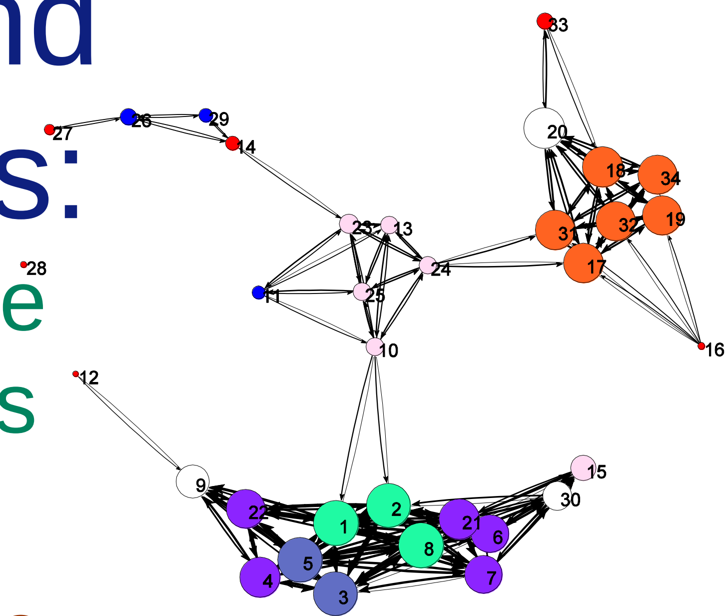
Imperial College London

Complexity Science

Uncertainty and Spatial Models:

The Effects of Space on Network Analysis



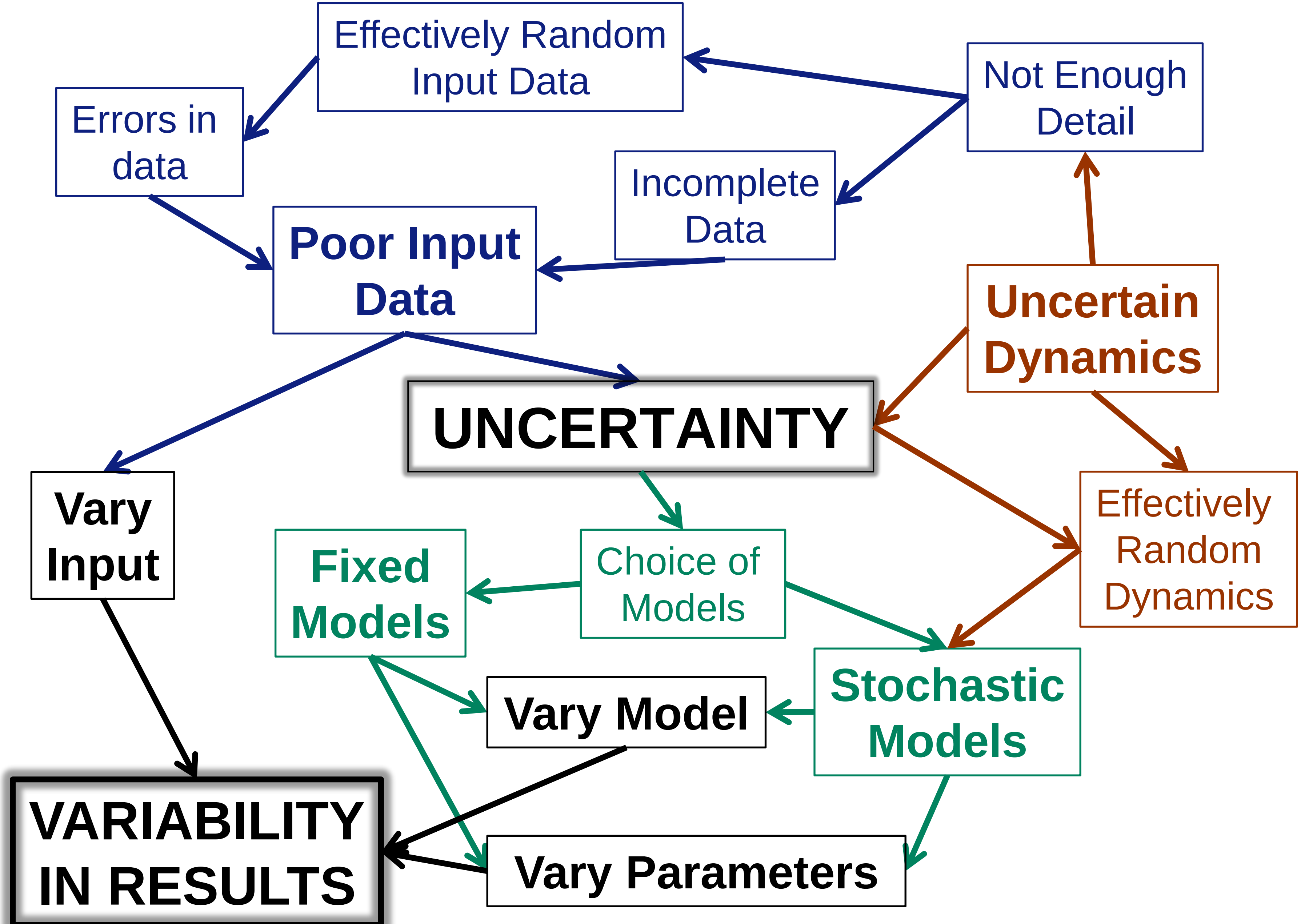
Tim Evans

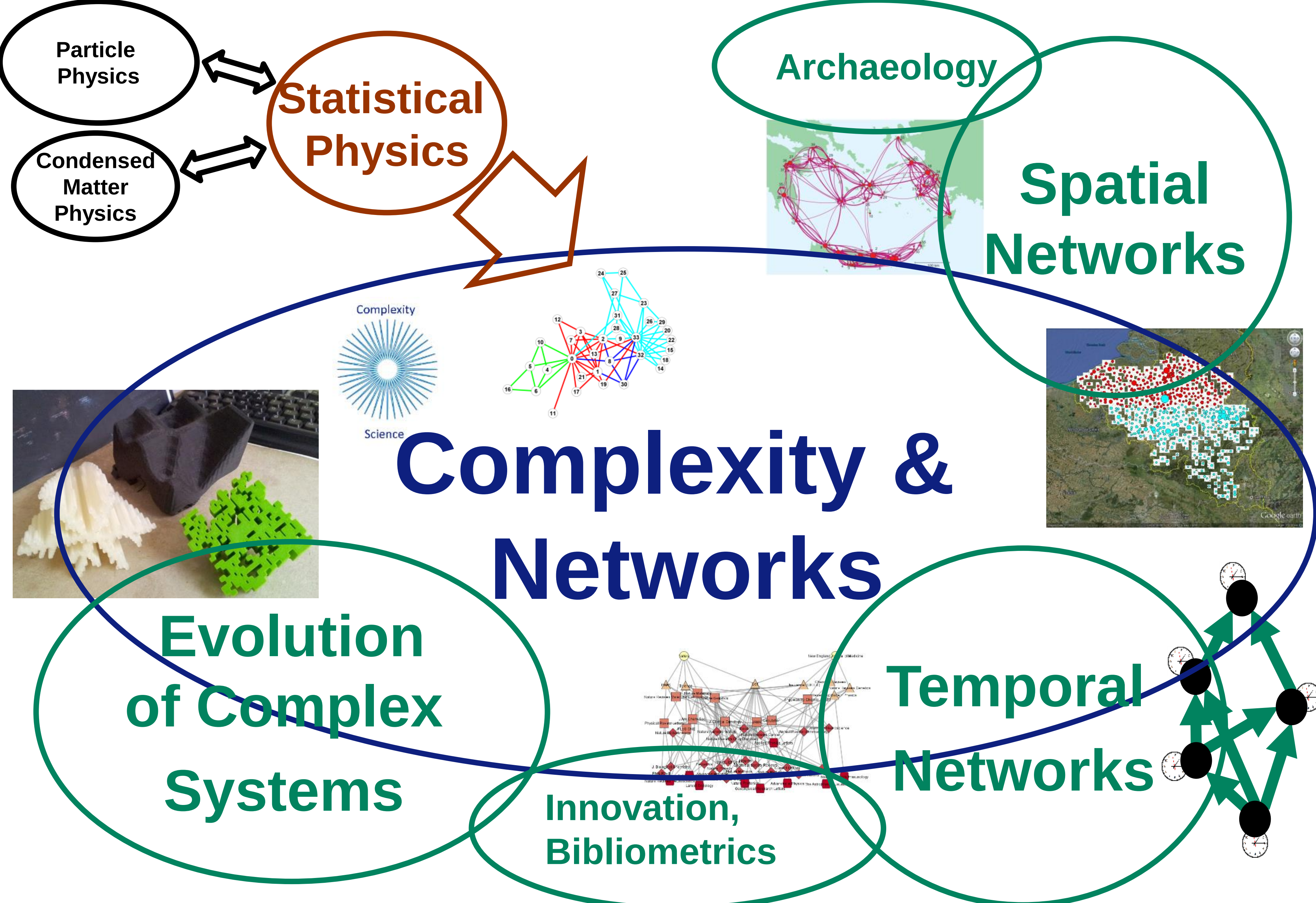
Centre for Complexity Science

http://netplexity.org

Slides: http://dx.doi.org/10.6084/m9.figshare.3840249

Workshop on « Archaeological Network Reconstruction »,
Univ. Konstanz, 20th Sept 2016


Effectively Random Input Data
Not Enough Detail
Errors in data
Incomplete Data
Poor Input Data
Uncertain Dynamics
UNCERTAINTY
Vary Input
Effectively Random Dynamics
Fixed Models
Choice of Models
Vary Model
Stochastic Models
VARIABILITY IN RESULTS
Vary Parameters


Particle Physics
Condensed Matter Physics
Statistical Physics
Archaeology
Spatial Networks
Complexity
Science
Complexity & Networks
Evolution of Complex Systems
Innovation, Bibliometrics
Temporal Networks

Rihll and Wilson Model

- Fixed Output Size Size O_i=1
- Variable Input Site Size I_i
- Dense Flow Matrix produced F_{ij}
  not a sparse binary network
- Zone of Control model,
  not a Bipartite Interaction model
- Standard Model Axioms/Properties?

