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Approximate Classical Simulation Of Quantum Architectures

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Introduction

A quantum computer is a computer that exploits quantum mechanical properties, such as superposition and entanglement to perform a computation. The difference between bits and qubits is that a qubit can be in a state other than 0 or 1.

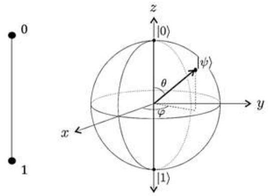

Fig.1.State of a classical deterministic bit and a quantum bit (qubit). A qubit can be in a superposition of states: $\psi = \alpha|0\rangle + \beta|1\rangle$, where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$.

Quantum Computational Supremacy is a milestone when a universal quantum computer performs a computational task that is beyond the capability of any classical computer.

Why does it matter? Showing Quantum Supremacy provides evidence that a classical computer can not perform any calculation that any other kind of computer can perform efficiently - refutes Church-Turing thesis.

Aim of Study: using computational complexity conjectures, we show that a particular type of quantum architecture cannot be exactly (and approximately) classically simulated.

Applications: Cryptography, Optimisation (weather forecasting, traffic problems etc.), Quantum chemistry (pharmaceuticals).

Background

Sampling Problems. Modern supremacy proposals are based around sampling problems. This is because quantum computers naturally produce probabilistic samples from a probability distribution p_x with probabilities determined by the Born rule:

$$p_x = Prob(outcome\ x) = |\langle x|C|0\rangle|^2. \qquad (1)$$

While such samples are inherently simple to perform on a Quantum computer, they are immensely difficult for classical computers to produce.

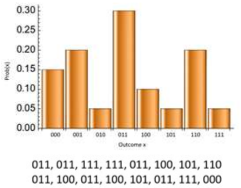

Fig.2.Sampling Problem: Given an input x, you are required to produce samples according to a probability distribution $D(x)$ over $\{0,1\}^n$.

An example probability distribution $D(x)$ over eight outcomes. The sequences of 0's and 1's are random samples according to the probability distribution above. The objective of a sampling problem is the compute samples like the sequences shown.

Exact & Approximate Sampling.
The task of sampling from a quantum computation can be viewed as a form of simulation. That is, a quantum circuit family is said to be (weakly) simulated if its output distribution p_x can be efficiently classically sampled.

Exact sampling: we require our classical distribution to be exactly equal to the target quantum distribution.
Approx. sampling: no quantum computer would exactly sample from the output distribution due to errors – need that no classical sampler can even approximately sample from the same distribution.

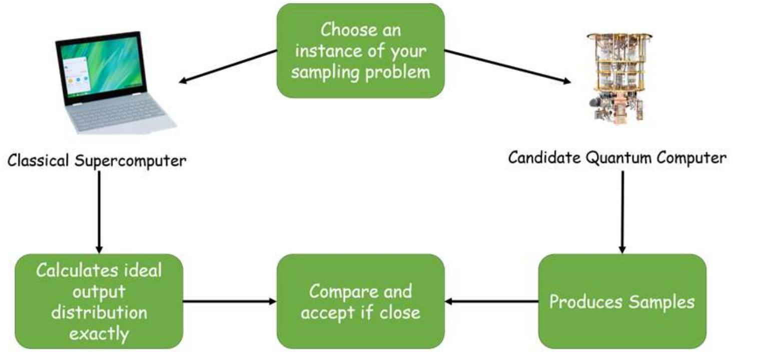

Fig.3. *Quantum Supremacy Sampling Proposal. 1. Choose an instance of your sampling problem – a quantum circuit C. 2. Classical computer calculates the ideal output distribution of our circuit C. 3. Quantum computer produces samples from probability distribution determined by circuit C. Assert that a classical computer should not be able to produce a probability distribution that matches our quantum probability distribution.*

Proving Quantum Architectures Are Hard To Simulate Classically

We are considering a specific quantum architecture in cluster states (see results section for further details). The aim is to prove that, using computational complexity conjectures ,our quantum architecture cannot be classically simulated. The method can be viewed as follows:

- Input quantum states parametrised (described) by surface of sphere and vary the radius.
- We develop a method (see Fig.4) based on sampling problems and computational complexity conjectures.
- Show that for our input states we can or cannot classically simulate our quantum architecture.

