Pseudo Panel Data Models with Cohort Interactive Effects
Online Supplementary Material

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1. Theoretical Results: Extensions

1.1. Unbalanced Samples

The quasi-differencing approach can be extended to allow for the number of time-series observations to be cohort specific (T_s), such that observations for cohort s are observed only at time periods $t \in \mathcal{T}_s$ (here $\mathcal{T}_s$ is the set of time indices for which observations are available for cohort s). This extension is important from an empirical point of view as some cohorts can disappear if the time span is sufficiently long, or alternatively, if irregularly spaced surveys from different countries are used, as in McKenzie (2001). The total number of cross-sectional averages is $T_{S'} = \sum_{s=1}^{S} T_s$, which in the case of a balanced panel is equal to ST. At first we define

$$\boldsymbol{P} = \mathrm{diag}(\boldsymbol{P}_1, \ldots, \boldsymbol{P}_S), \quad T = \left| \bigcup_{s=1}^{S} \mathcal{T}_s \right|,$$

where T is the total length of time series, while each $\boldsymbol{P}_s$ is a $[T_s \times T]$ group specific "selection matrix" of ones and zeros that is equal to $\boldsymbol{I}_T$ if the panel is balanced. Using this notation, the DGP in the stacked notations can be expressed as

$$\boldsymbol{y} = \boldsymbol{P}\,\mathrm{vec}\,(\boldsymbol{F}\boldsymbol{\Lambda}') + \boldsymbol{X}\boldsymbol{\theta} + \boldsymbol{u}, \tag{1.1}$$

where $\boldsymbol{X}$ now is of dimensions $[T_{S'} \times K]$. Assuming that the last L observations are observed for all cohorts, the QD matrix $\boldsymbol{M}(\boldsymbol{\phi})$ in this case can be written in the following way

$$\boldsymbol{M}(\boldsymbol{\phi}) = \boldsymbol{R}(\boldsymbol{I}_S \otimes (\boldsymbol{I}_{T-L}, \boldsymbol{\Phi}))\boldsymbol{P}', \tag{1.2}$$

where $\boldsymbol{R}$ is a $[(T_{S'} - SL) \times (T - L)S]$ block-diagonal selection matrix that selects only those rows of $(\boldsymbol{I}_S \otimes (\boldsymbol{I}_{T-L}, \boldsymbol{\Phi}))\boldsymbol{P}'$ that are available to the researcher.

In general, it is not necessary to use the last L observations for normalisation. The general necessary condition for normalisation to be feasible is

$$T_{min} = \left| \bigcap_{s=1}^{S} \mathcal{T}_s \right| \geq L, \tag{1.3}$$

which is obviously satisfied if the last L observations are observed. Furthermore, we define the set

$$\mathcal{S}^* = \{i, j, \ldots, k \in \{1, \ldots, S\} : \mathrm{rk}\,(\boldsymbol{\lambda}_i, \boldsymbol{\lambda}_j, \ldots, \boldsymbol{\lambda}_k) = L\}.$$

In other words, the set $\mathcal{S}^*$ contains all subsets of cohorts whose factor loadings span an L dimensional space. Using this definition we can formulate the identifying restriction as follows

$$\forall t \in \bigcup_{s=1}^{S} \mathcal{T}_s, \quad \exists S_t \subseteq \mathcal{S}^*, \quad s.t. \quad \forall s \in S_t : t \in \mathcal{T}_s, \quad |S_t| \geq L. \tag{1.4}$$

Combining these assumptions we can rewrite Assumption **(A.5)** as:

(A.5) a) $\sum_{s=1}^{S} T_s - (T-L+S)L > K$ with L_0 being the true number factors with non-zero mean factor loadings. b) $\forall t \in \bigcup_{s=1}^{S} \mathcal{T}_s, \quad \exists S_t \subseteq \mathcal{S}^*, \quad s.t. \quad \forall s \in S_t : t \in \mathcal{T}_s, \quad |S_t| \geq L_0$. c) $\left|\bigcap_{s=1}^{S} \mathcal{T}_s\right| \geq L_0$ and $\boldsymbol{F}_{L\times L}^{-1}$ exists. d) $\boldsymbol{F}$ and $\boldsymbol{\Lambda}$ are deterministic matrices.

In this case, $\boldsymbol{F}_{L\times L}$ denotes some $[L \times L]$ dimensional block of the $\boldsymbol{F}$ (not necessarily the last block), that is used for normalisation.

1.2. Dynamic Models

In the main text we assume that the vector of individual specific regressors $\boldsymbol{z}_{i,t}$ does not contain any lags of $y_{i,t}$. If p lags of $y_{i,t}$ are included among the regressors, then some of the results need to be adjusted. Note that unlike in genuine panel data models, the mere presence of $y_{i,t-1}, \ldots, y_{i,t-p}$ does not violate the consistency of the non-linear GMM estimator for fixed T, and consequently one does not have to use lagged values of $y_{i,t}$ as instruments. The fact that under Type I asymptotics dynamic pseudo panel data estimators have no "Nickell" bias (Nickell (1981)) is well documented in the literature (e.g. McKenzie (2004) and Verbeek and Vella (2005)).

For most pseudo panel datasets no information regarding the history of any individual i is observed, hence the historical averages $(y^*_{s,t-1}, \ldots, y^*_{s,t-p})$ are not observed either.[1] The following modified assumption **(A.2)** is sufficient to allow for the use of observed

[1] Here $y^*_{s,t-p} = (1/N_{s,t} \sum_{i\in\mathcal{I}_{s,t}} y_{i,t-p})$ and we use $*$ to distinguish between averages of the lagged dependent variables and lagged averages of dependent variables themselves $((1/N_{s,t} \sum_{i\in\mathcal{I}_{s,t}} y_{i,t-p})$ vs. $(1/N_{s,t-p} \sum_{i\in\mathcal{I}_{s,t-p}} y_{i,t-p}))$.

quantities ($\ddot{\boldsymbol{y}}_{s,t} = (\bar{y}_{s,t-1}, \ldots, \bar{y}_{s,t-p})'$) instead of their unobserved counterparts ($\boldsymbol{y}^*_{s,t} = (y^*_{s,t-1}, \ldots, y^*_{s,t-p})'$):

(A.2) $u_{i,t}$ are i.h.d. with finite $2+\delta$ moment, for $\delta > 0$, such that $\sqrt{N_{s,t}}(\bar{u}_{s,t} + (\boldsymbol{y}^*_{s,t} - \ddot{\boldsymbol{y}}_{s,t})'\boldsymbol{\alpha}_0) \xrightarrow{d} \mathcal{N}(0, \sigma^2_{s,t})$ jointly $\forall s, t \geq p$ with $0 < \min \sigma^2_{s,t} \leq \max \sigma^2_{s,t} < \infty$.

Here $\boldsymbol{\alpha}_0$ is a $[p \times 1]$ vector of corresponding true coefficients for lagged dependent variables. Due to the non-zero covariance between $\bar{u}_{s,t-1}$ and $\bar{y}_{s,t-1}$ (for $p = 1$ and analogously for $p > 1$), the variance-covariance matrix $\boldsymbol{\Sigma}$ has a block tri-diagonal structure (for further details please refer to *Assumption (2c)* of Inoue (2008) or *Theorem 2* of McKenzie (2004)).

Before considering the dynamic model for potentially unbalanced datasets in greater detail we define the following extended sets

$$\mathcal{T}^*_s = \mathcal{T}_s \bigcup \mathcal{T}^+_s, \quad s = 1, \ldots, S,$$

where $\mathcal{T}^+_s$ is the set of indices for observed "pre-sample" observations. For balanced samples, this definition implies that in total $T + p$ observations are observed (so that timing starts at $-p+1$). Using this definition, Condition (1.3) can be extended to allow dynamics in the model

$$\exists \mathcal{A}^{(l)} \subset \left(\bigcap_{s=1}^{S} \mathcal{T}^*_s \right), \quad s.t. \left| \mathcal{A}^{(l)} \right| > p, \quad l = 1, \ldots, L, \tag{1.5}$$

and each set $\mathcal{A}^{(l)}$ is distinct and connected. In other words, it is possible to find L distinctive time periods so that for all S cohorts the vector $(\bar{y}_{s,t}, \ddot{\boldsymbol{y}}'_{s,t}, \bar{\boldsymbol{x}}'_{s,t})'$ is observed. Although only L common subsets $\mathcal{A}^{(l)}$ are needed for normalisation, it is still necessary that for each cohort s the total number of complete observations be at least $L+1$. Making use of (1.5) we can reformulate Assumption **(A.5)** in the following way

(A.5) a) $\sum_{s=1}^{S} T_s - (T - L + S)L > K + p$ with L_0 being the true number factors with non-zero mean factor loadings. b) $\forall t \in \bigcup_{s=1}^{S} \mathcal{T}_s, \quad \exists S_t \subseteq \mathcal{S}^*, \quad s.t. \quad \forall s \in S_t : t \in \mathcal{T}_s, \quad |S_t| \geq L_0$. c) $\exists \mathcal{A}^{(l)} \subset \left(\bigcap_{s=1}^{S} \mathcal{T}^*_s \right), \quad s.t. \left| \mathcal{A}^{(l)} \right| > p, \quad l = 1, \ldots, L$ and $\boldsymbol{F}^{-1}_{L \times L}$ exists. d) $\boldsymbol{F}$ and $\boldsymbol{\Lambda}$ are deterministic matrices.

Note that for datasets with highly disconnected $\mathcal{T}_s^*$ this condition might be violated, thus observations of some cohorts need to be discarded for the estimation procedure to be feasible (for any given L). Otherwise, if it possible to group cohorts based on non-availability of data, one can opt for the estimator that estimates group specific $\boldsymbol{\phi}$. Assumption **(A.6)**, on the other hand, can be difficult to satisfy for some dynamic models, as we discuss in Section 1.2.2.

1.2.1. Local identification

The example with one regressor that is presented in the main text, is of particular importance if one considers the AR(1) model

$$y_{i,t} = \alpha y_{i,t-1} + u_{i,t}, \quad y_{i,0} = \frac{\mu_s}{N_{s,0}^{\gamma}} + u_{i,0}. \tag{1.6}$$

In this case, the only source of variation (that is denote by $\mu_{s,t}$ in the main text) is coming from the possible mean (effect) non-stationarity of the initial condition, as in this case

$$\mathrm{E}[y_{i,t-1}] = \alpha^t \left(\frac{\mu_s}{N_{s,0}^{\gamma}} \right).$$

Furthermore, as $y_{i,t-1}$ are generally not observed,[2] by construction the equation of interest contains an endogenous regressor with coefficient $\rho = -\alpha_0$, the estimator $\hat{\alpha}$ converges to a random limit centered at zero for $\gamma > 1/2$.

1.2.2. Global identification

Assumption **(A.6)** is particularly difficult to satisfy for linear dynamic models without any additional regressors. As we will see in the next two examples, a necessary condition for global identification of dynamic models is the presence of regressors (or initial condition $y_{i,0}$), that cannot be well approximated by the factor structure present in the model itself. To illustrate this point, consider a linear AR(1) model

$$y_{i,t} = \boldsymbol{\lambda}_s' \boldsymbol{f}_t + \alpha y_{i,t-1} + u_{i,t}, \quad i \in \mathcal{I}_{s,t}. \tag{1.7}$$

[2]As it is discussed in Section 1.2.

If the model only contains fixed effects, i.e. $\boldsymbol{f}_t = \boldsymbol{c}$ for all t, and the initial condition is mean-stationary, then

$$y_{i,t-1} = \frac{\delta_s}{1-\alpha_0} + \sum_{j=0}^{\infty} (\alpha_0)^j u_{i,t-1-j}, \quad i \in \mathcal{I}_{s,t}. \tag{1.8}$$

As it is discussed in the main text, the simplest DGP that fails to satisfy Assumption **(A.6)** is as follows

$$\mathrm{E}[\boldsymbol{\lambda}_i^z] = \kappa \boldsymbol{\lambda}_s, \forall i \in \mathcal{I}_{s,t}, \quad \kappa \neq 0. \tag{1.9}$$

For the AR(1) model described above, in terms of (1.9) we have $\kappa = 1/(1-\alpha_0)$, and thus irrespective of the value for $\boldsymbol{M}(\boldsymbol{\phi})$

$$\operatorname*{plim}_{N\to\infty} \boldsymbol{M}(\boldsymbol{\phi})(\boldsymbol{y} - \boldsymbol{X}\alpha) = \boldsymbol{0}_{S(T-L)}, \tag{1.10}$$

at $\alpha = 1$. As a result, unlike the linear GMM estimator of Inoue (2008), where the stationary initial condition violates the local identification assumption **(A.4)**, (as in Section 1.2.1), estimation of unobserved factors in this case also leads to the violation of the global identification assumption **(A.6)**.

The presence of unobserved factors in general is not sufficient for global identification. To illustrate that, we consider the process for $y_{i,t}$ with "infinite" initialization at "$-\infty$"

$$y_{i,t-1} = \boldsymbol{\lambda}_s' \left(\sum_{j=0}^{\infty} (\alpha_0)^j \boldsymbol{f}_{t-1-j} \right) + \sum_{j=0}^{\infty} (\alpha_0)^j u_{i,t-j} = \boldsymbol{\lambda}_s' \boldsymbol{f}_{t-1}^* + \sum_{j=0}^{\infty} (\alpha_0)^j u_{i,t-1-j}, \tag{1.11}$$

which is the special case of the model discussed in the main text. Particularly, defining $\boldsymbol{F}^* = (\boldsymbol{f}_0^*, \ldots, \boldsymbol{f}_{T-1}^*)'$ the global identification condition can then be formulated in the following way

$$\operatorname*{plim}_{N\to\infty} \boldsymbol{M}(\boldsymbol{\phi})(\boldsymbol{y} - \boldsymbol{X}\alpha) = \boldsymbol{M}(\boldsymbol{\phi}) \operatorname{vec}(((\alpha_0 - \alpha)\boldsymbol{F}^* + \boldsymbol{F})\boldsymbol{\Lambda}') = \boldsymbol{0}_{S(T-L)}. \tag{1.12}$$

If we can further assume that the appropriate $[L \times L]$ block of the $\tilde{\boldsymbol{F}} \equiv (\alpha_0 - \alpha)\boldsymbol{F}^* + \boldsymbol{F}$

matrix is invertible, then each parameter value from the set

$$\boldsymbol{\Gamma} = \{\boldsymbol{\gamma} = (\alpha, \boldsymbol{\phi}')' \in \mathbb{R}^{(T-L)L+1} : \alpha \in \mathbb{R}, \quad \boldsymbol{\Phi} = -\tilde{\boldsymbol{F}}_{[(T-L)\times L]}\tilde{\boldsymbol{F}}^{-1}_{[L\times L]}\}, \tag{1.13}$$

satisfies the moment conditions.