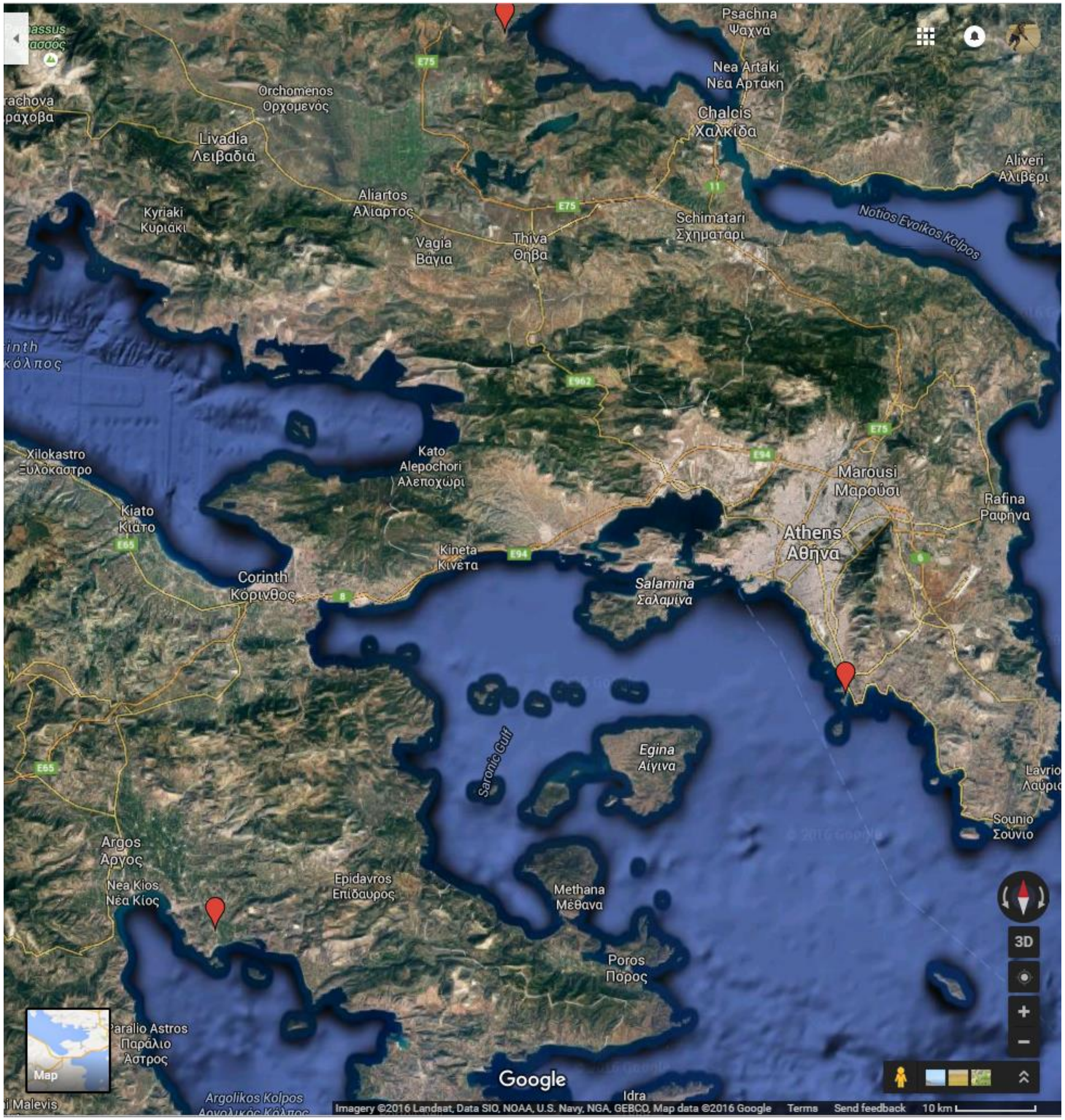
RW Model – recent uses

- Crete [Bevan and Wilson 2013]
- Dynamical growth – NE USA [Wilson et al]
- etc?

Data

- Same data set as **RW** Rihll and Wilson 1987, 1991 (**RW87** & **RW91**)
- Late Geometric Period
- Part of central southern mainland Greece
  - Boeotia, Attica, Isthmus, Argolid
- RW aim to look at rise of unequal cities from equal sized settlements
  - Thebes, Athens, Corinth, Argos

RW87 Fig 1


Psachna
Ψαχνά
Nea Artaki
Νέα Αρτάκη
Chalcis
Χαλκίδα
Aliveri
Αλιβέρι
Orchomenos
Ορχομενός
Livadia
Λειβαδιά
Aliartos
Αλίαρτος
Kyriaki
Κυριάκι
Vagia
Βάγια
Thiva
Θήβα
Schimatari
Σχηματάρι
Notios Evoikos Kolpos
Xilokastro
Ξυλόκαστρο
Kato
Alepochori
Αλεποχώρι
Kiato
Κιάτο
Marousi
Μαρούσι
Rafina
Ραφήνα
Athens
Αθήνα
Kineta
Κινέτα
Corinth
Κόρινθος
Salamina
Σαλαμίνα
Egina
Αίγινα
Saronic Gulf
Argos
Άργος
Nea Kios
Νέα Κίος
Epidavros
Επίδαυρος
Methana
Μέθανα
Poros
Πόρος
Sounio
Σούνιο
Argolikos Kolpos
Idra
Map
3D
Google
Imagery ©2016 Landsat, Data SIO, NOAA, U.S. Navy, NGA, GEBCO, Map data ©2016 Google
Terms
Send feedback
10 km

RW87 Fig 1: Distances

Digitised this figure by hand to produce 109 site locations

Includes missing point Vouliagmeni (64)