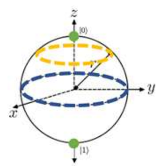

Fig.3. Modifying input states .to cluster state computation. Using the method described in Fig.4, for what values of r is it possible to simulate classically?

If we input states from the X-Y plane (blue line), then we get the full power of quantum computation and we cannot classically simulate. Inputting states $|0\rangle$ and $|1\rangle$ (green dots), we can classically simulate. For what radius ≤ 1 (yellow line) can we classically simulate?

> **Theorem [1] :** The non-collapse of the Polynomial hierarchy (a conjecture that is similar to $P \neq NP$)
> ⇒ Classically sampling quantum output distributions is impossible.

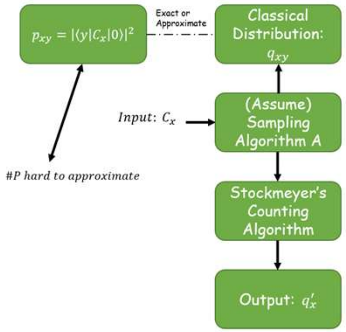

Fig.4 Outline of proof strategy for showing exact and approximate sampling hardness, based on an explanation from [3].

1. Show Quantum distribution p_{xy} is hard to exactly calculate but also hard to approximate. 2.Assume there exists a classical sampling algorithm that produces samples according to classical distribution q_{xy} which approximates p_{xy}. 3. Via Stockmeyer's algorithm produce approximation of q_{xy} ⇒ Polynomial Hierarchy Collapses ⇒Contradiction

Results, Future Work and Summary

Results: the classical computing model is restricted to only electronic circuits. However, with quantum computing there exist a diverse range of architectures:

- standard model is based on circuit models (similar to classical computers)
- alternative is measurement-based model, a computation is achieved via a sequence of measurements applied to a fixed state (known as a cluster state).

Our work explores the computational power of modifying the input state to a cluster state computation. In other words, we ask when can we classically simulate a 'modified' cluster state computation?

> **Theorem [2]:** there is a region of pure states (see Fig.3) such that a cluster state computation can be efficiently classically simulated.

Idea of proof: use generalised notion of separability to show that our state is not entangled and therefore can be classically simulated.

Future Work. We consider a different approach to rule out classical simulation for our quantum architecture [1]. But now we do make assumptions about separability, but instead base our methods on computational complexity conjectures, see Fig.4. In particular, we are interested in two notions of approximate classical simulation:

- Multiplicative approximation: $\frac{1}{c}\, p_x \leq \ q_x \leq \ c\, p_x,$
- Additive approximation: $\sum_x |p_x \ - q_x| \leq \epsilon,$

where $c \geq 1,\ \epsilon \geq 0,\ p_x$ is the target quantum distribution and q_x is the classical distribution.

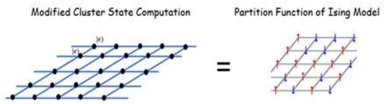

Fig.5 Correspondence between cluster states and Ising Partition Functions.

A key starting point to establishing a proof, as described in Fig.4, is to show that calculating quantum output probabilities is equivalent to classically intractable problems. An example of such a problem is calculating imaginary valued partition functions of the Ising model.

Summary.
- We consider a particular type of quantum architecture in cluster states.
- Our previous methods from [2] show that we can classically simulate our quantum architecture as long as our inputs (parametrised by a sphere) are restricted to the caps of the sphere.
- We now develop a new method, based on quantum supremacy arguments, to find obstacles to growing the caps of the sphere.

References.
[1] M.Bremner, A.Montanaro, D.Shepherd. Physical review letters,117(8):080501, 2016.

[2] S. Atallah, M.Garn, S.Jevtic, D.Tao, S.Virmani, In preparation. [3] D.Hangleiter, PhD thesis 2021.