthew Rimmer's technique [12] is used for computing relative illumination in OS. How does it work? The way I like to think about it is to first note that, for a Lambertian source of radiance L in air, the on-axis image irradiance E_o (in air) for an aberration-free optical system is proportional to the average of the squares of image space ray numerical apertures (NA) sampled around the lens's exit pupil [13]:

$$E_o = \pi L\frac{1}{N}\sum_{i=1}^{N} NA_i{}^2\,. \tag{B1}$$

For any off-axis ray bundle, a similar argument can be made if the local image surface at the image height of interest is orthogonal to the central ray in the ray bundle. Thus, at full field, if the chief ray is orthogonal to the local image surface at the full image height, then one may perform the sum in Eq. (B1) for those rays, provided that you know how to obtain the proper ray angles at the local image surface. Multiplying the result by $\cos\theta'$ (to "recline" the obliquity of the image plane, and where θ' is the chief ray in image space) yields the off-axis image irradiance. Finally, the ratio of the off-axis irradiance to the on-axis irradiance is the relative illumination. Note that if the optical system has aberrations, ray intercept data would provide lateral deviations from perfect focus, and one may apply these to correct the aberrated rays to determine ray angles for ideal rays. This is also pointed out by Rimmer in his paper [12].

One might wonder why, in this appendix, we talk about relative illumination in terms of ray data from the exit pupil. In contrast, in earlier parts of this paper (and in Appendix A), we talked about relative illumination in terms of ray data from the entrance pupil. Actually, they are equivalent, due to conservation of flux between rays entering the entrance pupil and rays exiting the exit pupil. In fact, Kingslake calls them the "Image Space Formula" and "Object Space Formula" for "Oblique Illuminance" [9]. It's just that only when you examine rays going into the entrance pupil would you be able to express relative illumination as an explicit function of image distortion and differential distortion (and of course, the ratio of the entrance pupil areas) as defined in Eqs. (1) – (3), which is convenient for practical lens design. And by the way, they are related to the coma of the exit pupil, which affects the size and shape of the exit pupil — and that is how the exit pupil affects relative illumination, besides the fact that the image plane is flat and orthogonal to the optic axis. Another advantage of examining rays at the object side is that all of the rays propagating from a field point towards the entrance pupil have no "focus aberration" at the field point (i.e., they all diverge from an ideal point at the object), so all of the rays surrounding a "diverging ray cone" graze perfectly around the entrance pupil. This is unlike the situation mentioned earlier concerning aberrated rays converging from the exit pupil towards the image plane. Therefore, in theory, I think that if one can obtain appropriate direction cosines for entrance pupil rays at the object plane, one can replace the quantity $(A/A_o)\cos^4\theta$ in Eqs. (1) and (3) by the type of sum given by Eq. (B1), which would enable lifting the restriction on having low NA for the lens system.

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