FIGURE 1

RW87 Fig 1: Regions

Three obvious regions:-

- Boeotia
- Attica
- Isthmus + Argolid

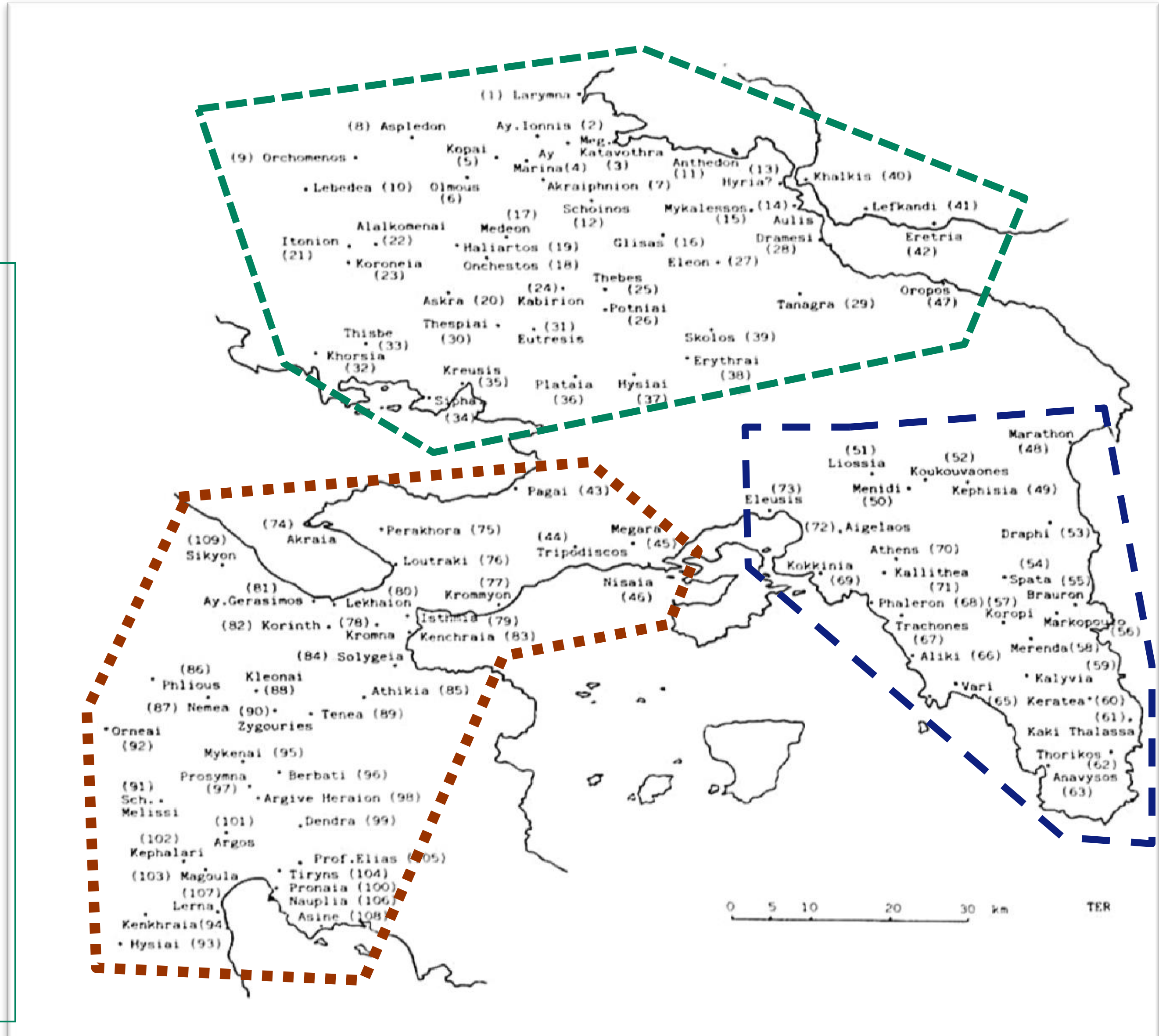


FIGURE 1

RW87 Fig 1: Distances

Distances are as-the-crow-flies

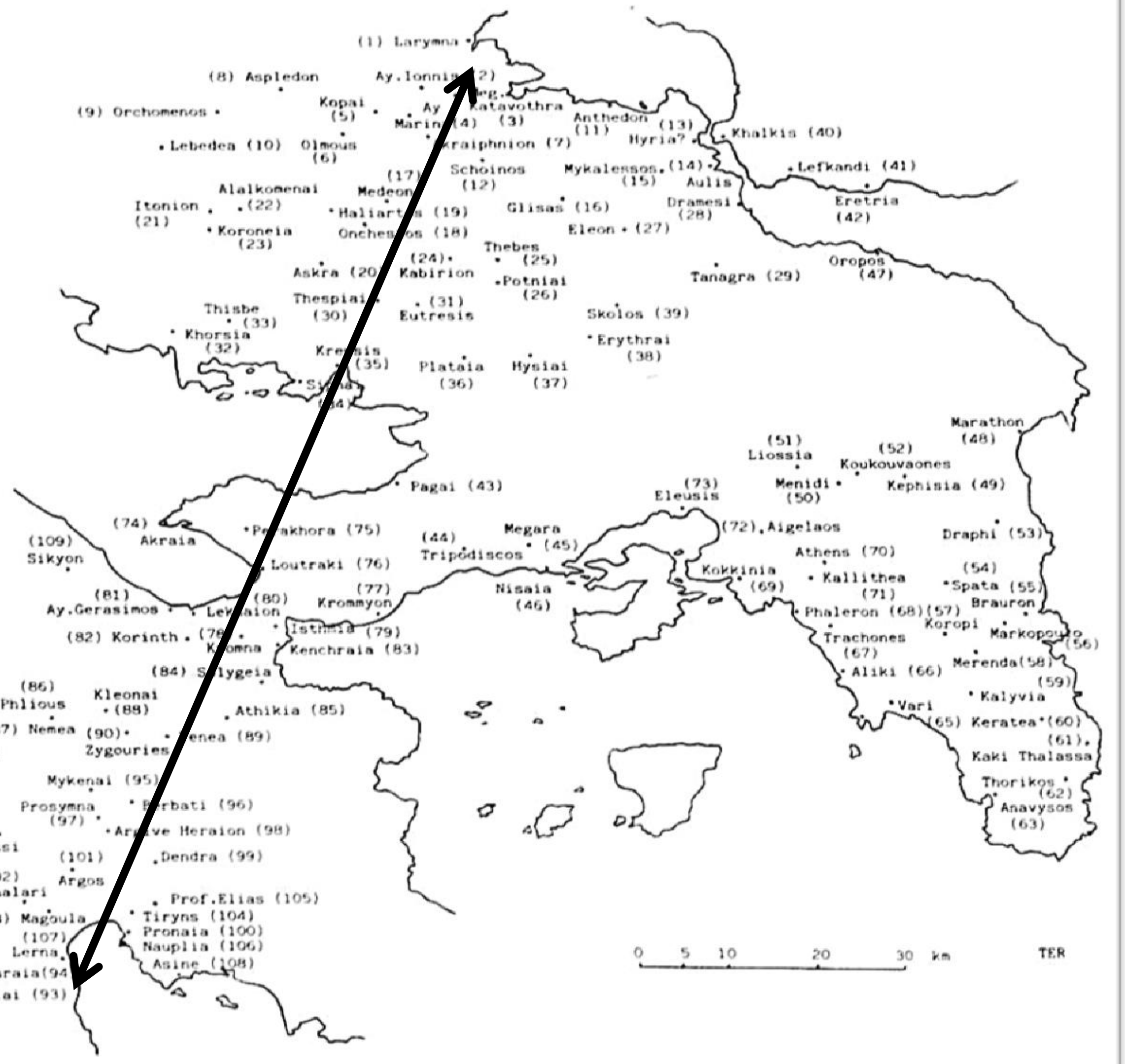
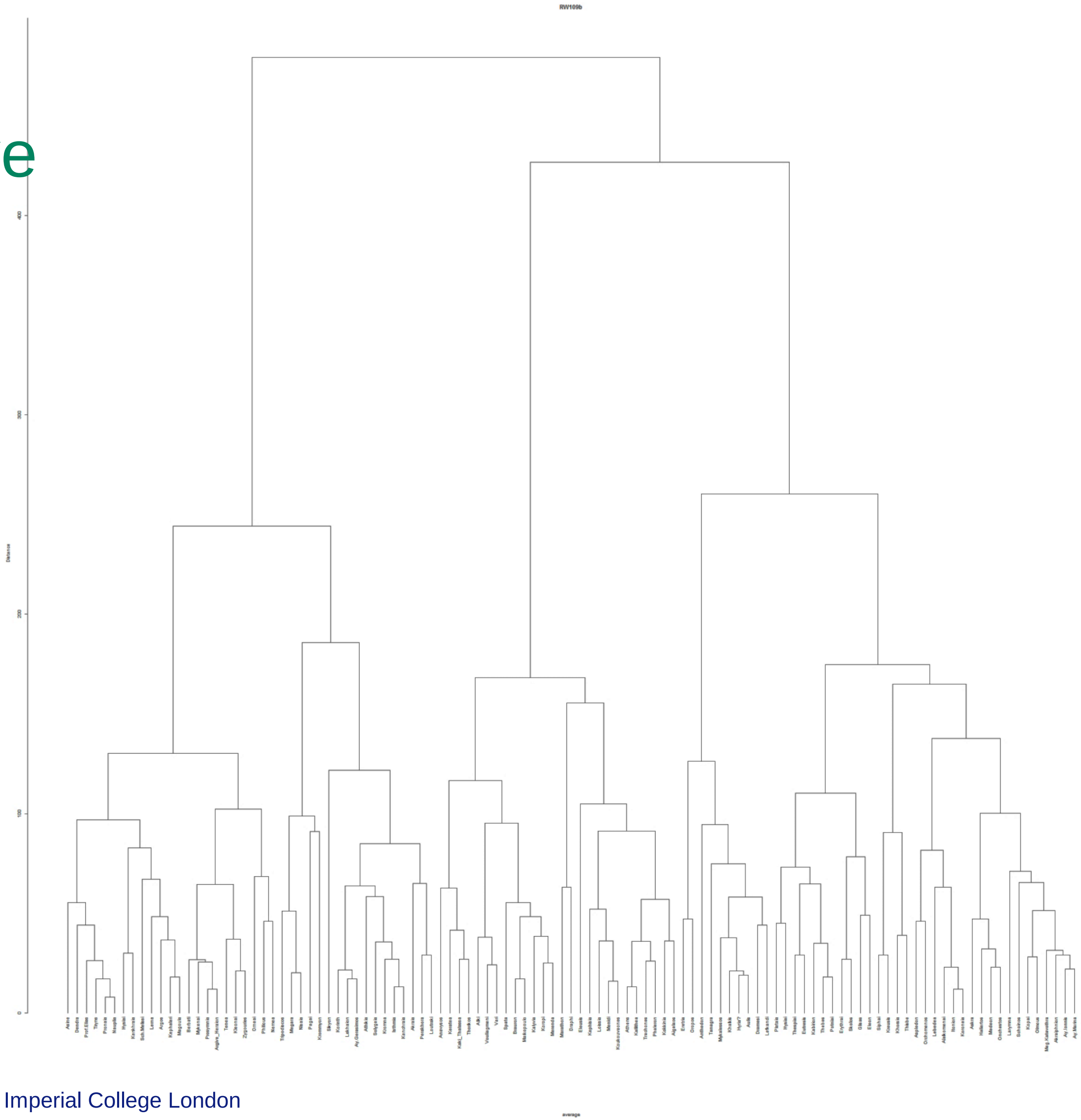


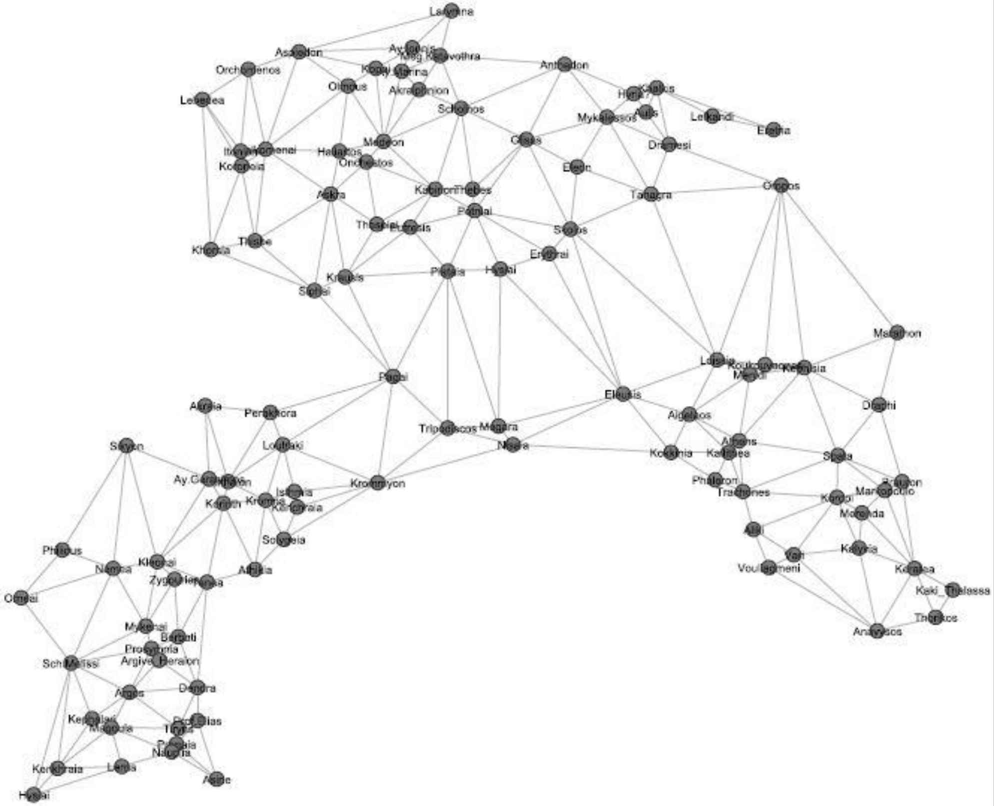
FIGURE 1

Hierarchical Agglomerative Clustering

- Average method
- 3 clear large scale regions
- No other clear scale?



Edited Delaunay Triangulation



- Produce Delaunay Triangulation
- Remove silly links
- Distances use shortest routes along remaining edges

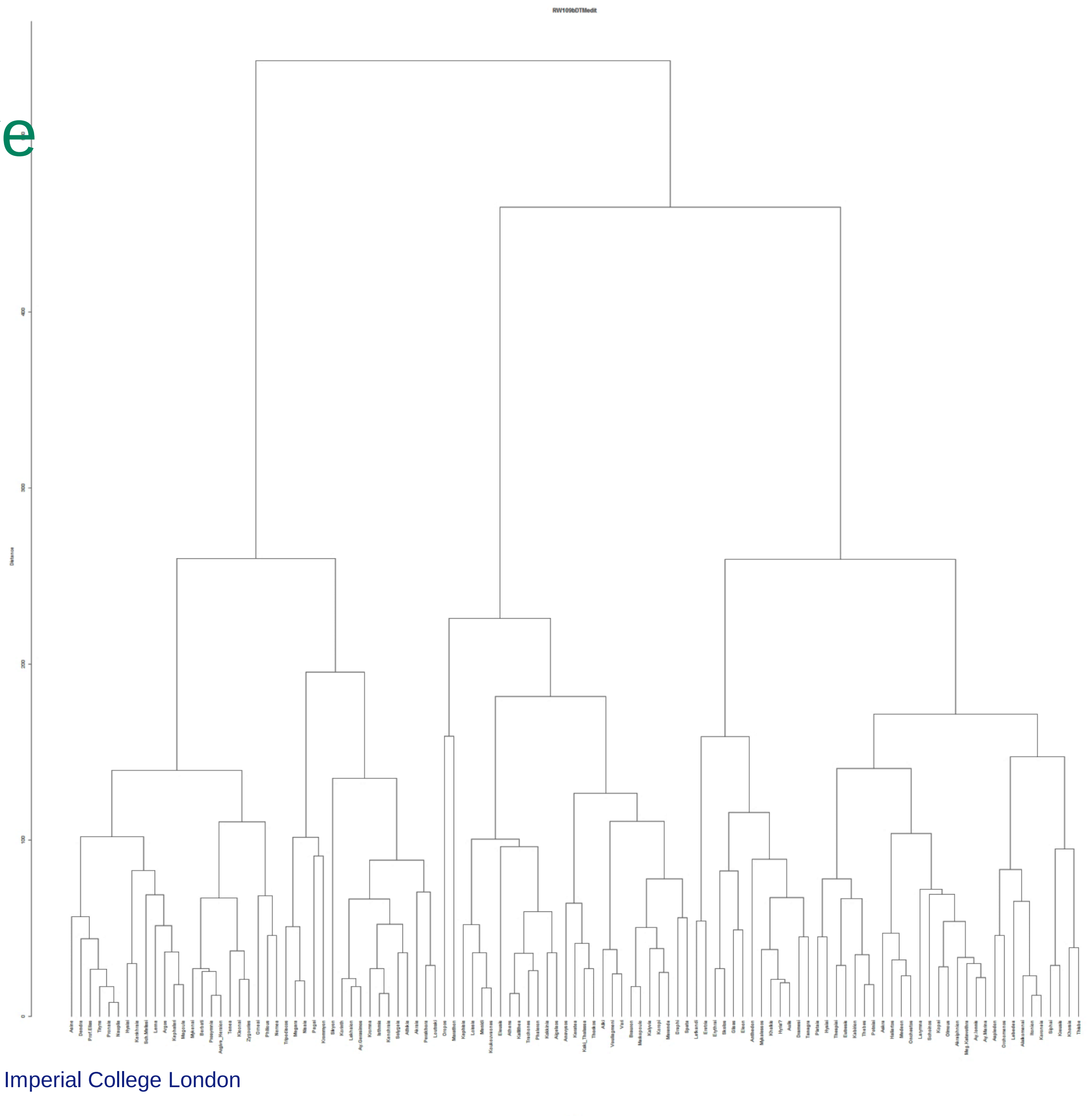
Result

- Immediate neighbour distance unchanged
- Other sites pushed further away

Hierarchical Agglomerative Clustering

DTMedit similar to before

- Average method
- 3 clear large scale regions
- No other clear scale?



Sparsification

How do you go from a **dense** matrix of flows

$$\begin{bmatrix} 0 & 0.1 & 0.2 & 0.7 \\ 0.1 & 0 & 0.1 & 0.8 \\ 0.1 & 0.1 & 0 & 0.8 \\ 0.2 & 0.4 & 0.4 & 0 \end{bmatrix}$$

$\boldsymbol{F_{ij} > 0}$ for all $\boldsymbol{i,j}$

to a sparse network?