Based on these two examples, we can see that a necessary condition for global identification of dynamic models is the presence of regressors (or initial condition $y_{i,0}$) that cannot be well approximated by the factor structure present in the model itself. Note that the non-linear dynamic pseudo panel data models as studied by e.g. Antman and McKenzie (2007) or more general models with regressors can still be globally identified.

2. Additional Monte Carlo studies

2.1. General Computational Remarks

All results are presented for the two-step estimators (where necessary) with the asymptotically optimal weighting matrix $\boldsymbol{\Omega}$ under the assumption that $\sigma^2_{s,t} = \sigma^2$. In this case for an estimator of σ^2 we use

$$\hat{\sigma}^2 = \frac{1}{ST}\sum_{t=1}^{T}\sum_{s=1}^{S}\hat{\sigma}^2_{s,t}$$

with $\hat{\sigma}^2_{s,t}$

$$\hat{\sigma}^2_{s,t} = \frac{1}{N_{s,t}}\sum_{i\in\mathcal{I}_{s,t}}(y_{i,t} - \boldsymbol{x}'_{i,s,t}\hat{\boldsymbol{\theta}}_1)^2 - \left(\frac{1}{N_{s,t}}\sum_{i\in\mathcal{I}_{s,t}}(y_{i,t} - \boldsymbol{x}'_{i,s,t}\hat{\boldsymbol{\theta}}_1)\right)^2. \tag{2.1}$$

Note that under these assumptions all linear GMM estimators are one-step efficient and we use this fact in estimation, whereas to obtain non-linear GMM estimators we perform estimation in two steps as the optimal weighting matrix depends on unknown parameters. Note that for non-linear GMM estimators we use "GMMl1" estimator for $\boldsymbol{\theta}$ and a vector of ones for factors.

Remark 1. All Monte Carlo results presented in this supplemental material are based on 4,000 Monte Carlo replications.

2.2. One Factor: Results without Normalizations

2.2.1. The Setup

In this section we present some Monte Carlo evidence in the model with one factor where we do not impose normalization (as in the main text) on the unobservables. To be precise let the main parameters be as follows

$$\begin{aligned}
&T = 5, \quad S = 10, \quad \boldsymbol{\theta}_0 = (1,1)', \quad \alpha_f = 0.5,\\
&\bar{N} = \{150; 300; 450\}, \quad \sigma_\varepsilon^2 = \{0.5; 0.9\},\\
&\sigma_\mu^2 = \{0.3; 0.5\}, \quad \rho = \{0; 0.3\}, \quad \sigma_f = \{0.1; 0.5; 1.0; 2.0\}.
\end{aligned}$$

For unobserved components we consider two possible DGPs. The first DGP ($DGP1$)

$$\boldsymbol{\lambda}_s' \boldsymbol{f}_t = \lambda_s f_t, \quad \lambda_s \sim N(0,1), \tag{2.2}$$

$$f_t = \sqrt{\frac{11}{9}} + \sigma_f u_t^{(f)}, \tag{2.3}$$

where $u_t^{(f)}$s are generated as in the main text. The reason for the 11/9 term will become obvious once we discuss the second DGP. Hence we have

$$\mathrm{E}[\boldsymbol{\lambda}_s' \boldsymbol{f}_t] = 0, \quad \mathrm{var}[\boldsymbol{\lambda}_s' \boldsymbol{f}_t] = \left(\frac{11}{9} + \sigma_f^2\right).$$

In the second DGP ($DGP2$) we consider

$$\boldsymbol{\lambda}_s' \boldsymbol{f}_t = \lambda_s f_t, \quad \lambda_s \sim N(0,1), \tag{2.4}$$

$$f_t = \frac{t}{\left(\frac{1}{T}\sum_{t=1}^T t\right)} + \sigma_f u_t^{(f)}, \tag{2.5}$$

where $u_t^{(f)}$s are again generated as in the main text and as in the previous example. Hence we have

$$\mathrm{E}[\boldsymbol{\lambda}_s' \boldsymbol{f}_t] = 0, \quad \mathrm{var}[\boldsymbol{\lambda}_s' \boldsymbol{f}_t] = \left(\frac{t^2}{\left(\frac{1}{T}\sum_{t=1}^T t\right)^2} + \sigma_f^2\right).$$

Unlike in any of the previously considered DGPs, in this DGP var$[\boldsymbol{\lambda}_s' \boldsymbol{f}_t]$, is time varying. However, one can see that both DGPs are comparable in terms of the *average* variance of the error component terms, as in this case

$$\frac{1}{T}\sum_{t=1}^{T} \text{var}[\boldsymbol{\lambda}_s' \boldsymbol{f}_t] = \frac{11}{9} + \sigma_f^2. \tag{2.6}$$

Remark 2. To simplify the setup for both DGPs we assume that $\mathrm{E}[\lambda_s] = 0$ (unlike the main text), in this case the GMMl0 estimator is asymptotically unbiased (but inconsistent).

2.2.2. Results one factor: DGP1

Detailed results for this sections are presented in Tables 1, 2, 3 and 4. Here we briefly summarize the main trends.

- Results are mostly unaffected by ρ and σ_ε^2.
- Larger values of σ_μ^2 are associated with smaller values of RMSE.
- Results for GMMl1 and GMMl2 estimators are comparable in all respect. The GMML2 estimators tends to have slightly larger RMSE and slightly lower rejection frequency of the J and $t-$ statistics.
- For larger values of σ_f^2 the RMSE of GMMn1 is larger than the corresponding value for GMMn2. We suspect that these results are mostly driven by the numerical outliers. Any numerical problems can be mostly removed by using the larger set of starting values for numerical routines.
- The GMMo estimator has negligible bias and small RMSE (smaller than of GMMn2). The $t-$ and J have correct size.
- For larger values of σ_f^2, the BIC criterion tends to perform marginally worse. However, as it was discussed above we suspect that this is mostly driven by numerical rather than theoretical properties of the estimator.

Table 1: One factor DGP1. Estimation results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.5$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\rho; \sigma_\mu^2; \bar{N}; \sigma_f\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .0 ; .3 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.29	.03	.04	.02	.03	.02
{ .0 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.31	.14	.15	.04	.03	.02
{ .0 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.34	.28	.30	.07	.03	.01
{ .0 ; .3 ; 150 ; 2.0 }	.00	.02	.02	.00	.00	.00	.48	.56	.59	.17	.05	.04
{ .0 ; .3 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.28	.03	.03	.01	.02	.01
{ .0 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.31	.15	.15	.04	.03	.02
{ .0 ; .3 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.35	.29	.31	.06	.04	.02
{ .0 ; .3 ; 300 ; 2.0 }	.00	.01	.01	.00	.00	.00	.48	.58	.61	.13	.04	.02
{ .0 ; .3 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.29	.03	.03	.01	.02	.01
{ .0 ; .3 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.30	.14	.15	.03	.01	.01
{ .0 ; .3 ; 450 ; 1.0 }	.00	-.01	-.01	.00	.00	.00	.34	.30	.31	.07	.03	.01
{ .0 ; .3 ; 450 ; 2.0 }	-.01	-.01	.00	.00	.00	.00	.48	.59	.62	.12	.03	.02
{ .0 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.03	.03	.02	.02	.02
{ .0 ; .5 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.11	.12	.03	.02	.02
{ .0 ; .5 ; 150 ; 1.0 }	.00	.00	.01	.00	.00	.00	.31	.22	.23	.06	.05	.03
{ .0 ; .5 ; 150 ; 2.0 }	-.01	-.01	-.01	.00	.00	.00	.42	.46	.47	.10	.03	.02
{ .0 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.02	.03	.01	.01	.01
{ .0 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.11	.12	.03	.02	.01
{ .0 ; .5 ; 300 ; 1.0 }	-.01	.00	.00	.00	.00	.00	.32	.22	.23	.07	.03	.03
{ .0 ; .5 ; 300 ; 2.0 }	.00	.00	.00	.00	.00	.00	.43	.43	.47	.11	.06	.05
{ .0 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.02	.03	.01	.01	.01
{ .0 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.12	.12	.03	.01	.01
{ .0 ; .5 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.31	.22	.23	.05	.03	.01
{ .0 ; .5 ; 450 ; 2.0 }	.01	.00	.00	.00	.00	.00	.44	.44	.47	.11	.05	.05
{ .3 ; .3 ; 150 ; 0.1 }	.00	.01	.01	.01	.01	.01	.29	.03	.04	.02	.03	.02
{ .3 ; .3 ; 150 ; 0.5 }	.00	.01	.00	.00	.00	.00	.31	.15	.15	.04	.02	.02
{ .3 ; .3 ; 150 ; 1.0 }	.00	.01	.01	.00	.00	.00	.34	.29	.31	.09	.03	.02
{ .3 ; .3 ; 150 ; 2.0 }	-.01	-.01	-.01	.00	.00	.00	.49	.57	.61	.13	.03	.03
{ .3 ; .3 ; 300 ; 0.1 }	.01	.00	.00	.00	.00	.00	.29	.03	.03	.01	.02	.01
{ .3 ; .3 ; 300 ; 0.5 }	-.01	.01	.01	.00	.00	.00	.31	.14	.15	.03	.03	.01
{ .3 ; .3 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.35	.29	.31	.09	.02	.02
{ .3 ; .3 ; 300 ; 2.0 }	.00	.01	.00	.00	.00	.00	.49	.56	.61	.13	.04	.04
{ .3 ; .3 ; 450 ; 0.1 }	.01	.00	.00	.00	.00	.00	.29	.03	.03	.01	.02	.01
{ .3 ; .3 ; 450 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.30	.15	.16	.03	.02	.01
{ .3 ; .3 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.34	.29	.31	.06	.05	.04
{ .3 ; .3 ; 450 ; 2.0 }	.00	.00	.01	.00	.00	.00	.49	.58	.62	.11	.01	.01
{ .3 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.25	.03	.03	.02	.02	.02
{ .3 ; .5 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.11	.12	.03	.02	.02
{ .3 ; .5 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.31	.22	.24	.07	.02	.02
{ .3 ; .5 ; 150 ; 2.0 }	.00	.00	.00	.00	.00	.00	.44	.44	.46	.13	.03	.02
{ .3 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.02	.03	.01	.01	.01
{ .3 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.12	.12	.02	.01	.01
{ .3 ; .5 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.30	.23	.24	.06	.02	.01
{ .3 ; .5 ; 300 ; 2.0 }	-.01	-.01	-.01	.00	.00	.00	.42	.45	.47	.10	.02	.01
{ .3 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.02	.03	.01	.01	.01
{ .3 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.11	.12	.03	.02	.01
{ .3 ; .5 ; 450 ; 1.0 }	.01	.01	.01	.00	.00	.00	.31	.23	.25	.08	.01	.01
{ .3 ; .5 ; 450 ; 2.0 }	.01	-.01	-.01	.00	.00	.00	.43	.45	.48	.09	.02	.02

Notes. Here "L_0" is the "GMMl0" estimator; "L_1^{FE}" and "L_2^{FE}" are the linear estimator "GMMl1" and "GMMl2" estimators; "L_1" and "L_2" are the non-linear "GMMn1" and "GMMn2" estimators; "$\hat{L}$" is the "GMMo" estimator with optimal number of factors based on BIC.

