# Minimization of energy loss in small caliber ammunition using optimization algorithm techniques

Kari Beardsley, Dan Carpenter, and David Grimm
Brigham Young University

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**Abstract**

The effectiveness of a bullet is often specified as a measure of the magnitude of deliverable energy upon impact. In hunting, military, and recreational applications, there is a need to be able to deliver the most amount of energy possible at very long distances. Current bullet technology reflects the results of a trial-and-error approach to maximizing the magnitude of energy delivered at predetermined distances. The research shown in this report describes the models, processes, and results for determining a true global optimum for reducing the total amount of energy lost in flight and therefore maximizing total deliverable energy. Especially important is the optimization of bullet shape factor to maximize ballistic coefficient. The main function includes the following design variables: grain (weight), nose length, bullet caliber, total length, and ogive radius with set parameters of bullet material, bullet muzzle velocity, and tip diameter. Multiple optimization algorithms have been employed in determining the minimum energy reduction due to fluid drag on the bullet. Bullet diameter has been restricted to sizes smaller than .50 caliber to focus the application of the results to small, personal firearms and hand-held weaponry. The findings reported here could be extended to include larger caliber projectiles and used to optimize large scale artillery effectiveness as well. The results found here are a reflection of theoretical limits and a proposal to manufacture and test physical prototypes, but include only the mathematical relationships and results. The algorithms used converged on a single global minimum. We concluded that although many advances have been made in bullet shape and dimensions, a truly optimum bullet has yet to be manufactured. We propose that the design described in this article should be manufactured and tested.

## 1 Introduction

The problem of ballistic coefficient is not uniquely a problem of shape. Similarly, it is not uniquely a problem of weight or frontal area. The problem of ballistic coefficient is a combination of every measurable factor concerning a bullet. For this reason, bullets (and other projectiles like arrows, BBs, pellets, cannonballs, and even large artillery pieces) present a unique challenge in the world of multidisciplinary optimization.

In traditional fluid drag analysis, equations of pressure, area, and smoothness have been developed that are capable of precisely predicting drag and lift conditions. Knowing these conditions, engineers are able to model drag and lift forces, calculate efficiency, predict performance, and make judgments based on the resulting data. After a full fluid analysis has been completed, scale or full-size models and prototypes can be built and tested in wind tunnels and in actual operating conditions.

In wind tunnel testing, for example, models are fixed to a sensor "sting" and fluid data can be acquired. This data can be compared to calculated solutions and verified mathematically. This typical process can be repeated or iterated until a final design decision can be made.

In projectile design, however, this process cannot be used exclusively. Wind tunnel testing cannot take into account the inertia of a bullet, flight conditions, trajectory, flight path variation, or transient range conditions. For the purpose of bullet design, wind tunnel testing is only a single step in the design process.

Since the determination of a bullet's performance is so heavily weighted toward inertia, engineers have generated a universal parameter to quantify bullet efficiency. It is called the ballistic coefficient. In short, the ballistic coefficient is a ratio of a bullets mass (and therefore inertia) and its drag properties. It can be roughly described by the following equation:

$$BC = \frac{m}{d_i^2 \cdot i} \tag{1}$$

where $d$ is the bullet caliber measured in calibers and $i$ is the bullet's form factor. Much of this report will be dedicated to the intricacies of this form factor and the optimization involved will include the design parameters of shape.

# 2 Background

The ballistic coefficient is a measure of a bullet's ability to overcome air resistance while in flight. Bullets are just one example of ballistic bodies that must overcome air resistance: automobiles, arrows, trains, jets, missiles, and myriad of other things share the goal of improving aerodynamics in order to optimize performance. By focusing uniquely on bullets, we can optimize the shape of near-supersonic and supersonic bodies to achieve the maximum distances and deliverable energies. Bullets have a large variety of shapes that all result in varying aerodynamic conditions. By maximizing the ballistic coefficient, we simultaneously minimize long-range bullet drop, wind drift, and maximize deliverable inertial energy.

A typical ballistic coefficient will range from 0 to unity as a comparison to a standard projectile, will unusually high performing bullets having a value greater than 1. The ballistic coefficient also roughly estimates the amount of energy lost over 1000 yards (Harris, 13). If a bullet has a ballistic coefficient of 0.4, it can be estimated that it will have lost about half of its initial kinetic energy 400 yards downrange. With a ballistic coefficient of 1.1, a bullet will have lost about half of it's inertial energy after 1100 yards.