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Sparsification for RW model

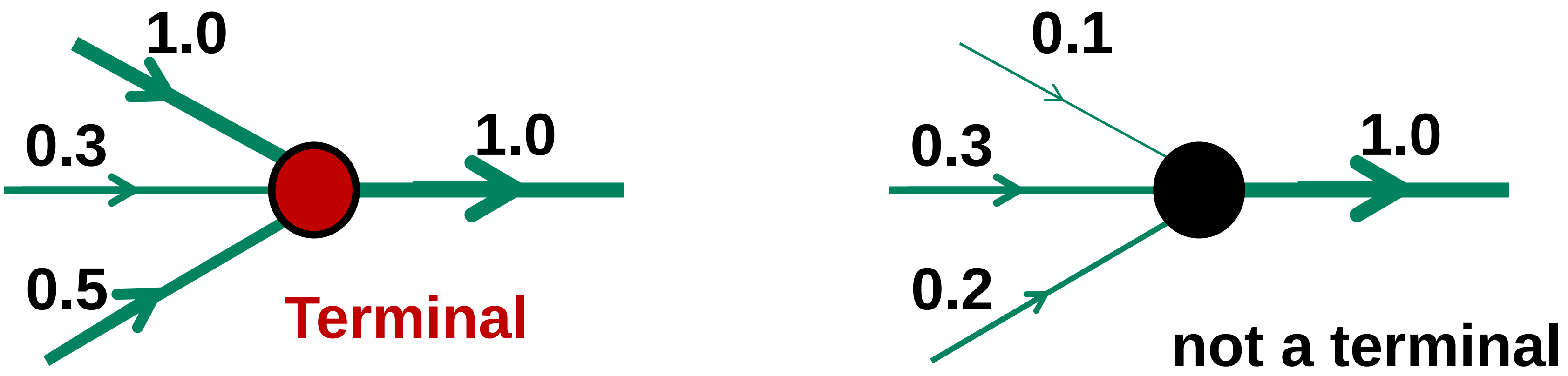
- Thresholding
  Edge present if flow bigger than some value F_0
  $$A_{ij} = \Theta(F_{ij} - F_0)$$

- **Not tried this**
  but may well work as
  - typical RW site has one strong edge out.
  - a few RW sites are star like with several equally weighted out edges

Sparsification using Terminals

[Nystuen & Dacey 1961, Rihll & Wilson 1991]

- Rihll & Wilson define **Terminal Sites**



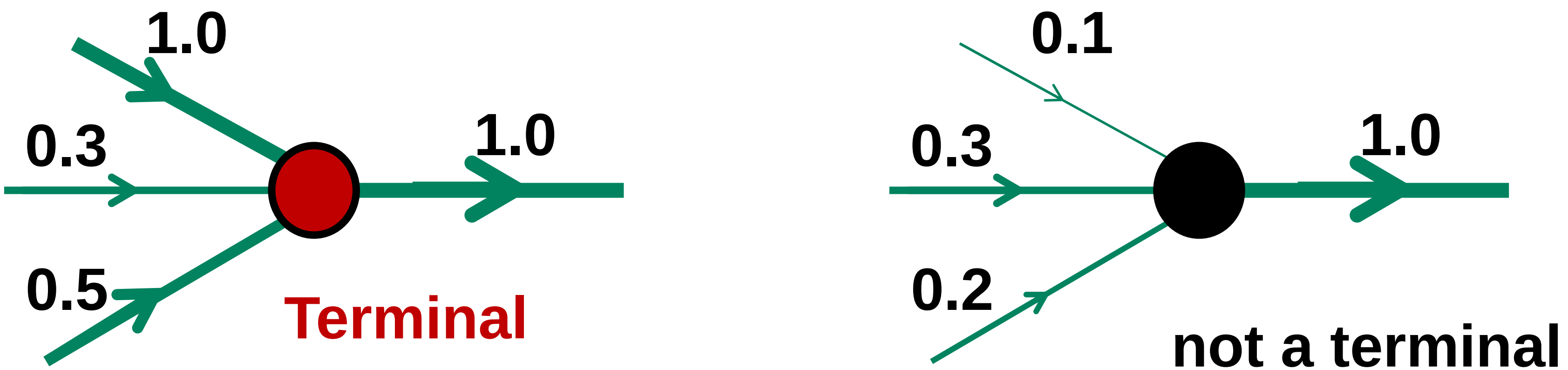
Terminal sites satisfy

Total Flow In > Largest Single flow out

More people owe you than you owe others

Terminals

- RW define **Terminal Sites**



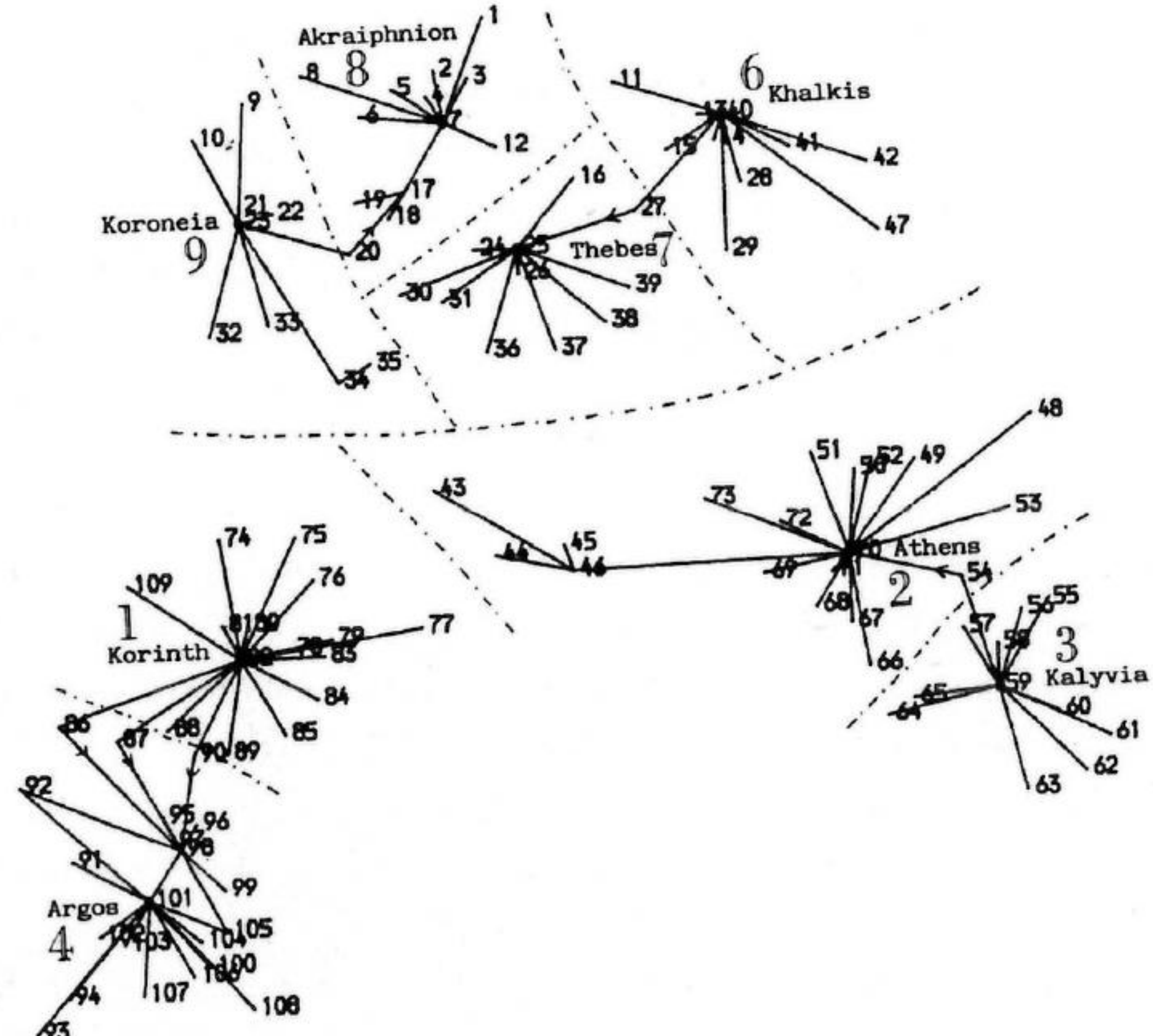
Terminal sites satisfy

Total Flow In > Largest Single flow out

More people owe you than you owe others

Terminals and Sparsification

Only show edges to and from terminals



[RW91, fig.6]

Figure 6: $\alpha = 1.05$ $\beta = 0.175$ $T = 8$ $\geq 75\%$ max. debit

Terminals as Dominant Sites

Rihll & Wilson identify terminal sites as dominant sites emerging from equal sized villages

