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Measurement of flow vorticity with helical beams of light: supplementary material

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Numerical experiments on the measurement of flow vorticity

When a set of independent scatterers, moving with velocity $\vec{U}$, passes the ring–like observation region given by Eq. (2), it generates a burst of optical echoes that contributes to the received optical signal. We use a superposition model for the scattering process that directly gives the complex amplitude of the return signal as the sum of the fields scattered by all the scatterers illuminated by the LG beam. After coherent detection and filtering to remove the carrier frequency and its harmonics, we obtain a detected signal (photocurrent) i characterizing the optical echo from the target, which can be written as

$$\boldsymbol{i} = \sum_{j=1}^{n_S} \boldsymbol{i_{0j}(\vec{\rho}_j)\, exp\{i\,[\psi_j + \Phi(\vec{\rho}_j)]\} + n_0}\,. \qquad \text{(S1)}$$

Here, the summation is carried out over all the n_S illuminated scatterers and $\vec{\rho}_j = (\rho_j, \phi_j)$ is the transverse position of the jth scatter. The complex amplitude of the electric signal at the detector $i_{0j}(\vec{\rho}_j)$ takes into account the radiation distribution $E_0(\vec{\rho}_j)$ of the illumination beam, the complex scattering amplitude of the jth scatter, and the efficiency of the heterodyne detection process. The phase $\Psi_j = \psi_j + \Phi(\vec{\rho}_j)$ of the return from the jth scatter considers two independent terms, with $\Phi(\vec{\rho}_j) = m\phi_j$ describing the scatter position across the beam, and ψ_j the random nature of the return arrival times. As the n_S randomly phased elementary contributions to the total observed field interfere with one another, the resultant intensity is affected by speckle. In Eq. (S1), n_0 is an additive noise term being determined by the intensity of the detected signal. In heterodyne receivers, we can consider shot-noise limited signals where the noise power level is proportional to the detected photocurrent i squared.

Equation (S1) above describes not only the instantaneous noisy return, but also the temporal evolution $i(t)$ of the complex amplitude and the dynamics of speckle phenomena. We only need to consider that the instantaneous scatter position vector $\vec{\rho}_j$ changes with the flow velocity $\vec{U}_j$ observed by the jth scatter as $\vec{\rho}_j(t) = \vec{U}_j\, t$.

Typically, an heterodyne receiver sample the output signals of a complex receiver with a specific sampling frequency F_S and temporal vector $\vec{\imath} = (i_1, i_2, \dots, i_M)$ describes the M complex data samples obtained from the return signal $i(t)$ at temporal sampling intervals $1/F_S$. The power Doppler spectrum $I(f_\perp)$ can be estimated from the linear discrete Fourier transform of the complex data vector $\vec{\imath}$ and requires a sufficiently long data sequence for the spectrum to be well defined. The resulting spectral data vector $\vec{I} = (I_1, I_2, \dots, I_M)$ describes the power levels obtained in M spectral channels $\vec{f}_\perp = (f_{\perp 1}, f_{\perp 2}, \dots, f_{\perp M})$. For a discretely sampled return, the bandwidth extends over frequencies limited by the Nyquist frequency, which is half the sampling frequency F_S. In actual heterodyne receivers aliasing effects are a major concern and signal bandwidth B is a fraction of the system sampling frequency. As the return spectrum is built of frequency components $f_\perp$ arising from sets of scatterers moving with different speeds and that are uncorrelated in position, these different frequency components are also uncorrelated.

As speckle noise on the spectral measurements degrades the quality of the spectral data, speckle is usually reduced by summing N independent unsmoothed sample spectrum, and the accumulated spectrum becomes $\vec{I} = \sum_{n=1}^{N} \vec{I}_n$. After spectral accumulation, the signal spectral data $\vec{I}$ can be used to calculate the frequency centroid integral as the weighted mean of the frequencies present in the signal

$$\langle f_\perp \rangle \equiv \int_{-F_S/2}^{F_S/2} I(f_\perp)\, f_\perp\, df_\perp = \sum_{k=1}^{M} f_{\perp k}\, I_k \ . \qquad \text{(S2)}$$

Equation (S2), in conjunction with Eq. (6), allows estimating the vorticity ω directly from the spectrum of the return signal. In our numerical experiments, typical parameters for a heterodyne receiver are chosen to be signal bandwidth normalized to the sampling frequency $B/F_S = 0.5$, and number of complex samples per estimate $M = 64$. The numerical experiments consider a signal return accumulation (spectral accumulation) of $N = 250$.