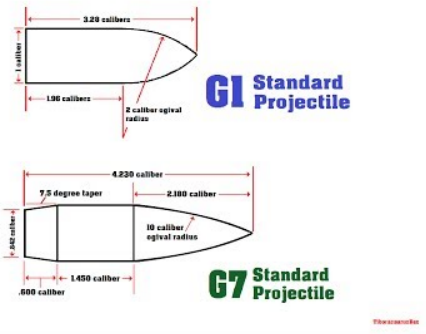



Figure 1: Measurements and shapes of G1 and G7 standard projectiles.

Bullets began the long journey of optimization with the Arab people. They would use flammable powder to shoot stiff arrows through metal-reinforced bamboo shoots. Over time, people began using steel and lead balls ejected by black powder in place of arrows. The more-commonly recognizable cylindrical shape was first implemented by a man named Claude-Étienne Minié in 1847. This new shape drastically improved the range of the bullets. It was said that Union and infantrymen [during the Civil War] hit their targets more often and at far greater distances. (Courtney, 2) In the 1840s to the 1850s, paper bullet cartridge were developed. This greatly improved the method for firing guns and opened the door for more sophisticated and efficient bullet shapes.

Now, to categorize different types of bullets, each design is compared to what are called G1 and G7 standard bullets. G1 standards are used as the benchmark for short range bullets and G7 standards correspond to long range bullets. These standards are based on dimensions and shapes that vary from bullet to bullet. The G1 Standard Projectile is commonly known as a flat base projectile and used to categorize the majority of commercial bullets. The G7 Standard Projectile has what is called a boat-tail. A boat-tail is the tapered portion at the back of some high performance bullets that is designed to reduce pressure drag and increases efficiency. Standard projectiles form a baseline for classifying and grading the performance of commercially manufactured bullets.

# 3 Objective

In this section, the derivation of the objection function used to maximize ballistic coefficient will be shown and commented on. As well, the design variables and their meanings will be discussed and constraints will be defined.

## Function Derivation

The complexity of this problem can be seen when determining a suitable objective function. In researching this topic, we considered no less than ten possible objective functions and variants. There is a virtually endless amount of data and sub-formulas available and sorting through each possible method is daunting. Despite the many variations of sub-equations, however, the major objective function defined by equation (1) is universal.

The mass of the bullet is calculated using simple volume-density equations. By multiplying the total bullet volume by the density, we find the bullet's mass. Dividing this number (in the units of pounds·force) by 7000 gives a bullet's weight in grains. Be aware that this weight is a gravitation based number and makes no reference to the amount of matter in a given bullet. Thus,

$$m = Volume * \rho * 7000 \tag{2}$$

Substituting equation (2) into equation(1), we arrive at

$$BC = \frac{Volume * \rho * 7000}{d_i^2 \cdot i} \tag{3}$$

By making the observation that the majority of bullets are made from solid lead and that jacketed bullets have a negligible volume contribution and density difference, we can take $\rho$ to be 0.410 $\frac{pounds}{in^3}$ and equation (3) becomes

$$BC = \frac{2870 \cdot Volume}{d_i^2 \cdot i} \tag{4}$$

A simplified volume calculation can be derived from adding a the volume of a cylinder to the volume of a truncated cone, which represent the bullet's body and nose respectively. This makes

$$Volume = \frac{\pi * d * (ol - l)^2}{4} + \frac{\pi * l}{3}\left(\frac{c^2 + c * t + t^2}{4}\right) \tag{5}$$

where $d$ = ogive diameter, $ol$ = ogive length, $l$ = total bullet length, $c$ = bullet diameter, and $t$ = tip diameter. Substituting $c$ for $d_i^2$, this yields

$$BC = \frac{2870 \cdot \frac{\pi * d * (ol-l)^2}{4} + \pi * l\left(\frac{c^2 + c * t + t^2}{12}\right)}{c^2 \cdot i} \tag{6}$$