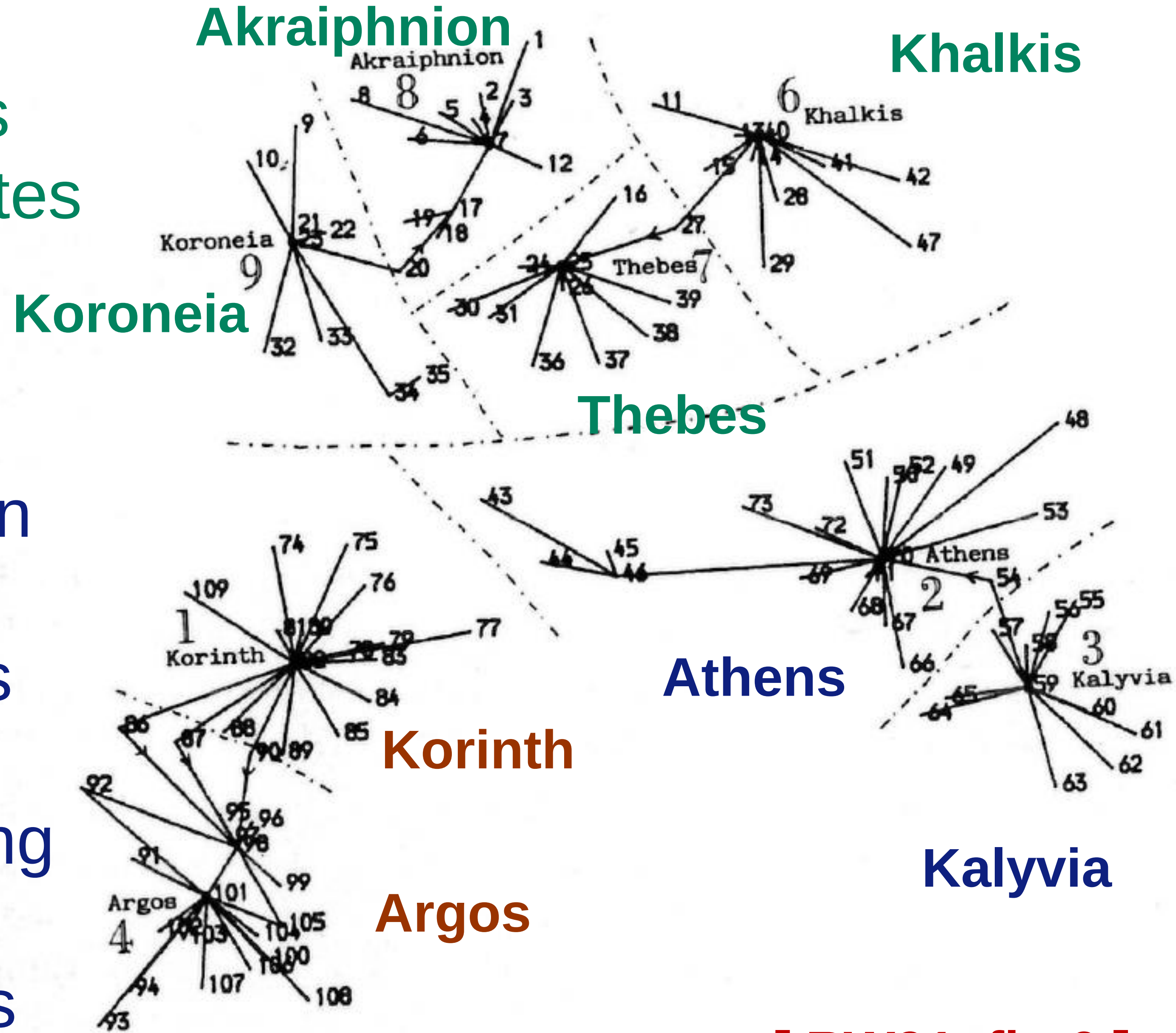
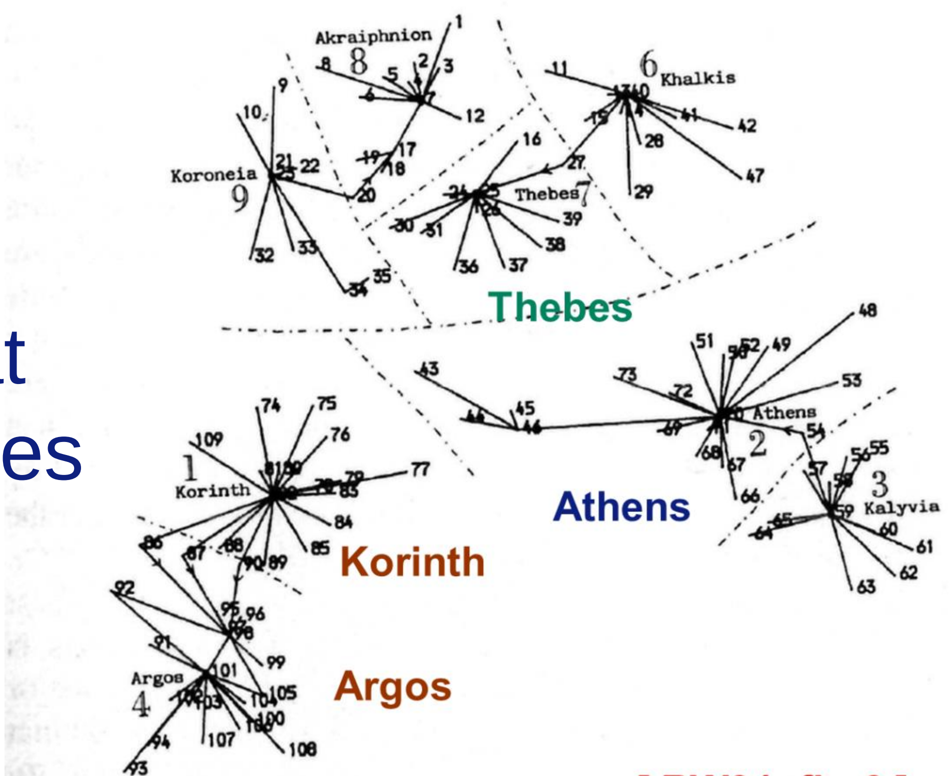


Figure 6: α = 1.05 β = 0.175 T = 8 ≥ 75% max. debit

[RW91, fig.6]

Uncertainty

- Rihll & Wilson looked at several parameter values but typically found similar results
  - Thebes and Athens always identified
  - Korinth and Argos or close neighbours often identified
  - Other centres also found in addition



[RW91, fig.6]

Figure 6: $\alpha = 1.05$ $\beta = 0.175$ $T = 8$ $\geq 75\%$ max. debit

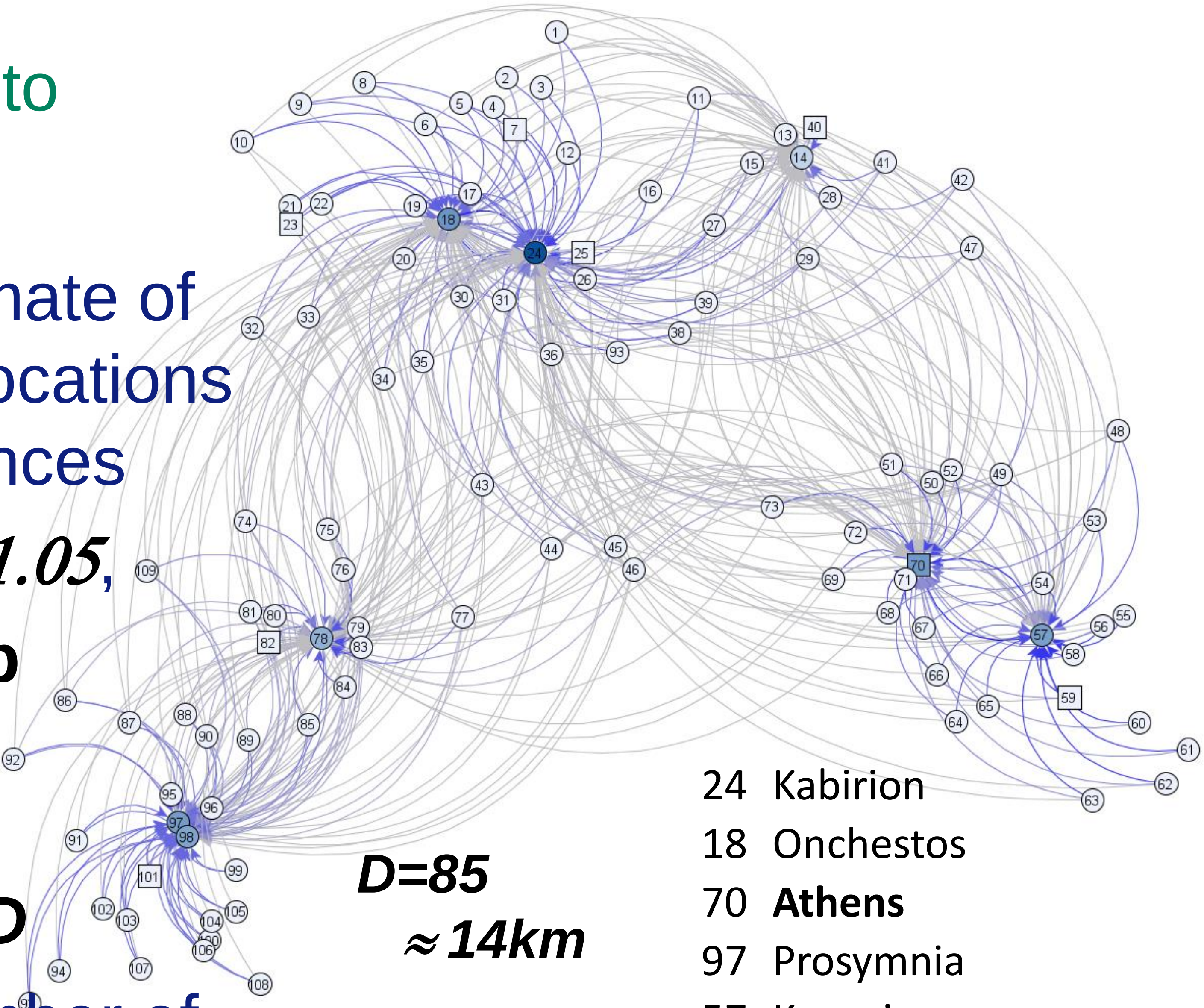
Uncertainty in Results

- Remarkably stable key results
- Only qualitative assessment of uncertainty
- Only one form of distance potential used pure exponential $\mathbf{exp}\,(-\boldsymbol{d_{ij}}/\boldsymbol{D})$
- No uncertainty in site locations of distances
- Unverifiable
  - no comprehensive list of site locations and distances used

until now

Best match to RW91 Fig 6

- Best estimate of RW site locations and distances
- Same β=*1.05*,
- Same **exp** potential
- vary distance ***D***
- same number of terminals ***8***

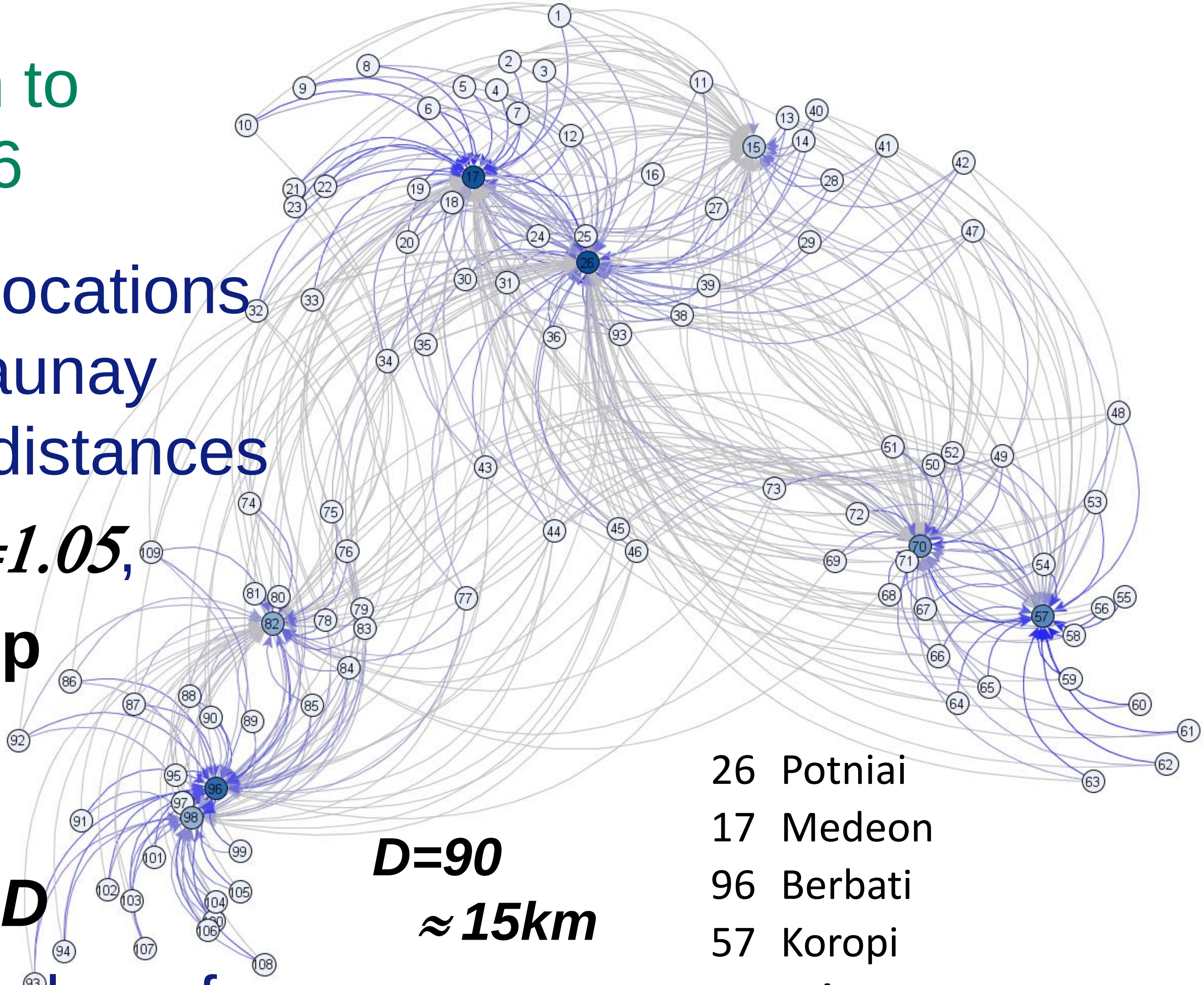


D=85
≈14km

24 Kabirion
18 Onchestos
70 **Athens**
97 Prosymnia
57 Koropi
98 Argive_Heraion
78 Kromna
14 Aulis

Best match to RW91 Fig 6

- RW site locations plus Delaunay inspired distances
- Same β=*1.05*,
- Same **exp** potential
- vary distance ***D***
- same number of terminals ***8***