Lab experiments on the measurement of flow vorticity

We extract the Doppler frequency shift imparted by the moving particles via an interferometric technique using the modified Mach-Zehnder interferometer shown in figure 3 (a). A 15-mW continuous wave He-Ne laser (Melles-Griot, $\lambda = 632.8$ nm) is spatially filtered and expanded to a 5 mm diameter beam. This by using a combination of lenses L_1 and L_2 of focal lengths $f_1 = 50$ mm and $f_2 = 200$ mm respectively, and a 30-μm pinhole (PH_1) placed at the middle focus. Afterwards the beam is split into two beams, a reference (red line) and a proof (green line), using a polarizing beam splitter (PBS_1). A mirror (M) redirects the proof beam to a Spatial Light Modulator (SLM) that imprints the beam with the desired phase profile. The first diffracted order of a fork-like hologram encoded into the SLM is used to illuminate a digitally emulated flow, as explained below. The remaining diffracted orders are spatially filtered using two lenses (L_3 and L_4) of focal lengths $f_3 = 150$ mm and $f_4 = 35$ mm respectively and a pinhole (PH_2), 200 μm in diameter, placed at the middle focus. The outer diameter of the resulting Laguerre-Gauss beam is $1160\ \mu m$ when the winding number is $m = 10$. A second polarizing beam splitter (PBS_2) in combination with a quarter waveplate (QWP) collects light reflected from the simulated flow back into the interferometer (blue line). These reflections are afterwards interfered with the reference signal using a beam splitter (BS). A balanced detection scheme is implemented with two photodetectors PD_1 and PD_2 connected to an oscilloscope (TDS2012 from Tektronix). To eliminate the low-frequency noise, a Phase Shifter (PS) connected to a Frequency generator (FG), placed in the path of the signal beam, shifts our detected frequency to 1 kHz.

We emulated particle flows with different velocity profiles using a DMD (DLP3000 Lightcrafter evaluation mode). The RGB LED light engine was removed to expose the DMD display. The DLP3000 is composed of 415,872 micromirrors (with a diagonal side length of $10.8\ \mu m$) arranged in a diamond pattern geometry 608×684. Each mirror can be tilted individually from $+12°$ to $-12°$. Hence, by carefully aligning the DMD such that $+12°$ coincides with the plane perpendicular to the incident beam, only mirrors tilted $+12°$ will reflect light parallel to the incident beam, defining our "on" state. Mirrors tilted at $-12°$ will reflect light at an angle that is blocked, defining our "off" state. The time in which the micromirrors are in the "on" or "off" state can be controlled by the software provided with the DMD. In order to simulate a particle moving in an specific direction, a set of 1-bit depth images is uploaded in the DMD software, where a "0" corresponds to the "on" state and a "1" to the "off" state. For example, to simulate a particle moving from left to right, we start with an image in which only one mirror close to the left edge is in the "on" state, in subsequent images this "on" state is displaced to the right by turning "off" this mirror and turning "on" the mirror to its right.

In our experiment we simulated different velocity profiles by creating a set of images, so that in the first images approximately 40,000 of the 608×684 available micromirrors are randomly set to an "on" state. In subsequent images these micromirrors are changed to the "off" state and new mirrors to the "on" state. Which mirrors are turned "on" depends on the velocity profile we want to simulate. For example, in the supplementary movie we show a parabolic velocity profile. Notice how the "on" states (in white) close to the edge of the image moves much slower than those at the center of the image. Those particles that leave the image area are replaced by the same amount following the same velocity profile.

Data analysis

Both the numerical and the lab experiments share the same data analysis. Understanding how the strength of our signal is distributed in the frequency domain is central to the design of any vorticity sensor intended to use the signal Doppler shift. Our data processing and analysis is rather straightforward and computes a spectrum or spectral density starting from a digitized time series, typically measured in Volts at the input of the A/D-converter. We use the overlapped segmented averaging of modified periodograms for power spectrum estimation. In our case, a periodogram is the discrete Fourier transform (DFT) of one segment of the signal time series that has been modified by the application of a time-domain window function. The practical implementation involves equal binning of frequencies, Hanning windowing, filtering of unwanted residual amplitude modulations, and spectrum averaging to reduce the variance of the spectral estimates. Finally, an estimation of the mean Doppler frequency, or Doppler centroid, operates in the power spectrum of the data by applying Eq. (S2).

Flow Visualization

The supplementary movie visualizes one flow generated by the DMD in the experimental setup. The elapsed time is displayed in the top right corner of the movie. Here, x and y are the transversal and longitudinal dimensions of the flow channel, respectively. The illuminating beam intensity distribution is shown as a bright yellow ring of radius $\rho_0 \approx 500$ μm. The parabolic profile of velocities used to describe the flow is exposed in the visualization (blue line, right axis).