From this point, the problem reduces to calculating the shape factor. We begin by calculating the ogive radius, frontal area, frontal nose area, and meplate area as follows:

$$OgiveRadius = \frac{\sqrt{NoseLength}}{Diameter - TipDiameter} + \frac{Diameter - TipDiameter}{4} \tag{7}$$

$$FrontalArea = \frac{Diameter^2 * pi}{4} \tag{8}$$

$$NoseArea = \frac{pi * NoseDiameter^2}{4} \tag{9}$$

$$MeplateArea = \frac{pi * TipDiameter^2}{4} \tag{10}$$

We can then begin to make a weighted average of each component of the form from equations 7 through 10 as follows:

$$OvigePercentage = \frac{(NoseArea - FrontalArea)}{FrontalArea} \tag{11}$$

$$MeplatePercentage = \frac{MeplateArea}{FrontalArea} \tag{12}$$

$$ShoulderPercentage = \frac{FrontalArea - NoseArea}{FrontalArea} \tag{13}$$

We can calculate the effect the ogive radius has on the form with the equation below:

$$AverageRadius = OgivePercentage * OgiveRadius + ShoulderArea * 0.5 \tag{14}$$

Having laid the ground work calculations for calculating the form factor, we will begin by calculating a rough form factor:

$$InitialFactor = \frac{2}{OgiveRadius} * \sqrt{\frac{4 * AverageRadius - 1}{6.85}} \tag{15}$$

The 6.85 in the denominator is known as the manufacturing factor and accounts for surface conditions in the bullet and is a correction factor for bullets of different design. For cast bullets, the value for the manufacturing factor is 7. We can use this initial factor and correct it by weighing it with its shoulder percentage:

$$ModifiedFactor = InitialFactor + ShoulderPercentage * 2.3 \tag{16}$$

We can then finalize the form factor by adding in final contributing effects from the diameter and length:

$$FinalFormFactor = ModifiedFormFactor * (1 - \frac{Diameter^2}{Length^2}) \tag{17}$$

Equation 17 is the final form factor which we will use in the calculation of the final Ballistic Coefficient. By substituting it in for $i$ in equation 6, the final formula yields:

$$BC = \frac{2870 \cdot \frac{\pi * d * (ol - l)}{4}^2 + \pi * l \left(\frac{c^2 + c * t + t^2}{12}\right)}{c^2 \cdot ModifiedFormFactor * (1 - \frac{c^2}{l^2})} \tag{18}$$

Equation 18 is the objective function. It is this function with which we are working to maximize the Ballistic Coefficient in this report. The following sections will discuss exactly how this has been done and summarize the details of the solution.

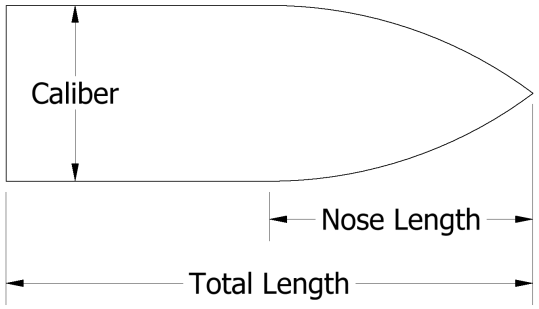
Further information on this derivation can be found at the web address found described in the bibliography of this article. Please refer to the source provided by Tom Myers for additional resources (Myers, 1).

# 4 Method

## Design Variables

The three major design variables are total length, nose length, and caliber. Total length spans the total parallel distance between both extreme ends of the bullet. Nose length spans the parallel distance between the beginning of the ogive and the bullet point. Caliber is a measure of the largest diameter of the bullet. As simple drawing has been included here for clarification.

Figure 2: Simple Bullet Dimensions.




While there are only three inputs to the optimizer, the objective function manipulates and calculates several other residual functions that affect the final function value. Ogive radius is an important calculation used in the objective function. It basically weighs the effect of added fluid drag because of the curved nature of the tip of the bullet. Naturally, because the ogive has a perpendicular component to its shape with respect to the flow direction and thus adds more resistance. The ogive radius is used in the calculation and weighed against other components of drag. It is a function of total length, nose length, and tip diameter and reflects an arbitrary radius that defines the curvature of the ogive.

Also calculated is the meplate area. This area is the frontal area created only by the tip of the bullet. Some bullets are rounded or pointed at the end and thus have no meplate area. Other bullets are hollowed at their point or expansive in nature and still others are flat. In all cases, the meplate area is simply the frontal projection of the area swept out by the radius of the tip of the bullet. The formula behind the meplate are found in equation 10 of this report.

Another important parameter calculated within the objective function is the shoulder form factor. This is displayed in equation 13 and represents the weighted effect of the discontinuity where the ogive connects with the main cylinder of the bullet. At bullet speeds approaching mach 1, an increase in drag occurs at discontinuous locations. Formulations for this are complex and imprecise. By making simple assumptions about atmospheric conditions, this value can be computed relatively easily.