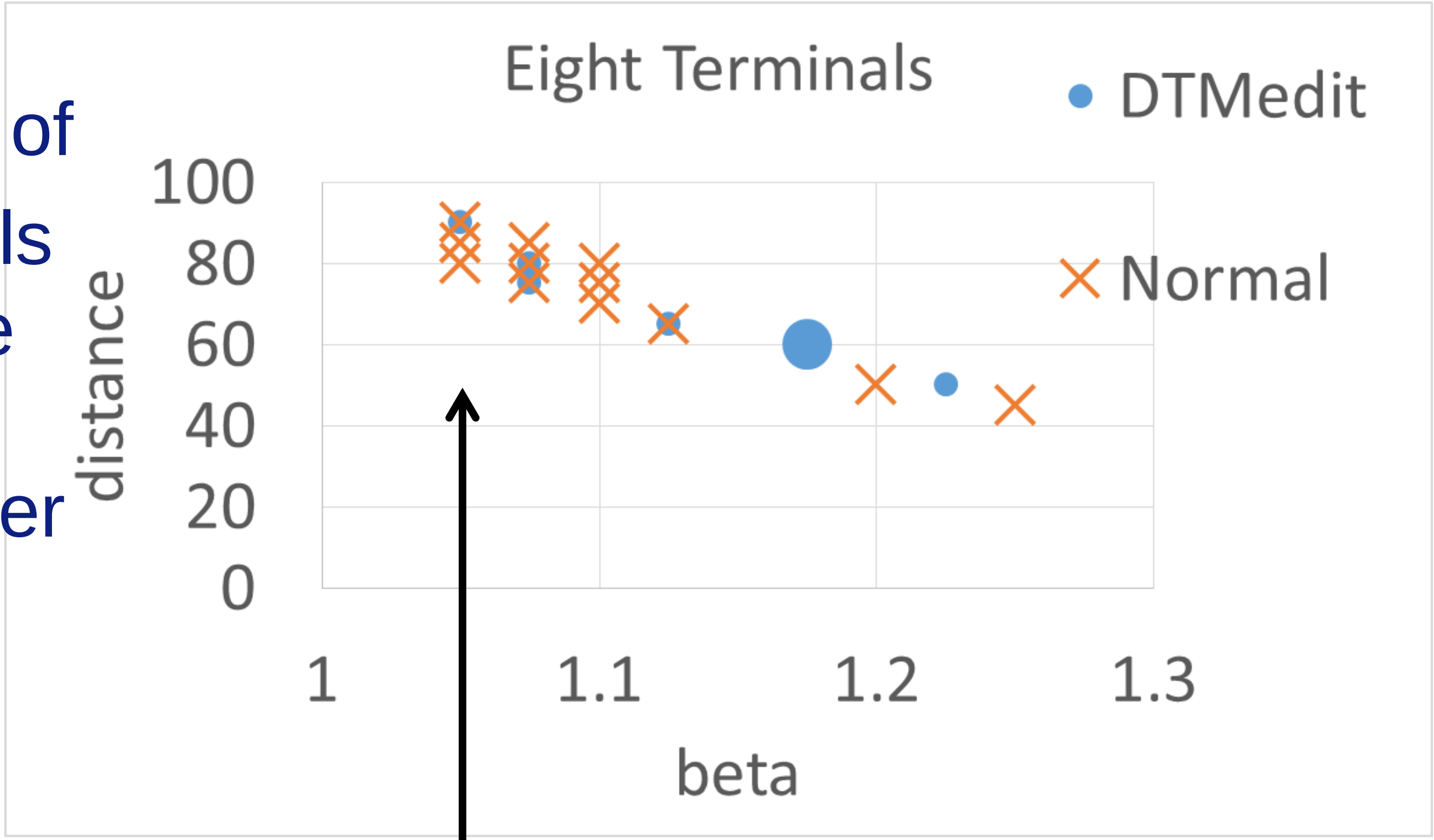


D=90 ≈15km

26 Potniai
17 Medeon
96 Berbati
57 Koropi
70 **Athens**
82 **Korinth**
98 Argive Heraion
15 Mykalessos

Large terminal number regions

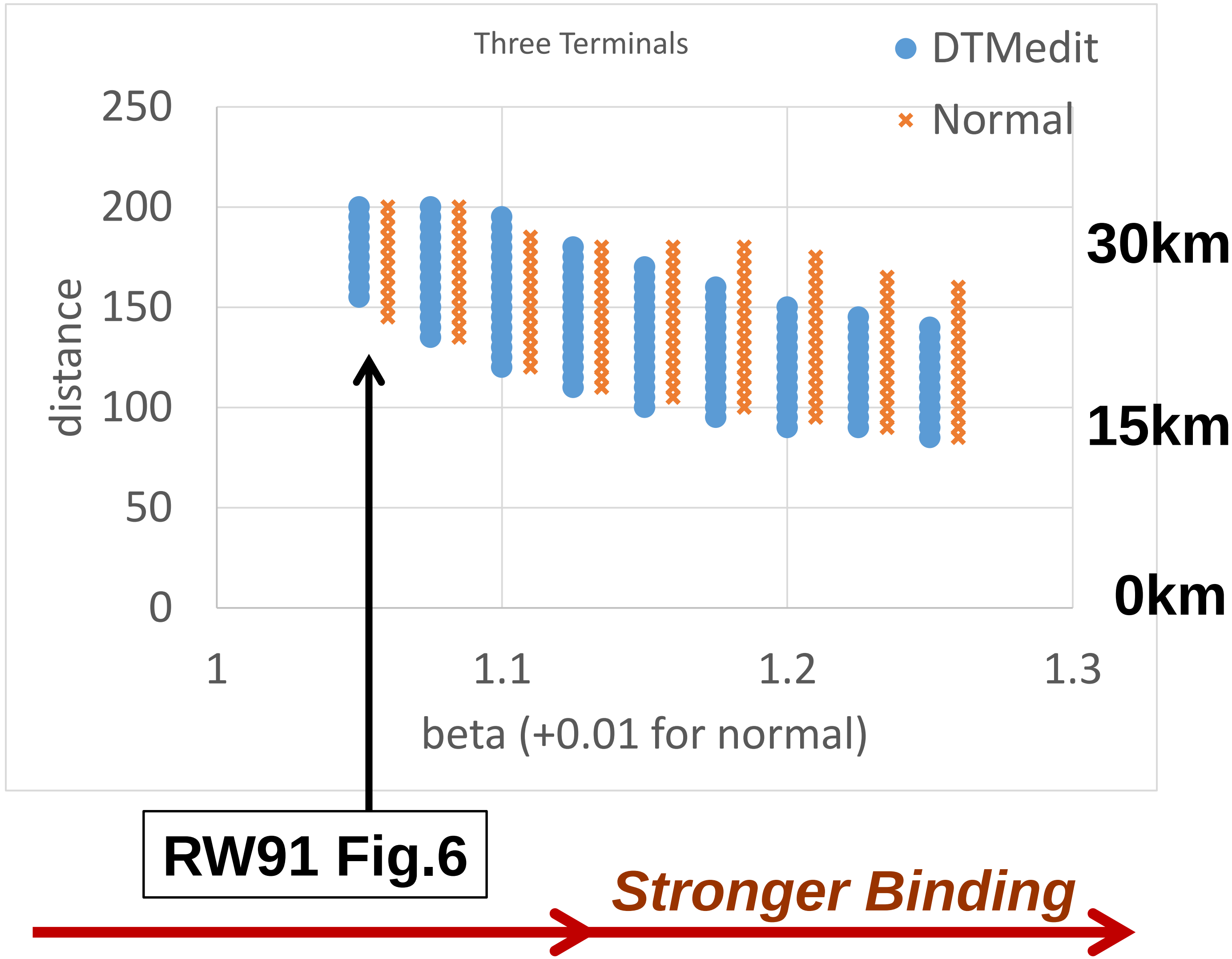
Number of Terminals sensitive to parameter values



RW91 Fig.6

Large terminal number regions

- Only 3 terminals stable
- DTMedit distance scales slightly larger



8 Terminals – in order of ‘hierarchy’

DTMedit D90	Normal D90	Normal D85	Normal D80	RW91 Fig 6
Potniai 26	Kabirion	Kabirion	Kabirion	Thebes (25)
Medeon 17	Athens	Onchestos	Onchestos	Akraiphnion (7), Koroneia (23)
Berbati 96	Prosymnia	Athens	Athens	Athens
Koropi 57	Koropi	Prosymnia	Koropi 57	Kalyvia 59
Athens 70	Argive Heraion	Koropi	Prosymnia 97	Argos (101)
Korinth 82	Onchestos	Argive Heraion	Argive Heraion 98	Argos (101)
Argive Heraion 98	Kromna	Kromna	Kromna 78	Korinth (82)
Mykalessos 15	Aulis	Aulis	Aulis 14	Khalkis (40)

8 Terminals – ordered by location

DTMedit D90	Normal D80, D85, D90	RW91 Fig 6	Wider Location
Mykalessos 15	Aulis 14	Khalkis 40	Eubaea environs
Potniai 26	Kabirion 24	Thebes 25	Neighbourhood of Thebes
Medeon 17	Onchestos 9	Akraiphnion 7 Koroneia 23	Northern Boeotia
Athens 70	Athens 70	Athens	Athens
Koropi 57	Koropi 57	Kalyvia 59	S.Attica
Korinth 82	Kromna 78	Korinth 82	Neighbourhood of Corinth
Berbati 96	Prosymnia 97	Argos (101)	Neighbourhood of Argos
Argive Heraion 98	Argive Heraion 98		

Largely consistent on scale of about 10km

8 Terminals

Now we have ‘error bars’