Table 2: One factor DGP1. Estimation results for $T = 5, S = 10$ and $\sigma^2_\varepsilon = 0.9$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\rho; \sigma^2_\mu; \bar{N}; \sigma_f\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .0 ; .3 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.29	.04	.04	.03	.04	.03
{ .0 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.30	.15	.15	.03	.03	.02
{ .0 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.35	.30	.31	.08	.03	.03
{ .0 ; .3 ; 150 ; 2.0 }	.00	-.02	-.01	.00	.00	.00	.47	.57	.61	.13	.07	.05
{ .0 ; .3 ; 300 ; 0.1 }	.01	.00	.00	.00	.00	.00	.29	.03	.04	.02	.03	.02
{ .0 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.31	.15	.16	.05	.03	.02
{ .0 ; .3 ; 300 ; 1.0 }	.00	-.01	.00	.00	.00	.00	.33	.29	.31	.08	.03	.02
{ .0 ; .3 ; 300 ; 2.0 }	-.01	.00	.00	.00	.00	.00	.49	.58	.61	.15	.08	.05
{ .0 ; .3 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.30	.03	.03	.01	.02	.01
{ .0 ; .3 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.31	.15	.16	.04	.02	.01
{ .0 ; .3 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.34	.29	.31	.08	.03	.02
{ .0 ; .3 ; 450 ; 2.0 }	-.02	.00	-.01	.00	.00	.00	.47	.57	.61	.13	.05	.02
{ .0 ; .5 ; 150 ; 0.1 }	.01	.00	.00	.00	.00	.00	.26	.03	.03	.02	.03	.02
{ .0 ; .5 ; 150 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.27	.11	.12	.03	.03	.02
{ .0 ; .5 ; 150 ; 1.0 }	.00	-.01	.00	.00	.00	.00	.31	.23	.24	.08	.03	.02
{ .0 ; .5 ; 150 ; 2.0 }	.01	.01	.01	.00	.00	.00	.43	.45	.48	.10	.04	.03
{ .0 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.03	.03	.01	.02	.01
{ .0 ; .5 ; 300 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.27	.11	.12	.03	.03	.03
{ .0 ; .5 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.32	.23	.24	.06	.02	.02
{ .0 ; .5 ; 300 ; 2.0 }	.00	-.01	-.01	.00	.00	.00	.42	.46	.48	.09	.03	.03
{ .0 ; .5 ; 450 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.25	.02	.03	.01	.02	.01
{ .0 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.11	.12	.04	.04	.03
{ .0 ; .5 ; 450 ; 1.0 }	.00	-.01	-.01	.00	.00	.00	.31	.22	.24	.07	.01	.01
{ .0 ; .5 ; 450 ; 2.0 }	.01	.00	.00	.00	.00	.00	.43	.45	.48	.11	.05	.02
{ .3 ; .3 ; 150 ; 0.1 }	.00	.01	.01	.01	.01	.01	.29	.04	.04	.03	.04	.03
{ .3 ; .3 ; 150 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.30	.14	.15	.04	.03	.02
{ .3 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.36	.28	.30	.07	.03	.02
{ .3 ; .3 ; 150 ; 2.0 }	.01	-.01	-.01	.00	.00	.00	.48	.56	.59	.14	.07	.04
{ .3 ; .3 ; 300 ; 0.1 }	.01	.00	.00	.00	.00	.00	.29	.03	.04	.02	.03	.02
{ .3 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.31	.15	.16	.04	.03	.02
{ .3 ; .3 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.35	.29	.31	.09	.04	.04
{ .3 ; .3 ; 300 ; 2.0 }	.00	.01	.01	.00	.00	.00	.47	.57	.60	.14	.05	.05
{ .3 ; .3 ; 450 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.29	.03	.03	.01	.02	.01
{ .3 ; .3 ; 450 ; 0.5 }	.01	.00	.00	.00	.00	.00	.30	.15	.15	.05	.02	.01
{ .3 ; .3 ; 450 ; 1.0 }	.02	.01	.01	.00	.00	.00	.34	.28	.30	.07	.02	.02
{ .3 ; .3 ; 450 ; 2.0 }	-.01	-.02	-.02	.00	.00	.00	.47	.56	.60	.12	.07	.07
{ .3 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.03	.03	.02	.03	.02
{ .3 ; .5 ; 150 ; 0.5 }	.01	.00	.00	.00	.00	.00	.27	.11	.12	.03	.03	.02
{ .3 ; .5 ; 150 ; 1.0 }	.00	.01	.01	.00	.00	.00	.31	.23	.24	.05	.03	.02
{ .3 ; .5 ; 150 ; 2.0 }	.00	.00	.01	.00	.00	.00	.43	.45	.47	.12	.02	.02
{ .3 ; .5 ; 300 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.26	.03	.03	.01	.02	.01
{ .3 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.11	.12	.02	.02	.01
{ .3 ; .5 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.31	.22	.23	.06	.03	.01
{ .3 ; .5 ; 300 ; 2.0 }	.01	.00	.00	.00	.00	.00	.41	.45	.47	.11	.07	.07
{ .3 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.02	.03	.01	.02	.01
{ .3 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.27	.11	.12	.03	.02	.01
{ .3 ; .5 ; 450 ; 1.0 }	-.01	.00	.00	.00	.00	.00	.32	.23	.23	.05	.01	.01
{ .3 ; .5 ; 450 ; 2.0 }	.01	-.01	-.01	.00	.00	.00	.43	.44	.46	.12	.05	.05

Notes. See Table 1.

Table 3: One factor DGP1. Testing results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.5$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\sigma_{f\lambda}^2; \rho; N; \sigma_f\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L = 1$
{ .0 ; .3 ; 150 ; 0.1 }	1	.96	.85	.27	.83	.27	.07	.05	.03	.07	.05	.06	.94	1	.81	.98
{ .0 ; .3 ; 150 ; 0.5 }	1	.95	1	.74	1	.74	.09	.06	.03	.06	.06	.05	1	.99	.98	.95
{ .0 ; .3 ; 150 ; 1.0 }	1	.95	1	.87	1	.87	.13	.09	.03	.07	.06	.06	1	.99	.99	.91
{ .0 ; .3 ; 150 ; 2.0 }	1	.95	1	.93	1	.93	.12	.09	.03	.07	.06	.06	1	1	1	.92
{ .0 ; .3 ; 300 ; 0.1 }	1	.97	.94	.36	.92	.35	.06	.06	.03	.07	.06	.06	.99	1	.90	.99
{ .0 ; .3 ; 300 ; 0.5 }	1	.97	1	.82	1	.81	.07	.06	.02	.07	.05	.05	1	1	.99	.98
{ .0 ; .3 ; 300 ; 1.0 }	1	.96	1	.90	1	.90	.11	.08	.03	.07	.06	.05	1	1	1	.95
{ .0 ; .3 ; 300 ; 2.0 }	1	.97	1	.95	1	.95	.10	.08	.03	.07	.06	.05	1	1	1	.95
{ .0 ; .3 ; 450 ; 0.1 }	1	.98	.97	.43	.96	.43	.06	.05	.02	.06	.05	.05	.99	1	.92	1
{ .0 ; .3 ; 450 ; 0.5 }	1	.97	1	.84	1	.84	.08	.06	.03	.07	.06	.05	1	1	.99	.98
{ .0 ; .3 ; 450 ; 1.0 }	1	.97	1	.92	1	.92	.10	.08	.03	.06	.06	.05	1	1	1	.95
{ .0 ; .3 ; 450 ; 2.0 }	1	.97	1	.96	1	.96	.10	.07	.03	.06	.06	.05	1	1	1	.96
{ .0 ; .5 ; 150 ; 0.1 }	1	.96	.86	.27	.84	.26	.06	.06	.02	.06	.05	.06	.96	1	.82	.98
{ .0 ; .5 ; 150 ; 0.5 }	1	.95	1	.75	1	.75	.09	.07	.03	.07	.06	.06	1	1	.99	.95
{ .0 ; .5 ; 150 ; 1.0 }	1	.94	1	.87	1	.87	.13	.09	.04	.08	.07	.06	1	.99	.99	.92
{ .0 ; .5 ; 150 ; 2.0 }	1	.96	1	.94	1	.94	.12	.09	.03	.07	.05	.06	1	1	1	.92
{ .0 ; .5 ; 300 ; 0.1 }	1	.97	.94	.37	.93	.36	.06	.05	.02	.07	.05	.05	.99	1	.89	.99
{ .0 ; .5 ; 300 ; 0.5 }	1	.97	1	.83	1	.82	.08	.06	.03	.07	.06	.06	1	1	.99	.98
{ .0 ; .5 ; 300 ; 1.0 }	1	.96	1	.91	1	.90	.11	.09	.03	.07	.06	.06	1	1	1	.94
{ .0 ; .5 ; 300 ; 2.0 }	1	.97	1	.95	1	.95	.11	.09	.03	.07	.06	.06	1	1	1	.94
{ .0 ; .5 ; 450 ; 0.1 }	1	.97	.97	.45	.97	.44	.06	.05	.03	.06	.06	.05	.99	1	.93	1
{ .0 ; .5 ; 450 ; 0.5 }	1	.97	1	.86	1	.85	.07	.06	.02	.07	.06	.05	1	1	1	.98
{ .0 ; .5 ; 450 ; 1.0 }	1	.97	1	.91	1	.92	.10	.08	.03	.07	.06	.06	1	1	1	.96
{ .0 ; .5 ; 450 ; 2.0 }	1	.98	1	.96	1	.96	.09	.08	.02	.06	.05	.05	1	1	1	.96
{ .3 ; .3 ; 150 ; 0.1 }	1	.96	.85	.25	.83	.25	.07	.06	.03	.07	.05	.06	.95	1	.83	.98
{ .3 ; .3 ; 150 ; 0.5 }	1	.95	1	.74	1	.74	.09	.08	.03	.08	.05	.07	1	.99	.98	.96
{ .3 ; .3 ; 150 ; 1.0 }	1	.94	1	.86	1	.86	.14	.10	.04	.08	.07	.07	1	.99	.99	.91
{ .3 ; .3 ; 150 ; 2.0 }	1	.96	1	.94	1	.94	.12	.10	.03	.07	.05	.06	1	1	1	.92
{ .3 ; .3 ; 300 ; 0.1 }	1	.97	.93	.36	.92	.35	.07	.06	.03	.07	.06	.06	.98	1	.88	.99
{ .3 ; .3 ; 300 ; 0.5 }	1	.97	1	.80	1	.81	.08	.06	.02	.07	.06	.06	1	1	.99	.98
{ .3 ; .3 ; 300 ; 1.0 }	1	.96	1	.91	1	.90	.11	.08	.03	.07	.06	.06	1	1	1	.95
{ .3 ; .3 ; 300 ; 2.0 }	1	.97	1	.95	1	.95	.10	.09	.03	.07	.05	.06	1	1	1	.95
{ .3 ; .3 ; 450 ; 0.1 }	1	.98	.97	.42	.96	.41	.06	.05	.03	.07	.05	.05	.99	1	.92	1
{ .3 ; .3 ; 450 ; 0.5 }	1	.97	1	.85	1	.84	.07	.07	.02	.07	.06	.06	1	1	1	.98
{ .3 ; .3 ; 450 ; 1.0 }	1	.97	1	.92	1	.91	.11	.08	.03	.06	.07	.06	1	1	1	.96
{ .3 ; .3 ; 450 ; 2.0 }	1	.98	1	.96	1	.96	.10	.08	.03	.07	.06	.05	1	1	1	.95
{ .3 ; .5 ; 150 ; 0.1 }	1	.95	.86	.27	.84	.27	.07	.06	.03	.07	.05	.06	.95	1	.82	.98
{ .3 ; .5 ; 150 ; 0.5 }	1	.96	1	.75	1	.75	.10	.07	.03	.08	.05	.07	1	.99	.98	.95
{ .3 ; .5 ; 150 ; 1.0 }	1	.94	1	.85	1	.87	.13	.08	.03	.07	.07	.05	1	.99	.99	.92
{ .3 ; .5 ; 150 ; 2.0 }	1	.95	1	.94	1	.93	.12	.08	.03	.07	.06	.05	1	1	1	.92
{ .3 ; .5 ; 300 ; 0.1 }	1	.97	.94	.37	.93	.36	.06	.06	.03	.06	.06	.06	.98	1	.89	.99
{ .3 ; .5 ; 300 ; 0.5 }	1	.97	1	.82	1	.81	.09	.06	.03	.07	.06	.05	1	1	.99	.97
{ .3 ; .5 ; 300 ; 1.0 }	1	.96	1	.90	1	.90	.10	.09	.03	.07	.06	.06	1	1	1	.95
{ .3 ; .5 ; 300 ; 2.0 }	1	.97	1	.95	1	.94	.10	.08	.03	.06	.06	.05	1	1	1	.95
{ .3 ; .5 ; 450 ; 0.1 }	1	.98	.97	.45	.96	.43	.06	.06	.02	.07	.05	.06	.99	1	.93	1
{ .3 ; .5 ; 450 ; 0.5 }	1	.97	1	.85	1	.85	.07	.06	.03	.07	.05	.05	1	1	1	.98
{ .3 ; .5 ; 450 ; 1.0 }	1	.97	1	.92	1	.92	.10	.08	.02	.06	.06	.05	1	1	1	.95
{ .3 ; .5 ; 450 ; 2.0 }	1	.98	1	.97	1	.96	.10	.08	.03	.07	.06	.05	1	1	1	.95

Notes. See Table 1.