Several subroutines also calculate the weighted significance of ogive effects, shoulder effects, meplate effects, skin drag, and inertia. Each of these are used in turn according to the equations shown in the objective section to calculate the final shape factor. This shape factor corrects the ballistic coefficient in such a way as to make it consistent with the reference projectiles as explained in the background section. Please refer to that section for clarification on this matter.

## Explanation of Constraints

As explained in the abstract of this report, bullet designs have been limited to small, personal firearms and hand-help weaponry. As such, the problem is limited only by practicality. Since firearms are manufactured not only to be effective, but also comfortable, portable, easy to maintain, and fit human anthropometrics reasonably well, all applied constraints are “soft” meaning that they can be manipulated to include designs outside their current ranges. Presently, this problem was optimized to include bullet designs only between 0.220 and 0.500 caliber. These sizes represent the current extreme industry standards for nominal bullets.

As mentioned before, for larger or smaller bullet designs, constraints could be relaxed outside the limits prescribed here.

The length constraint is what might be called the "master" constraint. Limiting projectiles to this size effectively limits the other design variables. Total length cannot reasonably be much longer that about 3 inches. Bullets which are proportionally much longer than their diameter are difficult to load and impractical to carry. The added weight from their increased length means they require much larger amounts of gunpowder to accelerate which in turn takes the casing larger as well. Long bullets like this are difficult to pack and are generally impractical for easy field use. For this reason, we limited the total length to between 0 inches and 3 inches to accommodate this practical constraint. Again, this, too could be relaxed to include unproportionally long bullets.

Finally, the nose length was constrained to limit the possible ogive radius. For all bullets, a sweeping radius is needed to facilitate loading and cycling of individual rounds. Radii that are too shallow (meaning the nose is too long) can get caught in the cycling action of the bolt and receiver of a firearm and cause jamming. To eliminate this problem, we constrained the maximum length of the nose to 1.5 inches. It is jamming and other mechanical malfunctions are of course still possible with a length this long, but the probability is greatly reduced.

Table 1: Upper and Lower Bound Constraints

| | Diameter (in) | Length (in) | Nose Length (in) |
|---|---|---|---|
| Lower Bounds | 0.22 | 0.001 | 0.001 |
| Upper Bounds | 0.50 | 3.00 | 1.50 |

Starting points were chosen for all three design variables to be at random between zero and one. This was to ensure that a global and not only a local maximum was found. As is described in the results section, this caused the optimizer to only solve a fraction of the time. The same optimized value was obtained often enough, however, to confirm that a maximum actually had been discovered.

Initial efforts in obtaining a global maximum used four design variables. This approach, however, led to unrealistically large results which is likely due to physics in the real world that the model does not account for. An assumption was made for the tip diameter and it was set to zero. After this change was made, the solver consistently converged.

Attempts were made to use the Nelder-Mead and godlike algorithms to maximize the objective function. These algorithms, however, took advantage of the model equations in a way that fmincon did not. The Nelder-Mead and godlike algorithms found a maximum by setting the diameter and length to be essentially the same number. In this scenario, a formula used in calculating the ballistic coefficient divides a number by the difference of diameter and length causing the resultant ballistic coefficient to be incredibly large. This is not how the formulas were intended to be used and thus, these results were faulty. We are confident that both of these algorithms would work and obtain the same maximum values if additional constraints were applied. This was not done in the interest of time and so that our efforts could be spend improving the results obtained from fmincon.

# 5 Results

As the superior optimization technique, we will report on the results of the sequential quadratic programming algorithm. As an implementation,we used MATLAB's commercially available 'fmincon' function. This algorithm uses sequential quadratic programming for major iterations, finite differences for gradient and Hessian calculations, and simple line search for step size determination. It converged with the following results:

Table 2: Optimized Solution Results

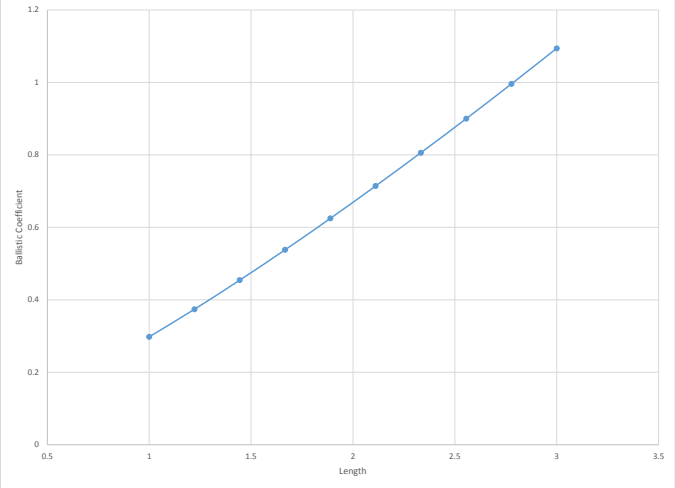
| Maximized Ballistic Coefficient | Diameter | Length | Nose Length |
|---|---|---|---|
| 1.094 | 0.220 | 3.00 | 0.845 |