Ranges of uncertainty

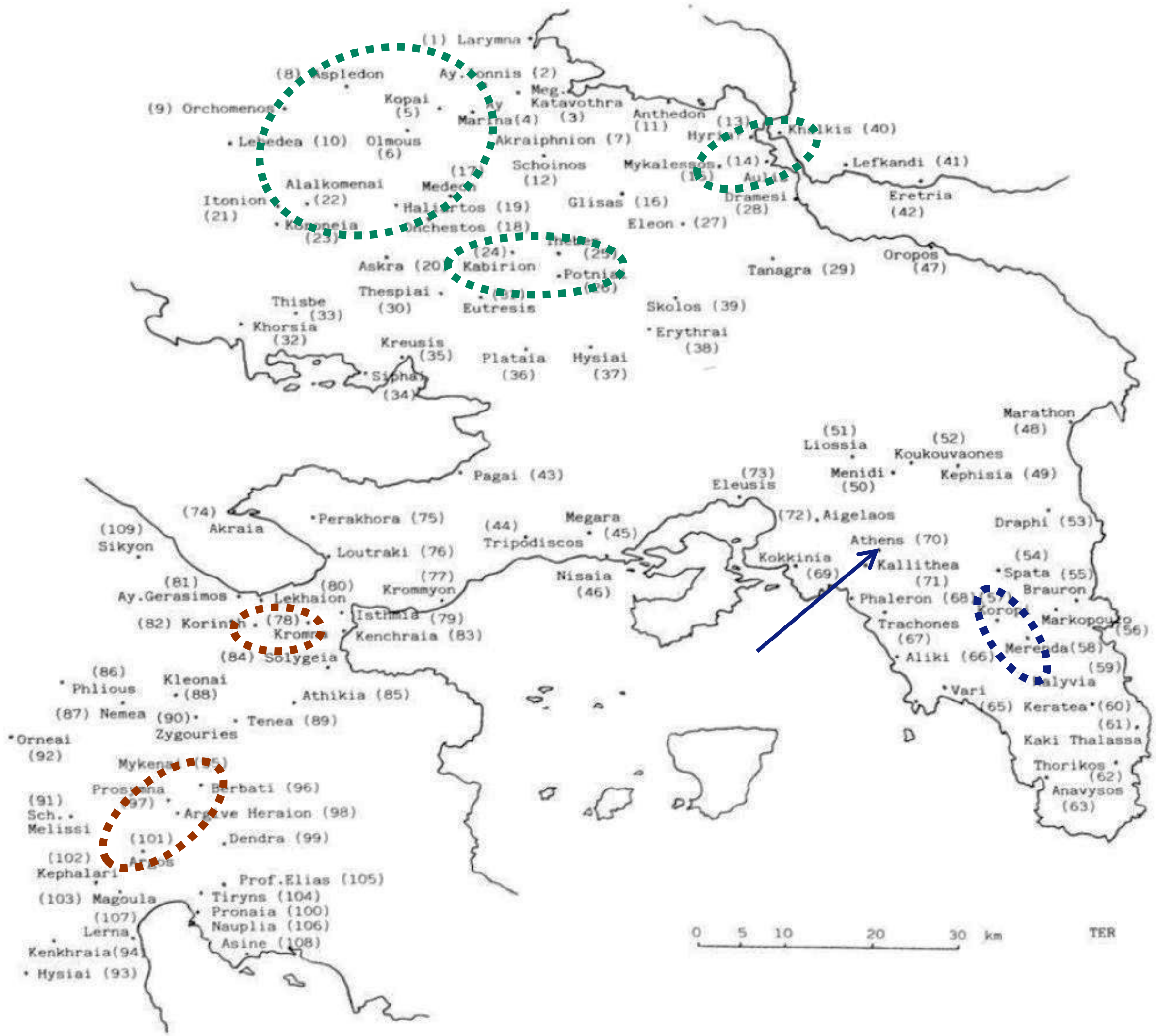
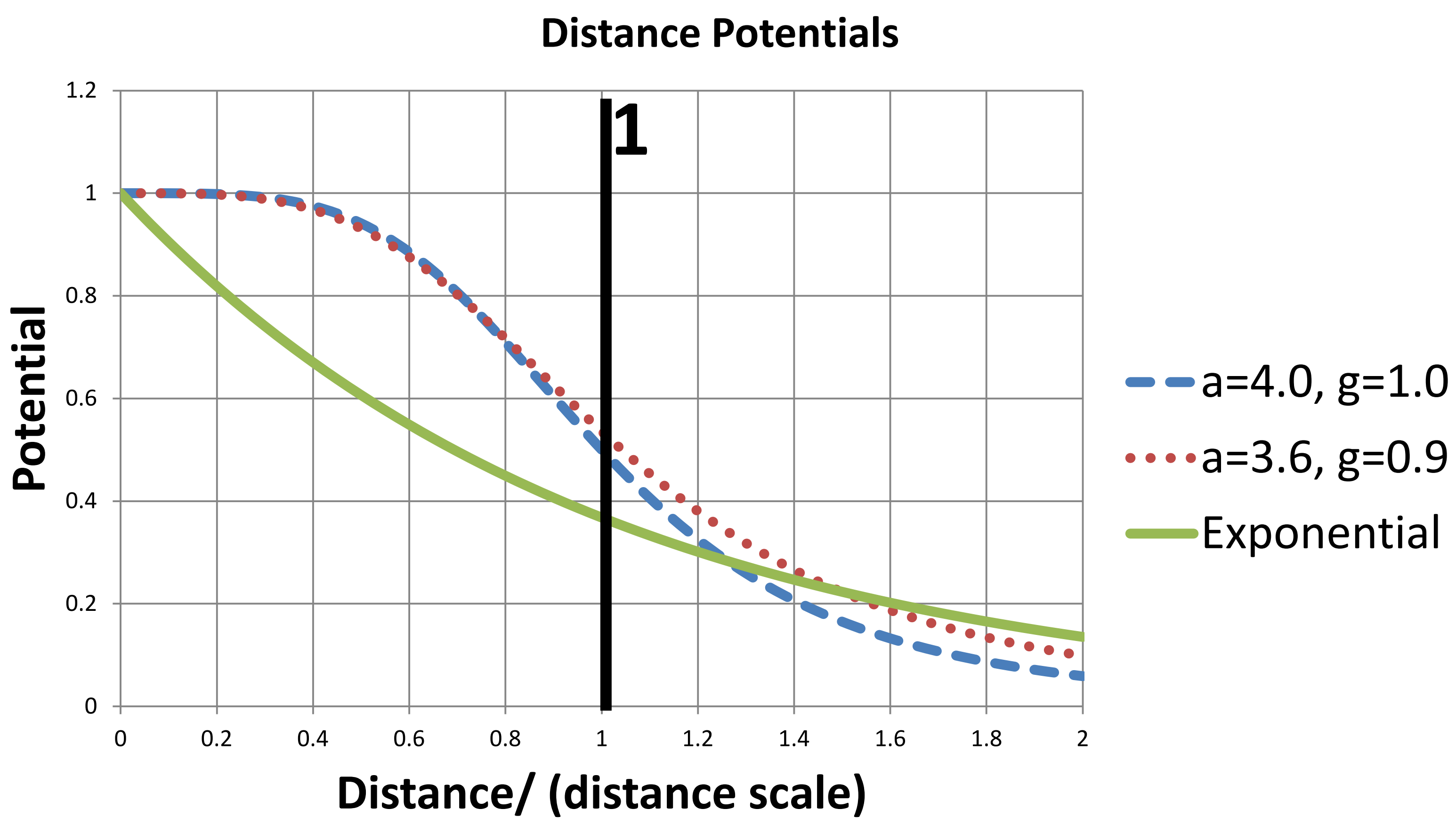


FIGURE 1

Different Distance Potentials

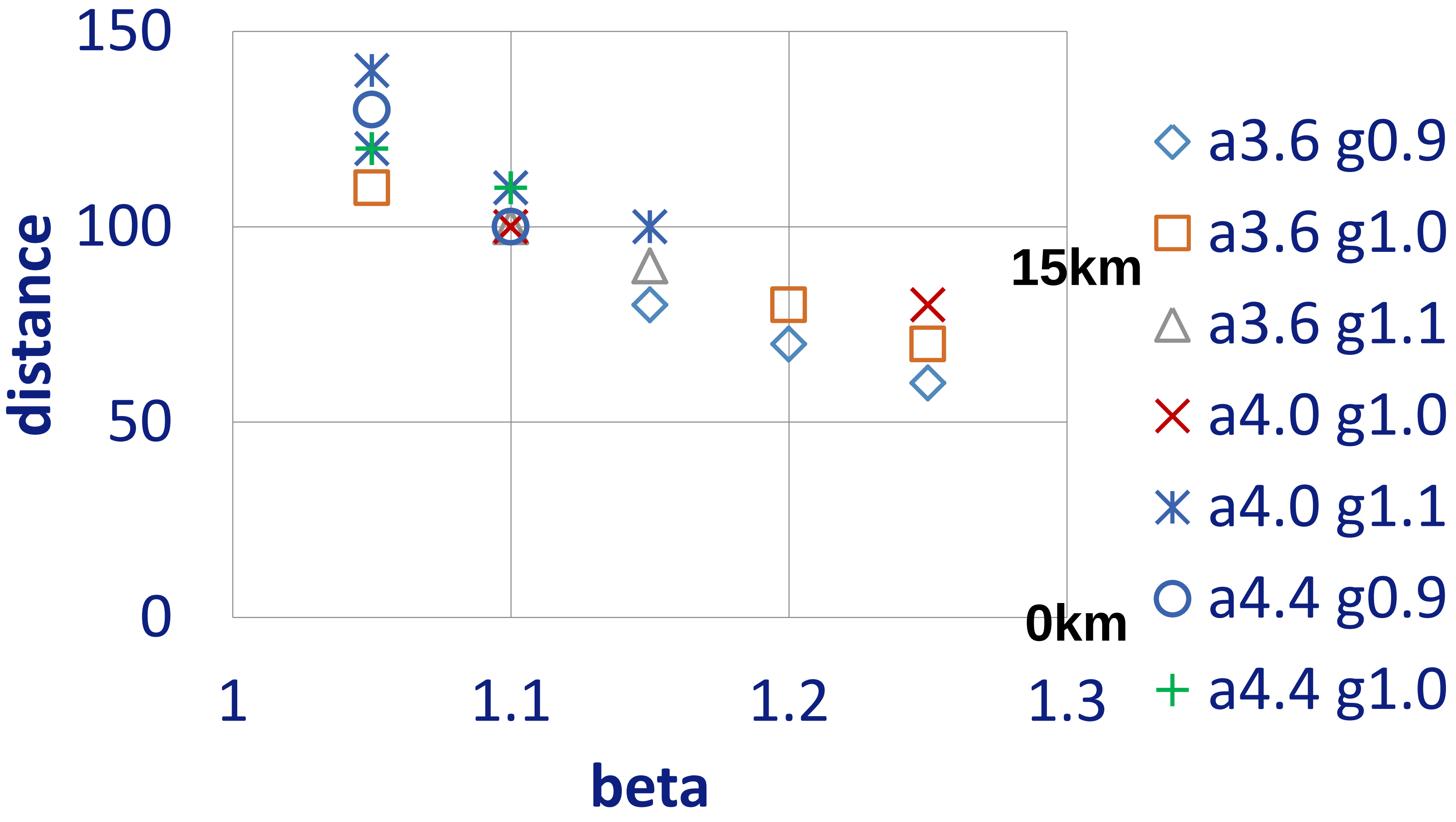
Ariadne	Exponential
$\dfrac{1}{(1+x^{\gamma})^{\alpha}}$	e^{-x}



