APPENDIX: The Relationship Between the *J*(*p*) and *p*\*(*q*) Curves and Lorentz Ordering

In [Section 3](#sec3), where the proposed curves and related measures were obtained for the Pareto family of distributions, it was seen that as the parameter alpha decreased, the Gini index and the new area measures increased. When comparing the measures AJ, AP\* and the Gini index calculated on two distributions, this is true whenever the Lorentz curves (and corresponding underlying distributions) are ordered in the following sense (Arnold 2015b, p. 201):

Def: The distribution *F*1 is at least as *unequal* in the Lorentz sense as a distribution *F*2 if the corresponding Lorentz curves, defined in [Section 2](#sec2), satisfy

<EQ> (A1)</EQ>

From formula ([5](#eqn3Z1)), it follows that the Pareto family is Lorentz ordered because as alpha increases, *L*(*p*) increases at all *p*. Similarly, the values of the Gini index, (2α−1) −1 decrease as alpha increases. Whenever, the relationship of the corresponding Lorentz curves implies that the Gini index is of F1 is *greater* *or equal* to the Gini index of F2 and that *J*1(*p*)  *J*2(*p*). Thus, AJ1 AJ2, i.e.,  implies that both the Gini index and AJ of *F*1 are both larger than the Gini index and AJ of *F*2. Also, when *L*1(*p*) *L*2(*p*) for *p* in (0,1), *L*1−1(t) *L*2−1(t) and *L*1(1−*q*)*L*2(1−*q*) for all *q* in (0,1). Thus, 1− *L*1(1−*q*) 1−*L*2 (1−*q*). From Equation ([3](#eqn2Z3)), *p*1\*(*q*)= *L*1−1(1− *L*1(1−*q*) ) *L*2−1(1− *L*1(1−*q*)) *L*2−1(1− *L*2(1−*q*) ) = *p*2\*(*q*). Consequently, whenever *L*1(*p*) *L*2(*p*) or , *p*1\*(*q*)  *p*2\*(*q*), which implies that the area AP\* corresponding to *F*1 is at least as large as the area AP\* for *F*2. Thus, when the income distribution underlying a Lorentz curves changes over time to a more unequal one, under the Lorentz order, the Gini index, AJ and AP\* all increase.