Although you cannot tell from this tabulated data, the function is actually rather misbehaved. There are several local maximums that cause the optimizer to converge before finding the global maximum. By using a multi-start subroutine, we were able to converge on the true global maximum for ballistic coefficient 66% of the time. It required many trials to collect all the data reported on here and as the trials grew in number, it became more and more obvious that the reported optimum is in fact the constrained global optimum.

It was very difficult to retrieve the useful data from the different multi-start solutions. Like stated before, one third of the time the data recorded from the optimizer was incorrect. Because of the ill-conditioned nature of the objective equation – Caused by several non-linear terms such as square roots and powers – significant noise was introduced when generating Pareto fronts. It should be noted that this "noise" was so distracting it took hours of troubleshooting to finally realize that the data actually was correct. By simply adding a small routine that filtered out all solutions with unfeasible objection function values or design parameters, the data became useful and incredibly meaningful. As an extension to the results that were tabulated above, much can be said about the nature of the data and the ballistic coefficient sensitivity to changes in design.

Perhaps the least surprising result was the effect total bullet length has on the ballistic coefficient. Simple fluid behavior intuition would indicate that the longer the bullet, the more fluid drag there will be on the body. In addition, however, more length adds more mass and thus more inertia to the system and increases the ballistic coefficient. This front is plotted here below and will be subsequently discussed:

Figure 3: Ballistic Coefficient vs. Length

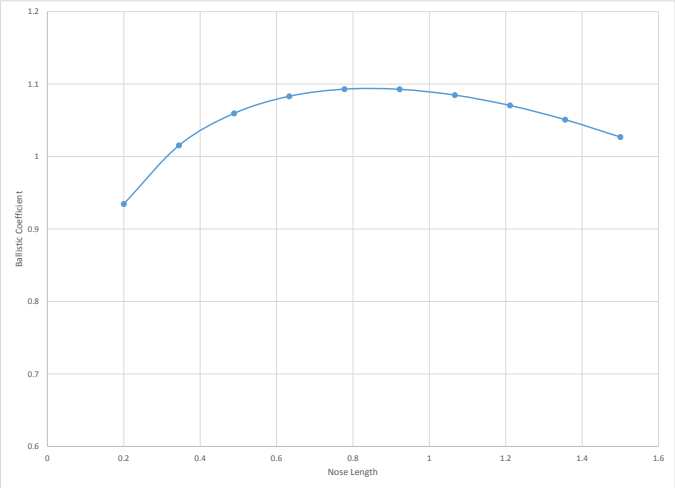



It is obvious here that any increase in air resistance is greatly outweighed by the additional inertial mass.

There is indeed no local optimum because were this problem unconstrained, total length would increase to approach infinity. The optimizer converges to the constrains and more experimental trials have shown that longer bullets *always* have higher ballistic coefficients than shorter bullets. This constraint was discussed in the "Constraints" section, but here it seems reasonable to mention once again that the longest practical bullet should be no longer than 3 inches in total length and thus 3 inches is the optimal length.

Nose length optimization provided a very interesting solution. It showed a definite optimal solution onto which the optimizer converged. The Pareto front for this parameter is shown here now.

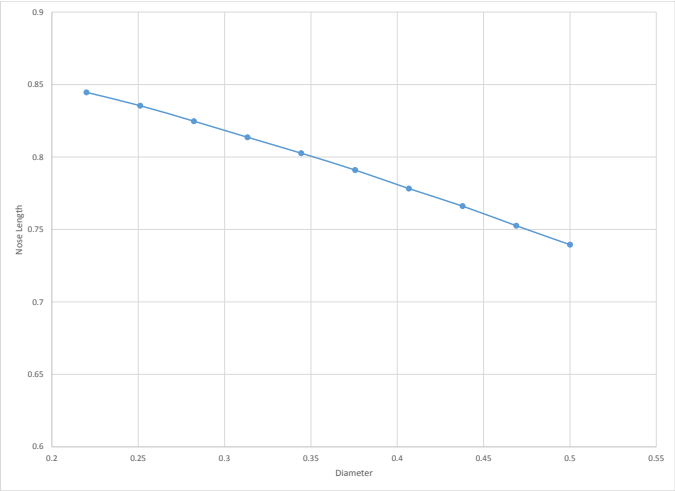
Figure 4: Ballistic Coefficient vs. Nose Length




It can be seen that the peak settles directly over the optimal length of 0.845 inches. Numeric optimization is crucial for this design variable because unlike the total length, it was impossible to generate a prediction based on intuition alone. No simple physical relationships exist that relate any type of cone length to air resistance and are impossible to visually predict.