8 Terminals

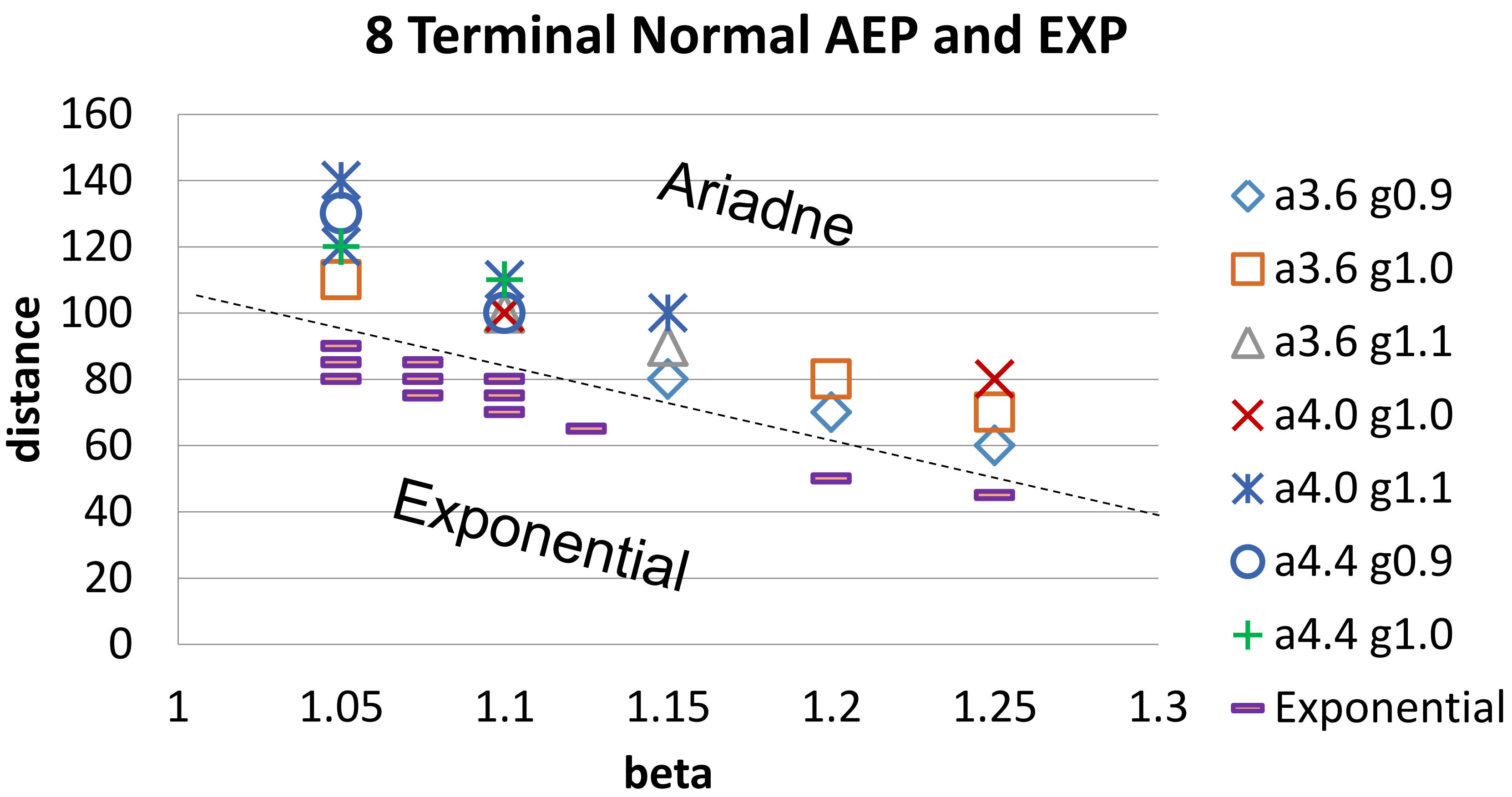
Normal data, ariadne potential

a$=\alpha$, ***g*** $=\gamma$



8 Terminals

Exponential and Ariadne Potential
Normal data, $\boldsymbol{a}=\alpha$, $\boldsymbol{g} = \gamma$



Note distance scales of different potentials are not directly compatible

3 Terminal Sites

Distance Data	Distance D	Terminal Sites		
		Boeotia	Attica	Isthmus/ Argolid
Normal	150	24 Kabirion	70 Athens	96 Berbati
Normal	155, 160, 165, 170, 175, 180	31 Eutresis	70 Athens	96 Berbati
Normal	185	31 Eutresis	70 Athens	89 Tenea
Normal	190,200	36 Plataia	70 Athens	89 Tenea
DTMedit	155,160,170, 175	26 Potniai	71 Kallithea	96 Berbati
DTMedit	180,190,200	26 Potniai	71 Kallithea	89 Teneai

Exponential potential

3 & 8 Terminals

Ranges of uncertainty

Grey =
8 Terminals

Coloured =
3 Terminals

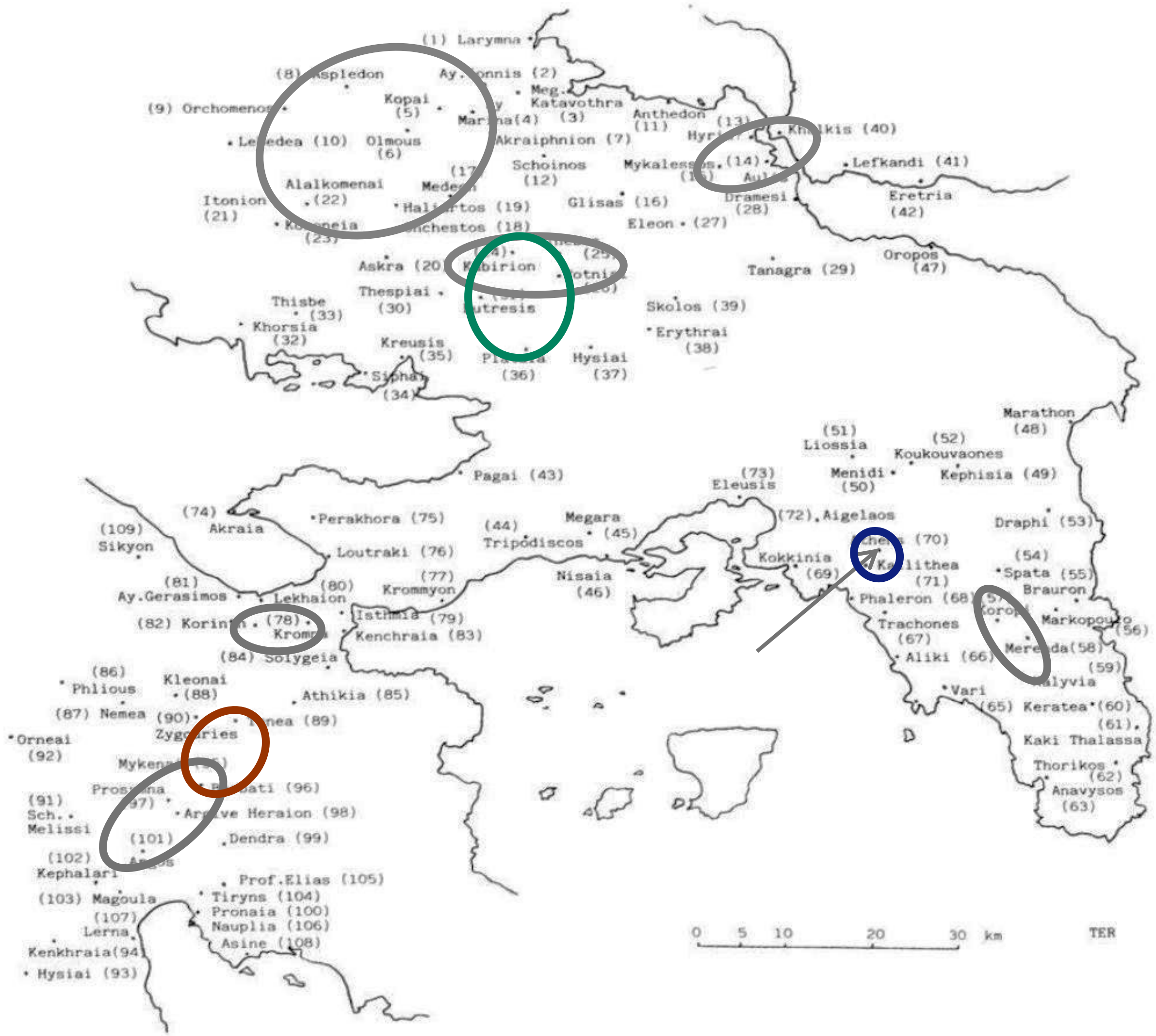


FIGURE 1

Summary

- Uncertainty should be reflected in results
- Should see “errors” – variability measures
  - Vary initial conditions ✓
  - Stochastic dynamics ✗ (but `ariadne/ERG`)
  - Vary parameters ✓
  - Different Models ✓
- Illustration with Rihll and Wilson data (1987, 1991)

Bibliography

- Ilias Bamis, Constrained Gravity Models for Network Flows, MSc thesis, Imperial College, 2012
- Bevan, A.,Political Geography and Palatial Crete, *Journal of Mediterranean Archaeology,* **2010***, 23* .
- Bevan, A. & Wilson, A., Models of settlement hierarchy based on partial evidence, *J.Archaeological Science,* **2013***, 40*, 2415-2427 [http://dx.doi.org/10.1016/j.jas.2012.12.025]
- Barthélemy, M., Spatial Networks, *Physics Reports,* **2011***, 499*, 1-101
- KNAPPETT, C., EVANS, T. & R. RIVERS, 2008. Modelling maritime interaction in the Aegean Bronze Age, *Antiquity* 82: 1009-24.
- Knappett, C.; Evans, T. & Rivers, R., The Theran eruption and Minoan palatial collapse: new interpretations gained from modelling the maritime network, *Antiquity,* **2011***, 85*, 1008-1023
- Renfrew, C. & Level, E. Exploring dominance: predicting polities from centres, in *Transformations: Mathematical Approaches to Culture Change,* Renfrew, A. & Cooke, K. *(Eds.), Academic Press, London,* **1979**, 145-67
- RIHLL, T.E. & A.G. WILSON, 1987. Spatial interaction and structural models in historical analysis: some possibilities and an example, *Histoire & Mesure* 2: 5-32.
- RIHLL, T.E. & A.G. WILSON, 1991. Modelling settlement structures in ancient Greece: new approaches to the polis, in J. Rich & A. Wallace-Hadrill (eds.), *City and country in the ancient world:* 59-95. London: Routledge.
- A.G.Wilson, A statistical theory of spatial distribution models, Trans. Res. 1 (1967) 253-269

Bibliography (TSE)

- Evans, T.S., 2004. *Complex Networks*, Contemporary Physics*, 45*, 455-474 [doi: 10.1080/00107510412331283531]
- Evans, T.S., Knappett, C., & R. Rivers, 2009. *Using statistical physics to understand relational space: a case study from Mediterranean prehistory*, in D. Lane, S. van der Leeuw, D. Pumain & G. West (eds.), *Complexity perspectives in innovation and social change*: 451-79. Berlin: Springer Methodos Series, [doi: 10.1007/978-1-4020-9663-1].
- Evans, T.S., Knappett, C., & R.J. Rivers, 2012. *Interactions in Space for Archaeological Models*, Advances in Complex Systems [doi: 10.1142/S021952591100327X].
- Evans, T.S., Knappett, C., Rivers, R., 2012. *Thirty Nine Minoan Sites*, Figshare http://dx.doi.org/10.6084/m9.figshare.97395 .
- Evans, T.S., Knappett, C., Rivers, R., 2012. *ariadne*, Figshare http://dx.doi.org/10.6084/m9.figshare.97746
- Evans, T.S, (2014). *Which Network Should I Use?,* in T. Brughmans, A. Collar and F. Coward (eds), *The Connected Past: challenging networks in archaeology and history.* Cambridge: Oxford University Press (in press).
- Expert, P.; Evans, T. S.; Blondel, V. D. & Lambiotte, R. 2011. *Uncovering space-independent communities in spatial networks, PNAS, 108*, 7663-7668 [doi: 10.1073/pnas.1018962108]
- Knappett, C., Evans, T.S. & R. Rivers, 2008. *Modelling maritime interaction in the Aegean Bronze Age*, Antiquity 82: 1009-24.
- Knappett, C.; Evans, T.S. & Rivers, R., 2011. *The Theran eruption and Minoan palatial collapse: new interpretations gained from modelling the maritime network*, Antiquity*, 85*, 1008-1023.
- Rivers, R.; Knappett, C. & Evans, T.S., 2013. *Network Models and Archaeological Spaces* in *Computational Approaches to Archaeological Spaces,* A.Bevan & Lake, M. *(Eds.)*, Left Coast Press*.*