Table 4: One factor DGP1. Testing results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.9$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\sigma^2_{f\lambda}; \rho; N; \sigma_f\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L = 1$
{ .0 ; .3 ; 150 ; 0.1 }	1	.95	.72	.18	.68	.17	.08	.06	.03	.07	.06	.06	.89	.99	.72	.97
{ .0 ; .3 ; 150 ; 0.5 }	1	.94	1	.67	1	.67	.10	.07	.04	.08	.06	.06	1	.99	.97	.94
{ .0 ; .3 ; 150 ; 1.0 }	1	.94	1	.82	1	.82	.14	.09	.04	.07	.06	.06	1	.99	.99	.90
{ .0 ; .3 ; 150 ; 2.0 }	1	.94	1	.91	1	.91	.13	.09	.03	.07	.05	.06	1	.99	1	.91
{ .0 ; .3 ; 300 ; 0.1 }	1	.96	.87	.28	.84	.26	.06	.05	.02	.07	.05	.05	.96	1	.84	.99
{ .0 ; .3 ; 300 ; 0.5 }	1	.96	1	.75	1	.75	.09	.07	.03	.07	.06	.06	1	.99	.99	.97
{ .0 ; .3 ; 300 ; 1.0 }	1	.95	1	.86	1	.87	.13	.09	.03	.07	.07	.06	1	.99	.99	.93
{ .0 ; .3 ; 300 ; 2.0 }	1	.96	1	.93	1	.93	.11	.09	.03	.06	.05	.06	1	1	1	.94
{ .0 ; .3 ; 450 ; 0.1 }	1	.97	.92	.32	.91	.31	.06	.05	.03	.06	.06	.05	.98	1	.87	1
{ .0 ; .3 ; 450 ; 0.5 }	1	.97	1	.80	1	.80	.08	.07	.02	.06	.05	.06	1	.99	.99	.97
{ .0 ; .3 ; 450 ; 1.0 }	1	.96	1	.90	1	.90	.12	.09	.04	.08	.07	.06	1	.99	1	.94
{ .0 ; .3 ; 450 ; 2.0 }	1	.96	1	.95	1	.95	.11	.08	.02	.07	.06	.06	1	1	1	.95
{ .0 ; .5 ; 150 ; 0.1 }	1	.95	.72	.19	.70	.19	.08	.05	.03	.07	.06	.06	.89	.99	.72	.97
{ .0 ; .5 ; 150 ; 0.5 }	1	.93	1	.66	1	.65	.11	.07	.03	.08	.06	.06	1	.99	.97	.94
{ .0 ; .5 ; 150 ; 1.0 }	1	.92	1	.82	1	.81	.14	.09	.03	.07	.06	.07	1	.98	.99	.90
{ .0 ; .5 ; 150 ; 2.0 }	1	.94	1	.91	1	.91	.13	.09	.03	.07	.06	.06	1	.99	1	.91
{ .0 ; .5 ; 300 ; 0.1 }	1	.96	.86	.27	.85	.26	.06	.05	.02	.07	.05	.05	.95	1	.83	.99
{ .0 ; .5 ; 300 ; 0.5 }	1	.95	1	.75	1	.73	.08	.07	.03	.07	.06	.06	1	1	.99	.97
{ .0 ; .5 ; 300 ; 1.0 }	1	.94	1	.87	1	.86	.13	.09	.03	.07	.07	.06	1	.99	.99	.93
{ .0 ; .5 ; 300 ; 2.0 }	1	.96	1	.94	1	.93	.12	.09	.03	.06	.07	.05	1	1	1	.94
{ .0 ; .5 ; 450 ; 0.1 }	1	.97	.93	.32	.91	.32	.06	.04	.03	.06	.06	.04	.98	1	.88	1
{ .0 ; .5 ; 450 ; 0.5 }	1	.96	1	.79	1	.79	.07	.06	.03	.06	.06	.05	1	1	.99	.98
{ .0 ; .5 ; 450 ; 1.0 }	1	.96	1	.90	1	.89	.11	.08	.03	.06	.07	.05	1	1	1	.95
{ .0 ; .5 ; 450 ; 2.0 }	1	.97	1	.95	1	.94	.11	.10	.03	.08	.06	.06	1	1	1	.95
{ .3 ; .3 ; 150 ; 0.1 }	1	.95	.73	.21	.70	.20	.07	.07	.03	.08	.05	.07	.89	.99	.73	.97
{ .3 ; .3 ; 150 ; 0.5 }	1	.94	1	.69	1	.68	.10	.08	.03	.07	.06	.06	1	.99	.97	.94
{ .3 ; .3 ; 150 ; 1.0 }	1	.93	1	.82	1	.82	.14	.10	.03	.08	.06	.07	1	.99	.99	.90
{ .3 ; .3 ; 150 ; 2.0 }	1	.94	1	.91	1	.91	.13	.09	.03	.07	.06	.06	1	.99	1	.92
{ .3 ; .3 ; 300 ; 0.1 }	1	.96	.87	.29	.85	.28	.06	.05	.03	.07	.06	.06	.95	1	.83	.99
{ .3 ; .3 ; 300 ; 0.5 }	1	.96	1	.77	1	.76	.09	.07	.03	.07	.07	.06	1	1	.98	.97
{ .3 ; .3 ; 300 ; 1.0 }	1	.96	1	.88	1	.87	.12	.09	.03	.07	.06	.06	1	.99	.99	.93
{ .3 ; .3 ; 300 ; 2.0 }	1	.96	1	.94	1	.94	.11	.09	.03	.07	.06	.06	1	1	1	.94
{ .3 ; .3 ; 450 ; 0.1 }	1	.97	.92	.34	.91	.34	.07	.05	.03	.07	.06	.05	.98	1	.88	1
{ .3 ; .3 ; 450 ; 0.5 }	1	.97	1	.80	1	.80	.08	.06	.03	.07	.06	.05	1	1	.99	.98
{ .3 ; .3 ; 450 ; 1.0 }	1	.96	1	.89	1	.89	.11	.09	.02	.07	.06	.06	1	1	1	.95
{ .3 ; .3 ; 450 ; 2.0 }	1	.97	1	.95	1	.95	.10	.09	.03	.06	.06	.05	1	1	1	.95
{ .3 ; .5 ; 150 ; 0.1 }	1	.94	.74	.20	.71	.19	.07	.05	.03	.07	.06	.05	.90	.99	.74	.97
{ .3 ; .5 ; 150 ; 0.5 }	1	.94	1	.68	1	.68	.09	.07	.03	.07	.05	.06	1	.99	.98	.95
{ .3 ; .5 ; 150 ; 1.0 }	1	.92	1	.83	1	.82	.14	.10	.03	.08	.07	.07	1	.99	.99	.90
{ .3 ; .5 ; 150 ; 2.0 }	1	.95	1	.91	1	.91	.13	.09	.03	.07	.05	.06	1	.99	1	.91
{ .3 ; .5 ; 300 ; 0.1 }	1	.96	.87	.28	.85	.27	.06	.05	.03	.07	.05	.06	.96	1	.84	.99
{ .3 ; .5 ; 300 ; 0.5 }	1	.95	1	.76	1	.76	.08	.07	.02	.07	.06	.06	1	1	.99	.97
{ .3 ; .5 ; 300 ; 1.0 }	1	.94	1	.88	1	.87	.12	.09	.03	.08	.06	.07	1	.99	1	.94
{ .3 ; .5 ; 300 ; 2.0 }	1	.95	1	.94	1	.94	.12	.09	.03	.06	.06	.06	1	1	1	.94
{ .3 ; .5 ; 450 ; 0.1 }	1	.97	.92	.35	.91	.34	.05	.05	.02	.07	.05	.06	.98	1	.88	1
{ .3 ; .5 ; 450 ; 0.5 }	1	.96	1	.81	1	.80	.08	.06	.03	.07	.06	.05	1	1	.99	.98
{ .3 ; .5 ; 450 ; 1.0 }	1	.96	1	.90	1	.90	.12	.09	.03	.07	.07	.06	1	1	1	.95
{ .3 ; .5 ; 450 ; 2.0 }	1	.96	1	.95	1	.95	.11	.08	.03	.07	.06	.05	1	1	1	.95

Notes. See Table 1.

2.2.3. Results one factor: DGP2

Detailed results for this sections are presented in Tables 5, 6, 7 and 8. As results are very similar to those presented for DGP1 we do not discuss them in details.

Table 5: One factor DGP2. Estimation results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.5$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\rho;\sigma_\mu^2;\bar{N};\sigma_f\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .0 ; .3 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.27	.19	.20	.01	.02	.01
{ .0 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.24	.25	.01	.02	.01
{ .0 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.32	.33	.35	.03	.02	.01
{ .0 ; .3 ; 150 ; 2.0 }	.00	.02	.02	.00	.00	.00	.46	.59	.63	.14	.02	.01
{ .0 ; .3 ; 300 ; 0.1 }	.00	-.01	-.01	.00	.00	.00	.26	.19	.20	.01	.01	.01
{ .0 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.23	.24	.02	.01	.01
{ .0 ; .3 ; 300 ; 1.0 }	.00	.01	.00	.00	.00	.00	.33	.35	.36	.06	.01	.01
{ .0 ; .3 ; 300 ; 2.0 }	.00	.01	.01	.00	.00	.00	.46	.62	.65	.09	.03	.02
{ .0 ; .3 ; 450 ; 0.1 }	.00	.00	-.01	.00	.00	.00	.27	.19	.20	.01	.01	.01
{ .0 ; .3 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.22	.24	.01	.01	.01
{ .0 ; .3 ; 450 ; 1.0 }	.00	-.01	-.01	.00	.00	.00	.32	.35	.37	.05	.01	.01
{ .0 ; .3 ; 450 ; 2.0 }	-.01	.00	.00	.00	.00	.00	.47	.62	.65	.12	.01	.01
{ .0 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.01	.02	.01
{ .0 ; .5 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.20	.01	.02	.01
{ .0 ; .5 ; 150 ; 1.0 }	.00	.01	.01	.00	.00	.00	.29	.26	.28	.04	.02	.01
{ .0 ; .5 ; 150 ; 2.0 }	-.01	.00	.00	.00	.00	.00	.41	.48	.50	.07	.03	.02
{ .0 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.15	.15	.01	.01	.01
{ .0 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.19	.01	.01	.01
{ .0 ; .5 ; 300 ; 1.0 }	-.01	.00	.00	.00	.00	.00	.30	.26	.28	.03	.01	.01
{ .0 ; .5 ; 300 ; 2.0 }	.00	.00	.00	.00	.00	.00	.42	.45	.49	.07	.02	.01
{ .0 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.01	.01	.01
{ .0 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.19	.01	.01	.01
{ .0 ; .5 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.30	.26	.28	.04	.01	.01
{ .0 ; .5 ; 450 ; 2.0 }	.01	.00	.00	.00	.00	.00	.43	.46	.49	.08	.01	.01
{ .3 ; .3 ; 150 ; 0.1 }	.00	.00	.01	.00	.00	.00	.27	.18	.19	.02	.02	.02
{ .3 ; .3 ; 150 ; 0.5 }	.00	.01	.00	.00	.00	.00	.29	.24	.25	.01	.02	.02
{ .3 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.32	.33	.36	.03	.02	.01
{ .3 ; .3 ; 150 ; 2.0 }	-.01	-.01	-.01	.00	.00	.00	.48	.60	.65	.11	.04	.04
{ .3 ; .3 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.27	.18	.19	.01	.01	.01
{ .3 ; .3 ; 300 ; 0.5 }	-.01	.01	.01	.00	.00	.00	.28	.23	.25	.01	.01	.01
{ .3 ; .3 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.32	.34	.36	.06	.02	.01
{ .3 ; .3 ; 300 ; 2.0 }	.00	.00	.00	.00	.00	.00	.48	.59	.63	.08	.02	.02
{ .3 ; .3 ; 450 ; 0.1 }	.01	.00	.00	.00	.00	.00	.27	.18	.19	.01	.01	.01
{ .3 ; .3 ; 450 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.28	.23	.25	.01	.01	.01
{ .3 ; .3 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.32	.35	.37	.05	.01	.01
{ .3 ; .3 ; 450 ; 2.0 }	.00	.00	.01	.00	.00	.00	.47	.62	.65	.13	.02	.02
{ .3 ; .5 ; 150 ; 0.1 }	.00	.01	.00	.00	.00	.00	.23	.14	.15	.01	.02	.01
{ .3 ; .5 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.19	.20	.01	.02	.01
{ .3 ; .5 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.29	.26	.28	.03	.02	.01
{ .3 ; .5 ; 150 ; 2.0 }	.00	.00	.00	.00	.00	.00	.43	.47	.49	.11	.04	.04
{ .3 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.01	.01	.01
{ .3 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.19	.01	.01	.01
{ .3 ; .5 ; 300 ; 1.0 }	.00	.00	-.01	.00	.00	.00	.29	.27	.29	.03	.01	.01
{ .3 ; .5 ; 300 ; 2.0 }	-.01	-.02	-.01	.00	.00	.00	.41	.47	.51	.06	.03	.03
{ .3 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.01	.01	.01
{ .3 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.26	.18	.19	.01	.01	.01
{ .3 ; .5 ; 450 ; 1.0 }	.01	.01	.01	.00	.00	.00	.29	.27	.29	.04	.01	.01
{ .3 ; .5 ; 450 ; 2.0 }	.00	-.01	.00	.00	.00	.00	.41	.47	.50	.09	.02	.01

Notes. See Table 1.

Table 6: One factor DGP2. Estimation results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.9$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\rho; \sigma_\mu^2; \bar{N}; \sigma_f\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .0 ; .3 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.27	.19	.20	.02	.03	.02
{ .0 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.22	.24	.02	.03	.02
{ .0 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.33	.34	.36	.04	.02	.02
{ .0 ; .3 ; 150 ; 2.0 }	.00	-.02	-.02	.00	.00	.00	.46	.60	.64	.09	.03	.03
{ .0 ; .3 ; 300 ; 0.1 }	.01	.00	.00	.00	.00	.00	.27	.18	.19	.01	.02	.01
{ .0 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.24	.26	.01	.02	.01
{ .0 ; .3 ; 300 ; 1.0 }	.00	-.01	.00	.00	.00	.00	.31	.34	.36	.03	.03	.02
{ .0 ; .3 ; 300 ; 2.0 }	-.01	.00	.00	.00	.00	.00	.47	.60	.63	.13	.04	.03
{ .0 ; .3 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.27	.18	.19	.01	.01	.01
{ .0 ; .3 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.29	.23	.24	.02	.01	.01
{ .0 ; .3 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.32	.35	.37	.05	.01	.01
{ .0 ; .3 ; 450 ; 2.0 }	-.01	.00	.00	.00	.00	.00	.45	.61	.65	.13	.02	.01
{ .0 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.02	.02	.02
{ .0 ; .5 ; 150 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.25	.18	.18	.02	.02	.02
{ .0 ; .5 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.29	.27	.29	.03	.02	.01
{ .0 ; .5 ; 150 ; 2.0 }	.01	.01	.01	.00	.00	.00	.42	.48	.50	.09	.03	.03
{ .0 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.15	.15	.01	.02	.01
{ .0 ; .5 ; 300 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.25	.18	.19	.01	.01	.01
{ .0 ; .5 ; 300 ; 1.0 }	.00	-.01	.00	.00	.00	.00	.30	.27	.28	.03	.01	.01
{ .0 ; .5 ; 300 ; 2.0 }	.00	-.01	-.01	.00	.00	.00	.41	.48	.51	.10	.02	.01
{ .0 ; .5 ; 450 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.23	.14	.15	.01	.01	.01
{ .0 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.19	.01	.01	.01
{ .0 ; .5 ; 450 ; 1.0 }	.00	.00	.00	.00	.00	.00	.29	.26	.28	.03	.01	.01
{ .0 ; .5 ; 450 ; 2.0 }	.01	.00	-.01	.00	.00	.00	.42	.47	.50	.08	.03	.03
{ .3 ; .3 ; 150 ; 0.1 }	.00	.01	.01	.00	.00	.00	.27	.19	.20	.02	.02	.02
{ .3 ; .3 ; 150 ; 0.5 }	-.01	.00	.01	.00	.00	.00	.28	.23	.24	.02	.02	.02
{ .3 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.34	.34	.36	.05	.03	.03
{ .3 ; .3 ; 150 ; 2.0 }	.01	-.01	.00	.00	.00	.00	.46	.59	.62	.09	.03	.02
{ .3 ; .3 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.26	.18	.19	.01	.02	.01
{ .3 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.28	.24	.25	.01	.02	.01
{ .3 ; .3 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.33	.35	.36	.05	.03	.01
{ .3 ; .3 ; 300 ; 2.0 }	.00	.00	.01	.00	.00	.00	.45	.61	.64	.09	.02	.01
{ .3 ; .3 ; 450 ; 0.1 }	-.01	.00	-.01	.00	.00	.00	.26	.19	.20	.01	.01	.01
{ .3 ; .3 ; 450 ; 0.5 }	.01	.00	.00	.00	.00	.00	.28	.23	.25	.01	.01	.01
{ .3 ; .3 ; 450 ; 1.0 }	.01	.00	.00	.00	.00	.00	.32	.33	.35	.03	.01	.01
{ .3 ; .3 ; 450 ; 2.0 }	-.01	-.02	-.02	.00	.00	.00	.46	.59	.63	.10	.02	.02
{ .3 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.02	.02	.02
{ .3 ; .5 ; 150 ; 0.5 }	.01	.00	.00	.00	.00	.00	.25	.18	.19	.02	.02	.02
{ .3 ; .5 ; 150 ; 1.0 }	.00	.01	.01	.00	.00	.00	.30	.27	.29	.03	.03	.03
{ .3 ; .5 ; 150 ; 2.0 }	.00	.00	.00	.00	.00	.00	.42	.47	.49	.10	.04	.03
{ .3 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.01	.02	.01
{ .3 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.19	.01	.02	.01
{ .3 ; .5 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.29	.27	.28	.04	.02	.01
{ .3 ; .5 ; 300 ; 2.0 }	.01	.00	.00	.00	.00	.00	.40	.48	.50	.10	.01	.01
{ .3 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.24	.14	.15	.01	.01	.01
{ .3 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.25	.18	.19	.01	.01	.01
{ .3 ; .5 ; 450 ; 1.0 }	-.01	-.01	.00	.00	.00	.00	.30	.27	.28	.03	.01	.01
{ .3 ; .5 ; 450 ; 2.0 }	.01	-.01	-.01	.00	.00	.00	.42	.46	.48	.11	.02	.02

Notes. See Table 1.