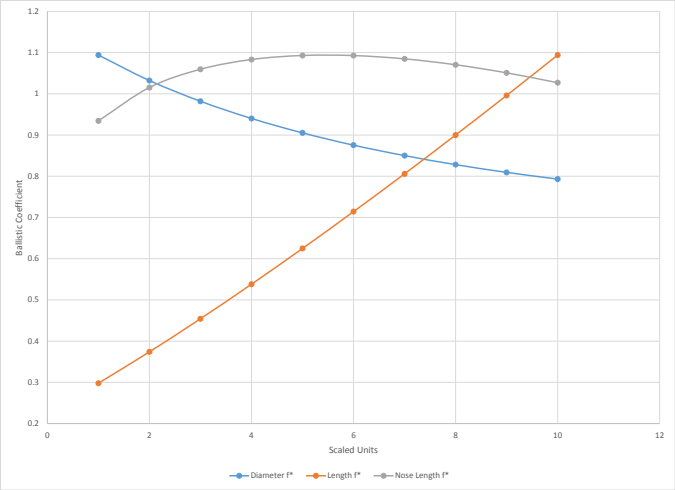
Intuitively, it would seem that there might be a strong skewed relationship between caliber and air resistance like length, but because the mass is increased exponentially compared to the diameter, the direction of that trend is unknown. Will the added mass of a larger diameter bullet contribute more inertia than the frontal area and shoulder interference can counteract? Or will the added air resistance contribute a much larger force then the additional massive inertia? It was this question the optimizer was used to solve. We discovered that indeed, smaller diameter bullets have higher ballistic coefficients than larger diameter bullets. The trend is shown here in the following plot:

Figure 5: Nose Length vs. Diameter




In order to provide a generalized viewpoint of the effects of design parameter change, a plot including all three design parameters is included here. Because the parameters are all scaled differently, the units have been modified to fit on the same plot. Each parameter is normalized between 1 and 10. For example, bullet diameter (caliber) is constrained between 0.220 inches and 0.500 inches. The plot seen here shows the first diameter of 0.220 inches falling at 1 on the x-axis and diameter 0.500 falling at 10 on the x-axis. In between are 10 evenly spaced points that represent the entire span of constrained designs. This creates a clear spline that is easy to compare with other parameters.

Figure 6: Comparative Solution Results



# 6 Conclusion

Bullet design up to this point in time has been subject to a trial and error approach. This process worked relatively well because different kinds of bullets serve different functions. Hollow point bullets are designed to break up on impact and cause massive internal damage. Armor piercing bullets are generally made of steel or depleted uranium and very pointy to penetrate steel plating or reinforced concrete. Still, some bullets are purely recreational and are designed to be cheap and reliable.

With this specialized process, manufacturers can choose the design variables for themselves and allow an optimization algorithm to optimize the remaining parameters for the highest ballistic coefficient possible. The algorithm can account for any number of constraints and design variability.

It is proposed that manufacturers take this approach moving forward to produce more effective bullets. We have designed the routines we used to be quickly and easily modifiable. Adding this simple step to the design and manufacturing stages of production would introduce only negligible cost to industry standard processes and result in bullets designed with optimality in mind.

# 7 Nomenclature

## Ogive

The roundly tapered end of a two-dimensional or three-dimensional object. For the purposes of this report, the ogive is the tapered portion of a bullet.

## Caliber

Caliber is the approximate internal diameter of the barrel, or the diameter of the projectile it fires, usually shown in millimeters, or in hundredths or thousandths of an inch. For the purposes of this report, the caliber considered is the bore-bearing diameter; the largest diameter of the bullet which would bear on the internal bore of the gun barrel.

## Meplate

From the French term for “flat surface” or tip. It is the technical term for the flat or open tip on the nose of a bullet.

## Ballistic Coefficient

A measure of a three-dimensional body’s ability to overcome air resistance in ballistic flight.

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