Table 7: One factor DGP2. Testing results for $T = 5, S = 10$ and $\sigma^2_\varepsilon = 0.5$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\sigma^2_{f\lambda}; \rho; N; \sigma_f\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L = 1$
{ .0 ; .3 ; 150 ; 0.1 }	1	.96	1	.83	1	.84	.06	.05	.02	.06	.05	.05	1	1	.99	.98
{ .0 ; .3 ; 150 ; 0.5 }	1	.95	1	.84	1	.84	.07	.06	.02	.06	.06	.06	1	.99	.99	.98
{ .0 ; .3 ; 150 ; 1.0 }	1	.94	1	.88	1	.89	.09	.06	.03	.06	.06	.05	1	.99	.99	.95
{ .0 ; .3 ; 150 ; 2.0 }	1	.96	1	.94	1	.92	.11	.08	.03	.06	.05	.06	1	1	1	.93
{ .0 ; .3 ; 300 ; 0.1 }	1	.97	1	.89	1	.88	.06	.05	.03	.06	.06	.05	1	1	.99	.99
{ .0 ; .3 ; 300 ; 0.5 }	1	.97	1	.88	1	.89	.05	.05	.02	.05	.05	.05	1	1	1	.99
{ .0 ; .3 ; 300 ; 1.0 }	1	.96	1	.92	1	.92	.09	.07	.02	.06	.06	.06	1	1	1	.97
{ .0 ; .3 ; 300 ; 2.0 }	1	.96	1	.95	1	.95	.09	.07	.03	.06	.06	.06	1	1	1	.96
{ .0 ; .3 ; 450 ; 0.1 }	1	.97	1	.90	1	.90	.06	.05	.02	.05	.06	.05	1	1	1	1
{ .0 ; .3 ; 450 ; 0.5 }	1	.97	1	.91	1	.91	.06	.06	.02	.06	.05	.05	1	1	1	.99
{ .0 ; .3 ; 450 ; 1.0 }	1	.97	1	.94	1	.94	.08	.06	.02	.05	.05	.05	1	1	1	.98
{ .0 ; .3 ; 450 ; 2.0 }	1	.97	1	.96	1	.96	.10	.08	.02	.06	.06	.05	1	1	1	.96
{ .0 ; .5 ; 150 ; 0.1 }	1	.95	1	.84	1	.84	.06	.05	.02	.05	.05	.05	1	1	.99	.98
{ .0 ; .5 ; 150 ; 0.5 }	1	.95	1	.83	1	.84	.07	.05	.03	.06	.05	.05	1	1	.99	.98
{ .0 ; .5 ; 150 ; 1.0 }	1	.94	1	.88	1	.88	.10	.06	.03	.06	.05	.05	1	.99	.99	.95
{ .0 ; .5 ; 150 ; 2.0 }	1	.96	1	.93	1	.93	.11	.08	.03	.07	.06	.06	1	1	1	.93
{ .0 ; .5 ; 300 ; 0.1 }	1	.97	1	.89	1	.88	.06	.05	.02	.05	.05	.05	1	1	1	.99
{ .0 ; .5 ; 300 ; 0.5 }	1	.97	1	.89	1	.88	.06	.05	.02	.06	.05	.05	1	1	1	.99
{ .0 ; .5 ; 300 ; 1.0 }	1	.96	1	.92	1	.92	.08	.07	.03	.06	.06	.06	1	1	1	.97
{ .0 ; .5 ; 300 ; 2.0 }	1	.97	1	.95	1	.95	.10	.08	.02	.06	.06	.06	1	1	1	.96
{ .0 ; .5 ; 450 ; 0.1 }	1	.97	1	.91	1	.91	.05	.05	.02	.06	.05	.05	1	1	1	1
{ .0 ; .5 ; 450 ; 0.5 }	1	.97	1	.90	1	.90	.06	.05	.02	.06	.05	.05	1	1	1	1
{ .0 ; .5 ; 450 ; 1.0 }	1	.97	1	.93	1	.93	.07	.06	.03	.06	.06	.05	1	1	1	.98
{ .0 ; .5 ; 450 ; 2.0 }	1	.97	1	.96	1	.96	.08	.08	.02	.06	.05	.05	1	1	1	.97
{ .3 ; .3 ; 150 ; 0.1 }	1	.96	1	.83	1	.83	.07	.06	.03	.07	.05	.06	1	.99	.99	.98
{ .3 ; .3 ; 150 ; 0.5 }	1	.95	1	.84	1	.83	.06	.06	.03	.07	.05	.06	1	.99	.99	.98
{ .3 ; .3 ; 150 ; 1.0 }	1	.93	1	.89	1	.88	.10	.09	.03	.07	.06	.07	1	.99	.99	.95
{ .3 ; .3 ; 150 ; 2.0 }	1	.96	1	.94	1	.94	.11	.09	.02	.07	.05	.06	1	1	1	.93
{ .3 ; .3 ; 300 ; 0.1 }	1	.97	1	.88	1	.87	.06	.05	.03	.06	.06	.05	1	1	.99	.99
{ .3 ; .3 ; 300 ; 0.5 }	1	.97	1	.88	1	.88	.06	.05	.02	.06	.05	.05	1	1	1	.99
{ .3 ; .3 ; 300 ; 1.0 }	1	.96	1	.92	1	.92	.08	.07	.03	.06	.06	.05	1	1	1	.97
{ .3 ; .3 ; 300 ; 2.0 }	1	.96	1	.96	1	.95	.09	.08	.03	.07	.06	.06	1	1	1	.96
{ .3 ; .3 ; 450 ; 0.1 }	1	.97	1	.90	1	.90	.06	.05	.03	.06	.06	.05	1	1	1	1
{ .3 ; .3 ; 450 ; 0.5 }	1	.97	1	.91	1	.90	.05	.05	.02	.06	.05	.05	1	1	1	1
{ .3 ; .3 ; 450 ; 1.0 }	1	.97	1	.93	1	.93	.08	.07	.03	.06	.06	.05	1	1	1	.97
{ .3 ; .3 ; 450 ; 2.0 }	1	.98	1	.96	1	.96	.09	.08	.02	.07	.06	.05	1	1	1	.96
{ .3 ; .5 ; 150 ; 0.1 }	1	.95	1	.83	1	.83	.06	.05	.02	.07	.05	.05	1	1	.99	.98
{ .3 ; .5 ; 150 ; 0.5 }	1	.95	1	.83	1	.83	.07	.06	.03	.06	.06	.06	1	.99	.99	.97
{ .3 ; .5 ; 150 ; 1.0 }	1	.95	1	.88	1	.88	.10	.07	.03	.06	.06	.05	1	.99	1	.95
{ .3 ; .5 ; 150 ; 2.0 }	1	.96	1	.93	1	.94	.10	.08	.03	.06	.05	.05	1	1	1	.94
{ .3 ; .5 ; 300 ; 0.1 }	1	.97	1	.88	1	.88	.06	.05	.02	.06	.05	.05	1	1	1	.99
{ .3 ; .5 ; 300 ; 0.5 }	1	.96	1	.88	1	.88	.06	.05	.03	.06	.05	.05	1	1	1	.99
{ .3 ; .5 ; 300 ; 1.0 }	1	.95	1	.92	1	.91	.08	.07	.03	.07	.06	.06	1	1	1	.97
{ .3 ; .5 ; 300 ; 2.0 }	1	.97	1	.95	1	.95	.08	.07	.03	.06	.05	.05	1	1	1	.96
{ .3 ; .5 ; 450 ; 0.1 }	1	.97	1	.91	1	.90	.06	.05	.02	.06	.05	.05	1	1	1	1
{ .3 ; .5 ; 450 ; 0.5 }	1	.97	1	.91	1	.90	.06	.05	.02	.06	.05	.05	1	1	1	1
{ .3 ; .5 ; 450 ; 1.0 }	1	.96	1	.93	1	.94	.07	.06	.02	.07	.05	.05	1	1	1	.98
{ .3 ; .5 ; 450 ; 2.0 }	1	.98	1	.96	1	.96	.09	.08	.02	.07	.06	.06	1	1	1	.96

Notes. See Table 1.

Table 8: One factor DGP2. Testing results for $T = 5, S = 10$ and $\sigma^2_\varepsilon = 0.9$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\sigma^2_{f\lambda};\rho;\bar{N};\sigma_f\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L=1$
{ .0 ; .3 ; 150 ; 0.1 }	1	.95	1	.78	1	.78	.07	.06	.02	.06	.05	.06	1	.99	.99	.98
{ .0 ; .3 ; 150 ; 0.5 }	1	.94	1	.78	1	.79	.07	.06	.02	.06	.05	.06	1	.99	.99	.98
{ .0 ; .3 ; 150 ; 1.0 }	1	.93	1	.85	1	.84	.10	.07	.03	.07	.06	.06	1	.99	.99	.94
{ .0 ; .3 ; 150 ; 2.0 }	1	.93	1	.91	1	.91	.12	.08	.03	.07	.06	.05	1	.99	1	.93
{ .0 ; .3 ; 300 ; 0.1 }	1	.97	1	.84	1	.83	.06	.05	.02	.06	.05	.05	1	.99	1	.99
{ .0 ; .3 ; 300 ; 0.5 }	1	.96	1	.85	1	.85	.06	.05	.03	.05	.06	.05	1	.99	.99	.99
{ .0 ; .3 ; 300 ; 1.0 }	1	.95	1	.89	1	.89	.09	.07	.02	.06	.06	.06	1	.99	1	.97
{ .0 ; .3 ; 300 ; 2.0 }	1	.95	1	.94	1	.94	.10	.08	.03	.06	.05	.06	1	1	1	.95
{ .0 ; .3 ; 450 ; 0.1 }	1	.97	1	.87	1	.86	.05	.05	.02	.05	.05	.05	1	1	1	1
{ .0 ; .3 ; 450 ; 0.5 }	1	.96	1	.87	1	.87	.05	.05	.02	.05	.05	.05	1	1	1	1
{ .0 ; .3 ; 450 ; 1.0 }	1	.95	1	.92	1	.91	.08	.07	.03	.06	.06	.06	1	.99	1	.98
{ .0 ; .3 ; 450 ; 2.0 }	1	.97	1	.95	1	.94	.10	.08	.02	.06	.06	.05	1	1	1	.96
{ .0 ; .5 ; 150 ; 0.1 }	1	.95	1	.78	1	.78	.07	.06	.03	.06	.06	.06	1	.99	.99	.98
{ .0 ; .5 ; 150 ; 0.5 }	1	.93	1	.78	1	.78	.08	.05	.03	.07	.06	.06	1	.99	.98	.97
{ .0 ; .5 ; 150 ; 1.0 }	1	.92	1	.85	1	.84	.10	.07	.03	.06	.06	.05	1	.98	.99	.95
{ .0 ; .5 ; 150 ; 2.0 }	1	.95	1	.92	1	.91	.11	.08	.03	.07	.06	.06	1	.99	1	.93
{ .0 ; .5 ; 300 ; 0.1 }	1	.97	1	.84	1	.84	.06	.06	.02	.06	.05	.06	1	1	1	.99
{ .0 ; .5 ; 300 ; 0.5 }	1	.95	1	.84	1	.85	.06	.06	.02	.06	.05	.06	1	.99	.99	.99
{ .0 ; .5 ; 300 ; 1.0 }	1	.95	1	.89	1	.89	.09	.07	.03	.06	.06	.05	1	.99	1	.97
{ .0 ; .5 ; 300 ; 2.0 }	1	.96	1	.93	1	.93	.11	.08	.03	.06	.06	.05	1	1	1	.95
{ .0 ; .5 ; 450 ; 0.1 }	1	.96	1	.87	1	.86	.06	.05	.02	.06	.05	.05	1	1	1	1
{ .0 ; .5 ; 450 ; 0.5 }	1	.96	1	.87	1	.87	.06	.05	.02	.06	.05	.05	1	1	1	1
{ .0 ; .5 ; 450 ; 1.0 }	1	.95	1	.90	1	.90	.08	.06	.02	.06	.06	.05	1	1	1	.98
{ .0 ; .5 ; 450 ; 2.0 }	1	.97	1	.95	1	.95	.09	.08	.03	.07	.06	.06	1	1	1	.96
{ .3 ; .3 ; 150 ; 0.1 }	1	.94	1	.79	1	.78	.07	.05	.02	.06	.05	.05	1	.99	.99	.97
{ .3 ; .3 ; 150 ; 0.5 }	1	.94	1	.80	1	.79	.07	.06	.03	.06	.05	.06	1	.99	.98	.97
{ .3 ; .3 ; 150 ; 1.0 }	1	.93	1	.85	1	.84	.10	.07	.03	.07	.05	.06	1	.99	.99	.94
{ .3 ; .3 ; 150 ; 2.0 }	1	.95	1	.91	1	.91	.12	.08	.04	.07	.06	.06	1	.99	1	.93
{ .3 ; .3 ; 300 ; 0.1 }	1	.96	1	.84	1	.83	.06	.06	.03	.06	.06	.06	1	.99	.99	.99
{ .3 ; .3 ; 300 ; 0.5 }	1	.96	1	.85	1	.85	.06	.05	.03	.06	.06	.05	1	.99	.99	.99
{ .3 ; .3 ; 300 ; 1.0 }	1	.95	1	.89	1	.89	.09	.07	.02	.06	.06	.06	1	.99	.99	.97
{ .3 ; .3 ; 300 ; 2.0 }	1	.96	1	.93	1	.93	.10	.09	.03	.07	.06	.06	1	1	1	.95
{ .3 ; .3 ; 450 ; 0.1 }	1	.96	1	.86	1	.86	.06	.05	.02	.06	.06	.05	1	1	1	1
{ .3 ; .3 ; 450 ; 0.5 }	1	.96	1	.87	1	.88	.06	.05	.02	.06	.05	.05	1	1	1	1
{ .3 ; .3 ; 450 ; 1.0 }	1	.95	1	.91	1	.91	.08	.07	.02	.06	.05	.06	1	.99	1	.97
{ .3 ; .3 ; 450 ; 2.0 }	1	.97	1	.95	1	.95	.09	.07	.02	.06	.06	.06	1	1	1	.96
{ .3 ; .5 ; 150 ; 0.1 }	1	.94	1	.78	1	.78	.06	.05	.03	.07	.05	.05	1	.99	.98	.98
{ .3 ; .5 ; 150 ; 0.5 }	1	.93	1	.79	1	.79	.06	.06	.03	.07	.05	.06	1	.99	.99	.98
{ .3 ; .5 ; 150 ; 1.0 }	1	.92	1	.86	1	.85	.10	.07	.03	.07	.06	.06	1	.99	.99	.94
{ .3 ; .5 ; 150 ; 2.0 }	1	.94	1	.91	1	.91	.11	.09	.03	.07	.05	.06	1	1	1	.92
{ .3 ; .5 ; 300 ; 0.1 }	1	.96	1	.85	1	.84	.05	.05	.03	.06	.05	.05	1	1	.99	1
{ .3 ; .5 ; 300 ; 0.5 }	1	.96	1	.85	1	.85	.06	.06	.02	.06	.05	.06	1	1	.99	.99
{ .3 ; .5 ; 300 ; 1.0 }	1	.95	1	.90	1	.89	.09	.07	.03	.07	.06	.06	1	.99	1	.96
{ .3 ; .5 ; 300 ; 2.0 }	1	.96	1	.94	1	.94	.11	.07	.03	.06	.06	.05	1	1	1	.95
{ .3 ; .5 ; 450 ; 0.1 }	1	.97	1	.86	1	.87	.05	.06	.02	.05	.05	.06	1	1	1	1
{ .3 ; .5 ; 450 ; 0.5 }	1	.96	1	.88	1	.87	.06	.05	.02	.06	.06	.05	1	1	1	.99
{ .3 ; .5 ; 450 ; 1.0 }	1	.96	1	.91	1	.91	.09	.07	.03	.06	.07	.06	1	.99	1	.98
{ .3 ; .5 ; 450 ; 2.0 }	1	.96	1	.95	1	.95	.09	.08	.02	.06	.06	.06	1	1	1	.96

Notes. See Table 1.

2.3. Two Factors

2.3.1. The Setup

In the Monte Carlo setup in the main text we focus only on the case with one time-varying (stochastic) factor. By focusing on this case we can more easily disentangle Monte Carlo parameters that drive consistency, local/global identification and possible problems with normalization used. However, there is always a trade-off between estimation of the correct number of factors and increase in the number of parameters and decrease in the number of moment conditions. In order to investigate this issues further, here we present two DGPs for the model with two factors. The first DGP ($DGP1$) is based on the design presented in the main text with

$$\boldsymbol{\lambda}_s' \boldsymbol{f}_t = \lambda_s f_t + \delta_s, \quad \delta_s \sim U(0, \delta_{f_2}), \quad \delta_{f_2} = \{0.2; 0.5\} \tag{2.7}$$

where λ_s, f_t are generated as in the model with one factor. We focus on small values δ_{f_2} so that parameters in the original setup and the normalisation used are still approximately satisfied and there is no need to use a totally different setup. Hence the only difference compared to the model presented in the main text is the presence of the second factors with relatively small variance.

In the second DGP ($DGP2$), we deviate from the original model and consider the following model

$$\boldsymbol{\lambda}_s' \boldsymbol{f}_t = \sum_{l=1}^{2} \lambda_s^{(l)} f_t^{(l)}, \quad \lambda_s^{(l)} \sim N(0,1), \tag{2.8}$$

$$f_t^{(l)} = 1 + \sigma_f u_t^{(l)}, \tag{2.9}$$

for $l = \{1; 2\}$. Other parameters are summarized below:

$$T = 5, \quad S = 10, \quad \boldsymbol{\theta}_0 = (1,1)', \quad \alpha_f = 0.5,$$
$$\bar{N} = \{150; 300; 450\}, \quad \sigma_\varepsilon^2 = \{0.5; 0.9\},$$
$$\sigma_\mu^2 = \{0.3; 0.5\}, \quad \rho = \{0; 0.3\}, \quad \sigma_f = \{0.1; 0.5; 1.0; 2.0\}.$$

Note that in this setup we do not impose any normalization on the relative variances. We do that as the identification in the model with two factors is conceptually more challenging, so that finding "sensible" normalizations in this case is not trivial.

Remark 3. Similarly, to the DGPs for the model with one factor (presented in this appendix) we assume that $\mathrm{E}[\lambda_s] = 0$ (unlike the main text). Hence, in this case the GMMl0 estimator remains asymptotically unbiased (but inconsistent).

2.3.2. Results two factors: DGP1

Here, we briefly summarise the results with two factors. More detailed results can be found in Tables 9 and 10.

- The results in terms of the bias for GMMn1 and GMMn2 are comparable, however, despite inconsistency GMMn1 performs no worse than the consistent GMMn2 estimator (in terms of RMSE).
- In some cases GMMl1 dominates both non-linear estimators.
- The $J-$ test of the GMMn1 estimator, as can be expected, has similar power to the nominal size for small values of δ_{f_2}, with increase in power visible as $\bar{N}$ and/or δ_{f_2} increase. The empirical size of the $J-$ statistic of the GMMn2 estimator is close to the nominal size, with minor size distortions present for larger values of $\sigma^2_{f\lambda}$.
- The performance of the BIC model selection procedure is sub-optimal. In the vast majority of cases, the optimal estimator is the estimator with one factor. Hence, despite the fact that this estimator is inconsistent, the magnitude of the inconsistency (as measured by the large values of the $J-$ statistic) is not sufficient and is dominated by the BIC penalty term.
- The optimal estimator dominates both GMMn1 and GMMn2 in terms of the RMSE criteria. Suggesting that also numerical optimization in the model with two factors is less straightforward as compared to the model with one factor.

2.3.3. Results two factors: DGP2

Detailed results for this sections are presented in Tables 11, 12, 13 and 14. Here we briefly summarize the main trends.

- Results for GMMl1 and GMMl2 estimators are comparable in all respect. The GMML2 estimators tends to have slightly larger RMSE and slightly lower rejection frequency of the J and $t-$ statistics.
- The BIC criterion selects the model with one factor only for small values of σ_f^2 and larger values of σ_ε^2.
- In some cases, for smaller values of σ_f^2 the model with one factor is also preferred as it also has smaller RMSE. However, the same cannot be said regarding testing as both tests clearly reject null hypothesis.
- The results for different values of ρ are similar. Increase of σ_μ^2, on the other hand reduces the RMSE of all estimator.
- The GMMn2 tends to perform better (and/or dominate other estimators) for larger values of σ_f^2 and $\bar{N}$. That suggests that in order to precisely estimate model with two time-varying factors, both of those factors have to exhibit sufficient variation. Otherwise, global identification of such model is difficult.

Table 9: Two factors DGP1. Estimation results for $T = 5, S = 10, \sigma_f = 0.1$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\sigma^2_{f\lambda}; \rho; \sigma^2_\mu; \bar{N}; \delta_{f_2}\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .1 ; .0 ; .00 ; 150 ; .2 }	.67	.00	.00	.04	.02	.03	.68	.19	.18	.21	.23	.20
{ .1 ; .0 ; .00 ; 300 ; .2 }	.71	.00	.00	.04	.02	.03	.71	.23	.20	.19	.22	.18
{ .1 ; .3 ; .00 ; 150 ; .2 }	.68	.29	.30	.26	.26	.26	.69	.35	.34	.32	.34	.31
{ .1 ; .3 ; .00 ; 300 ; .2 }	.61	.30	.30	.22	.22	.21	.62	.37	.35	.27	.30	.27
{ .5 ; .0 ; .00 ; 150 ; .2 }	.47	.00	.00	.03	.03	.03	.52	.18	.18	.12	.15	.12
{ .5 ; .0 ; .00 ; 300 ; .2 }	.62	-.01	-.01	.04	.04	.04	.66	.24	.21	.12	.14	.12
{ .5 ; .3 ; .00 ; 150 ; .2 }	.48	.30	.30	.22	.23	.22	.53	.34	.34	.24	.27	.24
{ .5 ; .3 ; .00 ; 300 ; .2 }	.57	.30	.30	.19	.21	.19	.61	.38	.37	.22	.24	.21
{ .1 ; .0 ; .00 ; 150 ; .5 }	.82	.00	.00	.05	.03	.03	.83	.19	.18	.23	.24	.21
{ .1 ; .0 ; .00 ; 300 ; .5 }	.86	.00	.00	.06	.04	.05	.86	.23	.20	.22	.24	.21
{ .1 ; .3 ; .00 ; 150 ; .5 }	.83	.29	.30	.29	.29	.29	.84	.35	.34	.35	.36	.34
{ .1 ; .3 ; .00 ; 300 ; .5 }	.76	.30	.30	.26	.26	.26	.77	.37	.35	.31	.33	.30
{ .5 ; .0 ; .00 ; 150 ; .5 }	.61	.00	.00	.07	.06	.07	.66	.18	.18	.15	.18	.15
{ .5 ; .0 ; .00 ; 300 ; .5 }	.77	-.01	-.01	.09	.08	.09	.81	.24	.21	.17	.18	.16
{ .5 ; .3 ; .00 ; 150 ; .5 }	.62	.30	.30	.27	.28	.27	.67	.34	.34	.29	.31	.29
{ .5 ; .3 ; .00 ; 300 ; .5 }	.72	.30	.30	.27	.27	.27	.75	.38	.37	.29	.30	.28
{ .1 ; .0 ; .05 ; 150 ; .2 }	.60	.00	.00	.01	.00	.00	.61	.07	.07	.07	.09	.06
{ .1 ; .0 ; .05 ; 300 ; .2 }	.59	.00	.00	.00	.00	.00	.60	.06	.05	.05	.06	.05
{ .1 ; .3 ; .05 ; 150 ; .2 }	.78	.04	.04	.05	.05	.04	.79	.09	.08	.12	.10	.08
{ .1 ; .3 ; .05 ; 300 ; .2 }	.60	.02	.02	.03	.03	.02	.61	.06	.06	.06	.07	.05
{ .5 ; .0 ; .05 ; 150 ; .2 }	.52	.00	.00	.01	.01	.01	.56	.07	.07	.04	.06	.05
{ .5 ; .0 ; .05 ; 300 ; .2 }	.68	.00	.00	.01	.01	.01	.72	.07	.06	.03	.05	.03
{ .5 ; .3 ; .05 ; 150 ; .2 }	.60	.04	.04	.04	.05	.04	.64	.08	.08	.07	.08	.06
{ .5 ; .3 ; .05 ; 300 ; .2 }	.68	.02	.02	.03	.03	.02	.71	.07	.07	.05	.05	.04
{ .1 ; .0 ; .05 ; 150 ; .5 }	.74	.00	.00	.01	.01	.01	.75	.07	.07	.08	.09	.07
{ .1 ; .0 ; .05 ; 300 ; .5 }	.73	.00	.00	.01	.01	.00	.74	.06	.05	.06	.07	.05
{ .1 ; .3 ; .05 ; 150 ; .5 }	.92	.04	.04	.05	.05	.04	.93	.09	.08	.14	.11	.09
{ .1 ; .3 ; .05 ; 300 ; .5 }	.74	.02	.02	.03	.03	.03	.75	.06	.06	.08	.07	.06
{ .5 ; .0 ; .05 ; 150 ; .5 }	.66	.00	.00	.01	.01	.01	.70	.07	.07	.05	.07	.05
{ .5 ; .0 ; .05 ; 300 ; .5 }	.82	.00	.00	.01	.01	.01	.85	.07	.06	.05	.05	.04
{ .5 ; .3 ; .05 ; 150 ; .5 }	.75	.04	.04	.05	.05	.05	.78	.08	.08	.08	.08	.07
{ .5 ; .3 ; .05 ; 300 ; .5 }	.82	.02	.02	.03	.03	.03	.85	.07	.07	.05	.06	.05
{ .1 ; .0 ; .30 ; 150 ; .2 }	.38	.00	.00	.00	.00	.00	.40	.03	.03	.04	.04	.03
{ .1 ; .0 ; .30 ; 300 ; .2 }	.47	.00	.00	.00	.00	.00	.48	.02	.02	.04	.03	.02
{ .1 ; .3 ; .30 ; 150 ; .2 }	.53	.01	.01	.01	.01	.01	.54	.03	.03	.05	.04	.03
{ .1 ; .3 ; .30 ; 300 ; .2 }	.45	.00	.00	.00	.00	.00	.46	.02	.02	.03	.03	.02
{ .5 ; .0 ; .30 ; 150 ; .2 }	.52	.00	.00	.00	.00	.00	.55	.03	.03	.02	.03	.02
{ .5 ; .0 ; .30 ; 300 ; .2 }	.54	.00	.00	.00	.00	.00	.57	.03	.03	.02	.02	.01
{ .5 ; .3 ; .30 ; 150 ; .2 }	.47	.00	.00	.01	.01	.01	.50	.03	.03	.02	.03	.02
{ .5 ; .3 ; .30 ; 300 ; .2 }	.46	.00	.00	.00	.00	.00	.50	.03	.03	.02	.03	.02
{ .1 ; .0 ; .30 ; 150 ; .5 }	.50	.00	.00	.00	.00	.00	.51	.03	.03	.05	.05	.03
{ .1 ; .0 ; .30 ; 300 ; .5 }	.59	.00	.00	.01	.00	.00	.60	.02	.02	.06	.04	.02
{ .1 ; .3 ; .30 ; 150 ; .5 }	.65	.01	.01	.01	.01	.01	.66	.03	.03	.07	.05	.03
{ .1 ; .3 ; .30 ; 300 ; .5 }	.56	.00	.00	.01	.01	.00	.57	.02	.02	.04	.04	.02
{ .5 ; .0 ; .30 ; 150 ; .5 }	.64	.00	.00	.00	.00	.00	.67	.03	.03	.02	.03	.02
{ .5 ; .0 ; .30 ; 300 ; .5 }	.65	.00	.00	.00	.00	.00	.68	.03	.03	.03	.02	.02
{ .5 ; .3 ; .30 ; 150 ; .5 }	.58	.00	.00	.01	.01	.01	.61	.03	.03	.03	.03	.02
{ .5 ; .3 ; .30 ; 300 ; .5 }	.58	.00	.00	.01	.01	.00	.61	.03	.03	.03	.03	.03

Notes. See Table 1.

Table 10: Two factors DGP1. Testing results for $T = 5, S = 10, \sigma_f = 0.1$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\sigma^2_{f\lambda}; \rho; \sigma^2_\mu; N; \delta_{f_2}\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L = 1$
{ .1 ; .0 ; .00 ; 150 ; .2 }	1	1	.50	.09	.20	.06	.09	.13	.02	.12	.04	.12	.71	.99	.57	.93
{ .1 ; .0 ; .00 ; 300 ; .2 }	1	1	.75	.16	.39	.09	.08	.12	.02	.12	.06	.11	.87	1	.67	.98
{ .1 ; .3 ; .00 ; 150 ; .2 }	1	1	.53	.52	.22	.49	.12	.55	.03	.43	.06	.53	.73	.95	.54	.92
{ .1 ; .3 ; .00 ; 300 ; .2 }	1	1	.71	.53	.40	.49	.09	.51	.03	.42	.07	.50	.85	.95	.60	.97
{ .5 ; .0 ; .00 ; 150 ; .2 }	1	.99	.75	.17	.65	.14	.06	.10	.02	.11	.04	.11	.90	.99	.67	.97
{ .5 ; .0 ; .00 ; 300 ; .2 }	1	1	.90	.28	.81	.22	.06	.13	.02	.13	.05	.13	.97	.99	.78	.99
{ .5 ; .3 ; .00 ; 150 ; .2 }	1	.99	.78	.71	.67	.68	.08	.70	.02	.55	.05	.69	.92	.97	.66	.96
{ .5 ; .3 ; .00 ; 300 ; .2 }	1	1	.92	.67	.85	.65	.07	.69	.03	.54	.06	.69	.98	.98	.78	.99
{ .1 ; .0 ; .00 ; 150 ; .5 }	1	1	.50	.09	.20	.06	.10	.13	.02	.12	.05	.12	.71	1	.60	.93
{ .1 ; .0 ; .00 ; 300 ; .5 }	1	1	.75	.16	.39	.09	.09	.14	.03	.14	.07	.14	.88	1	.70	.97
{ .1 ; .3 ; .00 ; 150 ; .5 }	1	1	.53	.52	.22	.49	.11	.58	.03	.45	.06	.56	.72	.98	.55	.93
{ .1 ; .3 ; .00 ; 300 ; .5 }	1	1	.71	.53	.40	.49	.10	.60	.03	.48	.07	.60	.85	.98	.60	.97
{ .5 ; .0 ; .00 ; 150 ; .5 }	1	1	.75	.17	.65	.14	.09	.20	.03	.18	.06	.21	.89	1	.70	.96
{ .5 ; .0 ; .00 ; 300 ; .5 }	1	1	.90	.28	.81	.22	.11	.29	.04	.23	.09	.28	.97	1	.80	.97
{ .5 ; .3 ; .00 ; 150 ; .5 }	1	1	.78	.71	.67	.68	.09	.83	.03	.68	.06	.82	.92	.98	.66	.96
{ .5 ; .3 ; .00 ; 300 ; .5 }	1	1	.92	.67	.85	.65	.11	.86	.04	.72	.10	.86	.98	.99	.78	.98
{ .1 ; .0 ; .05 ; 150 ; .2 }	1	1	.55	.13	.28	.09	.10	.07	.03	.08	.06	.07	.78	1	.63	.95
{ .1 ; .0 ; .05 ; 300 ; .2 }	1	1	.71	.18	.37	.10	.08	.07	.02	.08	.06	.07	.89	1	.71	.98
{ .1 ; .3 ; .05 ; 150 ; .2 }	1	1	.66	.21	.23	.14	.11	.14	.04	.12	.07	.12	.85	1	.68	.94
{ .1 ; .3 ; .05 ; 300 ; .2 }	1	1	.71	.22	.38	.13	.09	.12	.03	.11	.07	.11	.89	1	.73	.98
{ .5 ; .0 ; .05 ; 150 ; .2 }	1	1	.80	.21	.68	.19	.07	.06	.03	.08	.06	.06	.93	1	.76	.97
{ .5 ; .0 ; .05 ; 300 ; .2 }	1	1	.94	.35	.85	.27	.08	.07	.04	.08	.07	.07	.98	1	.89	.99
{ .5 ; .3 ; .05 ; 150 ; .2 }	1	1	.81	.30	.67	.25	.10	.20	.03	.18	.07	.19	.92	1	.76	.96
{ .5 ; .3 ; .05 ; 300 ; .2 }	1	1	.93	.36	.83	.28	.09	.15	.03	.14	.08	.15	.98	1	.87	.99
{ .1 ; .0 ; .05 ; 150 ; .5 }	1	1	.55	.13	.28	.09	.11	.07	.04	.09	.07	.07	.77	1	.62	.95
{ .1 ; .0 ; .05 ; 300 ; .5 }	1	1	.71	.18	.37	.10	.10	.07	.04	.08	.08	.07	.88	1	.70	.98
{ .1 ; .3 ; .05 ; 150 ; .5 }	1	1	.66	.21	.23	.14	.12	.14	.04	.13	.07	.13	.85	1	.67	.93
{ .1 ; .3 ; .05 ; 300 ; .5 }	1	1	.71	.22	.38	.13	.12	.12	.04	.12	.09	.11	.88	1	.72	.97
{ .5 ; .0 ; .05 ; 150 ; .5 }	1	1	.80	.21	.68	.19	.13	.09	.05	.10	.10	.09	.92	1	.76	.95
{ .5 ; .0 ; .05 ; 300 ; .5 }	1	1	.94	.35	.85	.27	.18	.11	.08	.11	.15	.10	.98	1	.88	.96
{ .5 ; .3 ; .05 ; 150 ; .5 }	1	1	.81	.30	.67	.25	.15	.23	.06	.20	.11	.22	.92	1	.76	.94
{ .5 ; .3 ; .05 ; 300 ; .5 }	1	1	.93	.36	.83	.28	.19	.20	.08	.18	.15	.19	.97	1	.87	.96
{ .1 ; .0 ; .30 ; 150 ; .2 }	1	1	.38	.11	.19	.08	.09	.06	.03	.07	.06	.06	.65	1	.52	.96
{ .1 ; .0 ; .30 ; 300 ; .2 }	1	1	.66	.18	.30	.10	.08	.06	.03	.07	.06	.06	.87	1	.68	.98
{ .1 ; .3 ; .30 ; 150 ; .2 }	1	1	.55	.15	.23	.09	.10	.07	.04	.08	.07	.07	.79	1	.61	.96
{ .1 ; .3 ; .30 ; 300 ; .2 }	1	1	.66	.16	.36	.10	.09	.06	.03	.07	.07	.06	.86	1	.69	.98
{ .5 ; .0 ; .30 ; 150 ; .2 }	1	1	.84	.24	.70	.19	.08	.06	.03	.08	.06	.06	.95	1	.81	.97
{ .5 ; .0 ; .30 ; 300 ; .2 }	1	1	.94	.36	.84	.28	.08	.06	.03	.07	.07	.06	.98	1	.89	.99
{ .5 ; .3 ; .30 ; 150 ; .2 }	1	1	.81	.24	.69	.19	.07	.07	.03	.08	.06	.07	.93	1	.79	.97
{ .5 ; .3 ; .30 ; 300 ; .2 }	1	1	.93	.33	.87	.28	.09	.07	.04	.09	.08	.07	.98	1	.88	.98
{ .1 ; .0 ; .30 ; 150 ; .5 }	1	1	.38	.11	.19	.08	.10	.07	.04	.08	.07	.06	.63	1	.49	.95
{ .1 ; .0 ; .30 ; 300 ; .5 }	1	1	.66	.18	.30	.10	.11	.07	.04	.08	.09	.06	.86	1	.66	.98
{ .1 ; .3 ; .30 ; 150 ; .5 }	1	1	.55	.15	.23	.09	.11	.08	.04	.09	.08	.07	.78	1	.60	.95
{ .1 ; .3 ; .30 ; 300 ; .5 }	1	1	.66	.16	.36	.10	.11	.07	.04	.08	.09	.07	.85	1	.68	.98
{ .5 ; .0 ; .30 ; 150 ; .5 }	1	1	.84	.24	.70	.19	.13	.07	.05	.09	.10	.07	.94	1	.80	.95
{ .5 ; .0 ; .30 ; 300 ; .5 }	1	1	.94	.36	.84	.28	.19	.08	.08	.09	.16	.08	.98	1	.89	.95
{ .5 ; .3 ; .30 ; 150 ; .5 }	1	1	.81	.24	.69	.19	.13	.09	.06	.10	.10	.09	.92	1	.77	.95
{ .5 ; .3 ; .30 ; 300 ; .5 }	1	1	.93	.33	.87	.28	.22	.10	.09	.10	.17	.09	.98	1	.87	.94

Notes. See Table 1.

Table 11: Two factors DGP2. Estimation results for $T = 5, S = 10$ and $\sigma_{\varepsilon}^2 = 0.5$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\rho; \sigma_\mu^2; \bar{N}; \sigma_f\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .0 ; .3 ; 150 ; 0.1 }	.01	.00	.00	.00	.00	.00	.38	.04	.05	.04	.04	.04
{ .0 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.40	.20	.21	.16	.06	.06
{ .0 ; .3 ; 150 ; 1.0 }	.00	-.01	-.01	-.01	.00	.00	.45	.39	.41	.29	.09	.09
{ .0 ; .3 ; 150 ; 2.0 }	.00	.00	-.01	-.01	.00	.00	.64	.80	.84	.53	.19	.20
{ .0 ; .3 ; 300 ; 0.1 }	.01	.00	.00	.00	.00	.00	.37	.04	.05	.03	.03	.03
{ .0 ; .3 ; 300 ; 0.5 }	.01	.00	.00	.00	.00	.00	.40	.20	.21	.17	.06	.06
{ .0 ; .3 ; 300 ; 1.0 }	.00	.01	.01	-.01	.00	.00	.46	.41	.43	.32	.12	.12
{ .0 ; .3 ; 300 ; 2.0 }	.00	-.01	-.01	.00	.00	.00	.64	.82	.86	.52	.21	.21
{ .0 ; .3 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.37	.04	.04	.03	.03	.02
{ .0 ; .3 ; 450 ; 0.5 }	.00	.01	.01	.00	.00	.00	.40	.20	.21	.17	.07	.06
{ .0 ; .3 ; 450 ; 1.0 }	.00	-.01	-.01	.00	.00	.00	.46	.41	.43	.31	.09	.09
{ .0 ; .3 ; 450 ; 2.0 }	.00	-.02	-.01	-.01	.00	.00	.66	.84	.90	.55	.21	.21
{ .0 ; .5 ; 150 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.33	.03	.04	.03	.03	.03
{ .0 ; .5 ; 150 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.35	.16	.17	.13	.05	.05
{ .0 ; .5 ; 150 ; 1.0 }	.00	.01	.01	-.01	.00	.00	.41	.32	.33	.25	.09	.09
{ .0 ; .5 ; 150 ; 2.0 }	.00	-.01	-.01	-.01	.00	.00	.57	.63	.66	.44	.13	.13
{ .0 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.34	.03	.04	.03	.02	.02
{ .0 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.35	.16	.17	.13	.05	.05
{ .0 ; .5 ; 300 ; 1.0 }	.00	-.01	-.01	.00	.00	.00	.41	.31	.33	.25	.08	.08
{ .0 ; .5 ; 300 ; 2.0 }	.00	.00	.00	.00	.00	.00	.60	.62	.66	.45	.16	.16
{ .0 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.34	.03	.04	.03	.02	.02
{ .0 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.35	.16	.17	.13	.05	.05
{ .0 ; .5 ; 450 ; 1.0 }	-.01	.00	.00	.00	.00	.00	.40	.31	.33	.25	.06	.07
{ .0 ; .5 ; 450 ; 2.0 }	.00	.00	.00	.00	.00	.00	.60	.64	.67	.44	.14	.14
{ .3 ; .3 ; 150 ; 0.1 }	.01	.01	.01	.01	.01	.01	.37	.04	.05	.04	.04	.04
{ .3 ; .3 ; 150 ; 0.5 }	.01	.01	.01	.01	.01	.01	.39	.20	.21	.16	.06	.06
{ .3 ; .3 ; 150 ; 1.0 }	.00	.01	.01	.00	.00	.00	.45	.41	.43	.31	.11	.11
{ .3 ; .3 ; 150 ; 2.0 }	-.01	-.01	-.02	.00	.00	.00	.67	.81	.86	.54	.19	.20
{ .3 ; .3 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.38	.04	.05	.03	.03	.03
{ .3 ; .3 ; 300 ; 0.5 }	-.01	.00	.01	.00	.00	.00	.40	.20	.21	.16	.06	.06
{ .3 ; .3 ; 300 ; 1.0 }	-.01	.00	.00	.00	.00	.00	.46	.41	.43	.30	.11	.11
{ .3 ; .3 ; 300 ; 2.0 }	.00	.00	.00	-.01	.00	.00	.66	.80	.86	.52	.17	.17
{ .3 ; .3 ; 450 ; 0.1 }	.01	.00	.00	.00	.00	.00	.37	.04	.04	.03	.03	.03
{ .3 ; .3 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.39	.21	.22	.17	.06	.06
{ .3 ; .3 ; 450 ; 1.0 }	.01	.01	.01	.00	.00	.00	.46	.41	.43	.29	.10	.10
{ .3 ; .3 ; 450 ; 2.0 }	-.02	.00	.00	-.01	.00	.00	.65	.82	.87	.52	.14	.14
{ .3 ; .5 ; 150 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.32	.04	.04	.03	.03	.03
{ .3 ; .5 ; 150 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.35	.16	.17	.13	.06	.06
{ .3 ; .5 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.40	.32	.34	.26	.10	.09
{ .3 ; .5 ; 150 ; 2.0 }	.00	-.01	-.01	.00	.00	.00	.61	.62	.66	.46	.14	.14
{ .3 ; .5 ; 300 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.33	.03	.04	.03	.02	.02
{ .3 ; .5 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.35	.16	.17	.14	.05	.05
{ .3 ; .5 ; 300 ; 1.0 }	.00	.00	.00	.01	.00	.00	.41	.32	.34	.25	.09	.08
{ .3 ; .5 ; 300 ; 2.0 }	-.02	-.02	-.01	-.01	.00	.00	.58	.62	.66	.44	.11	.11
{ .3 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.34	.03	.03	.03	.02	.02
{ .3 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.36	.16	.16	.14	.05	.04
{ .3 ; .5 ; 450 ; 1.0 }	.01	.01	.01	.00	.00	.00	.41	.32	.34	.26	.09	.08
{ .3 ; .5 ; 450 ; 2.0 }	.01	-.01	-.01	.00	.00	.00	.58	.63	.66	.44	.14	.14

Notes. See Table 1.

Table 12: Two factors DGP2. Estimation results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.9$. For $\zeta_0 = 1$.

	Bias						RMSE					
$\{\rho; \sigma_\mu^2; \bar{N}; \sigma_f\}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$	L_0	L_1^{FE}	L_2^{FE}	L_1	L_2	$\hat{L}$
{ .0 ; .3 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.38	.05	.05	.04	.05	.04
{ .0 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.39	.20	.21	.16	.07	.07
{ .0 ; .3 ; 150 ; 1.0 }	.00	.00	.00	-.01	.00	.00	.46	.41	.43	.31	.10	.10
{ .0 ; .3 ; 150 ; 2.0 }	.00	.00	.00	.00	.00	.00	.66	.80	.86	.53	.16	.17
{ .0 ; .3 ; 300 ; 0.1 }	.01	.00	.00	.00	.00	.00	.37	.04	.05	.04	.04	.03
{ .0 ; .3 ; 300 ; 0.5 }	.00	.00	.00	.00	.00	.00	.39	.20	.22	.17	.08	.08
{ .0 ; .3 ; 300 ; 1.0 }	.00	.00	.00	.00	.00	.00	.44	.41	.43	.31	.10	.10
{ .0 ; .3 ; 300 ; 2.0 }	-.01	-.01	-.01	.00	.00	.00	.66	.80	.85	.52	.15	.15
{ .0 ; .3 ; 450 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.37	.04	.05	.03	.03	.03
{ .0 ; .3 ; 450 ; 0.5 }	-.01	.00	.00	.00	.00	.00	.40	.20	.22	.18	.05	.06
{ .0 ; .3 ; 450 ; 1.0 }	-.01	-.01	-.01	.00	.00	.00	.46	.42	.44	.31	.11	.10
{ .0 ; .3 ; 450 ; 2.0 }	-.02	.00	.00	.00	.00	.00	.64	.82	.87	.52	.21	.21
{ .0 ; .5 ; 150 ; 0.1 }	.01	.00	.00	.00	.00	.00	.33	.04	.04	.03	.04	.03
{ .0 ; .5 ; 150 ; 0.5 }	.00	.00	.01	.00	.00	.00	.35	.16	.17	.13	.06	.06
{ .0 ; .5 ; 150 ; 1.0 }	.00	-.01	-.01	.00	.00	.00	.40	.32	.34	.25	.09	.09
{ .0 ; .5 ; 150 ; 2.0 }	.00	.01	.02	.00	.00	.00	.58	.63	.66	.45	.13	.13
{ .0 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.33	.03	.04	.03	.03	.03
{ .0 ; .5 ; 300 ; 0.5 }	-.02	.00	.00	.00	.00	.00	.36	.16	.17	.14	.05	.05
{ .0 ; .5 ; 300 ; 1.0 }	-.01	.00	.00	-.01	.00	.00	.41	.32	.33	.26	.09	.09
{ .0 ; .5 ; 300 ; 2.0 }	.00	-.01	.00	.00	.00	.00	.59	.65	.67	.42	.19	.19
{ .0 ; .5 ; 450 ; 0.1 }	-.01	.00	.00	.00	.00	.00	.33	.03	.04	.03	.02	.02
{ .0 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.36	.16	.17	.14	.05	.05
{ .0 ; .5 ; 450 ; 1.0 }	-.01	-.01	-.01	.00	.00	.00	.41	.32	.34	.24	.08	.08
{ .0 ; .5 ; 450 ; 2.0 }	.00	.00	.00	.01	.00	.00	.61	.65	.69	.45	.17	.16
{ .3 ; .3 ; 150 ; 0.1 }	.00	.01	.01	.01	.01	.01	.37	.05	.05	.04	.05	.04
{ .3 ; .3 ; 150 ; 0.5 }	.00	.00	.00	.00	.00	.00	.40	.20	.21	.17	.08	.08
{ .3 ; .3 ; 150 ; 1.0 }	.00	.00	.00	.00	.00	.00	.47	.40	.42	.30	.12	.11
{ .3 ; .3 ; 150 ; 2.0 }	.01	-.01	-.01	.00	.00	.00	.65	.79	.84	.51	.19	.19
{ .3 ; .3 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.37	.04	.05	.03	.04	.03
{ .3 ; .3 ; 300 ; 0.5 }	.00	.01	.01	.00	.00	.00	.39	.21	.22	.17	.06	.06
{ .3 ; .3 ; 300 ; 1.0 }	.00	.00	.01	.01	.00	.00	.46	.41	.43	.30	.09	.09
{ .3 ; .3 ; 300 ; 2.0 }	-.01	.00	.00	-.01	.00	.00	.65	.82	.86	.53	.15	.16
{ .3 ; .3 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.37	.04	.05	.04	.03	.03
{ .3 ; .3 ; 450 ; 0.5 }	.01	.01	.01	.01	.00	.00	.39	.21	.22	.17	.06	.06
{ .3 ; .3 ; 450 ; 1.0 }	.01	.00	.00	.00	.00	.00	.46	.40	.42	.31	.10	.10
{ .3 ; .3 ; 450 ; 2.0 }	-.02	-.04	-.03	-.01	.00	.00	.65	.79	.85	.52	.18	.18
{ .3 ; .5 ; 150 ; 0.1 }	.00	.00	.00	.00	.00	.00	.33	.04	.04	.03	.04	.03
{ .3 ; .5 ; 150 ; 0.5 }	.01	.00	.00	.00	.00	.00	.35	.16	.17	.13	.06	.06
{ .3 ; .5 ; 150 ; 1.0 }	.00	.01	.01	.00	.00	.00	.41	.32	.33	.25	.09	.09
{ .3 ; .5 ; 150 ; 2.0 }	.00	.01	.01	-.01	.00	.00	.59	.62	.66	.44	.13	.13
{ .3 ; .5 ; 300 ; 0.1 }	.00	.00	.00	.00	.00	.00	.34	.03	.04	.03	.03	.03
{ .3 ; .5 ; 300 ; 0.5 }	.01	.00	.00	.00	.00	.00	.35	.16	.17	.14	.05	.04
{ .3 ; .5 ; 300 ; 1.0 }	.01	.00	.00	.00	.00	.00	.42	.32	.34	.26	.07	.07
{ .3 ; .5 ; 300 ; 2.0 }	.00	.00	.00	.00	.01	.01	.57	.63	.66	.43	.15	.15
{ .3 ; .5 ; 450 ; 0.1 }	.00	.00	.00	.00	.00	.00	.33	.03	.03	.03	.02	.02
{ .3 ; .5 ; 450 ; 0.5 }	.00	.00	.00	.00	.00	.00	.35	.16	.17	.14	.05	.05
{ .3 ; .5 ; 450 ; 1.0 }	-.01	-.01	.00	.00	.00	.00	.41	.32	.34	.25	.10	.10
{ .3 ; .5 ; 450 ; 2.0 }	.00	-.01	-.02	.00	.00	.00	.60	.64	.67	.45	.16	.15

Notes. See Table 1.

Table 13: Two factors DGP2. Testing results for $T = 5, S = 10$ and $\sigma_\varepsilon^2 = 0.5$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\rho;\sigma_\mu^2;N;\sigma_f\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L=1$
{ .0 ; .3 ; 150 ; 0.1 }	1	.97	.98	.40	.98	.39	.83	.27	.37	.18	.42	.18	.96	1	.83	.32
{ .0 ; .3 ; 150 ; 0.5 }	1	.97	1	.85	1	.84	1	.76	.28	.16	.28	.16	1	1	.99	.00
{ .0 ; .3 ; 150 ; 1.0 }	1	.97	1	.92	1	.91	1	.88	.21	.14	.21	.14	1	1	1	.00
{ .0 ; .3 ; 150 ; 2.0 }	1	.98	1	.96	1	.96	1	.93	.17	.11	.17	.11	1	1	1	.00
{ .0 ; .3 ; 300 ; 0.1 }	1	.98	1	.51	1	.51	.93	.37	.41	.19	.44	.19	.99	1	.90	.19
{ .0 ; .3 ; 300 ; 0.5 }	1	.97	1	.88	1	.88	1	.84	.24	.15	.24	.15	1	1	.99	.00
{ .0 ; .3 ; 300 ; 1.0 }	1	.98	1	.95	1	.94	1	.91	.18	.12	.18	.12	1	1	1	.00
{ .0 ; .3 ; 300 ; 2.0 }	1	.98	1	.97	1	.97	1	.95	.14	.12	.14	.12	1	1	1	.00
{ .0 ; .3 ; 450 ; 0.1 }	1	.98	1	.59	1	.58	.96	.42	.41	.20	.43	.20	.99	1	.94	.13
{ .0 ; .3 ; 450 ; 0.5 }	1	.98	1	.90	1	.90	1	.85	.22	.14	.22	.14	1	1	1	.00
{ .0 ; .3 ; 450 ; 1.0 }	1	.98	1	.95	1	.94	1	.93	.16	.12	.16	.12	1	1	1	.00
{ .0 ; .3 ; 450 ; 2.0 }	1	.99	1	.97	1	.97	1	.97	.13	.10	.13	.10	1	1	1	.00
{ .0 ; .5 ; 150 ; 0.1 }	1	.97	.98	.40	.98	.38	.83	.27	.38	.19	.41	.18	.96	1	.84	.31
{ .0 ; .5 ; 150 ; 0.5 }	1	.97	1	.84	1	.83	1	.75	.29	.16	.29	.16	1	1	.99	.00
{ .0 ; .5 ; 150 ; 1.0 }	1	.97	1	.92	1	.92	1	.87	.23	.15	.23	.15	1	1	1	.00
{ .0 ; .5 ; 150 ; 2.0 }	1	.98	1	.96	1	.96	1	.94	.17	.12	.17	.12	1	1	1	.00
{ .0 ; .5 ; 300 ; 0.1 }	1	.98	1	.52	1	.51	.92	.37	.39	.20	.42	.20	.99	1	.91	.19
{ .0 ; .5 ; 300 ; 0.5 }	1	.98	1	.89	1	.88	1	.83	.23	.15	.23	.14	1	1	1	.00
{ .0 ; .5 ; 300 ; 1.0 }	1	.98	1	.94	1	.94	1	.91	.19	.13	.19	.13	1	1	1	.00
{ .0 ; .5 ; 300 ; 2.0 }	1	.98	1	.97	1	.97	1	.95	.14	.11	.14	.11	1	1	1	.00
{ .0 ; .5 ; 450 ; 0.1 }	1	.98	1	.59	1	.58	.96	.43	.42	.21	.45	.21	.99	1	.93	.14
{ .0 ; .5 ; 450 ; 0.5 }	1	.98	1	.91	1	.90	1	.86	.22	.14	.22	.14	1	1	1	.00
{ .0 ; .5 ; 450 ; 1.0 }	1	.98	1	.95	1	.95	1	.92	.17	.13	.17	.13	1	1	1	.00
{ .0 ; .5 ; 450 ; 2.0 }	1	.99	1	.98	1	.98	1	.96	.13	.11	.13	.11	1	1	1	.00
{ .3 ; .3 ; 150 ; 0.1 }	1	.97	.99	.38	.98	.38	.82	.26	.37	.18	.41	.17	.96	1	.83	.32
{ .3 ; .3 ; 150 ; 0.5 }	1	.97	1	.84	1	.83	1	.75	.28	.17	.28	.17	1	1	.99	.00
{ .3 ; .3 ; 150 ; 1.0 }	1	.97	1	.91	1	.91	1	.88	.22	.15	.22	.15	1	1	1	.00
{ .3 ; .3 ; 150 ; 2.0 }	1	.97	1	.97	1	.96	1	.94	.17	.13	.17	.13	1	1	1	.00
{ .3 ; .3 ; 300 ; 0.1 }	1	.98	1	.50	1	.50	.93	.35	.42	.20	.46	.20	.99	1	.90	.20
{ .3 ; .3 ; 300 ; 0.5 }	1	.98	1	.88	1	.88	1	.82	.25	.16	.25	.16	1	1	1	.00
{ .3 ; .3 ; 300 ; 1.0 }	1	.98	1	.94	1	.94	1	.90	.18	.12	.18	.12	1	1	1	.00
{ .3 ; .3 ; 300 ; 2.0 }	1	.98	1	.97	1	.97	1	.95	.14	.11	.14	.11	1	1	1	.00
{ .3 ; .3 ; 450 ; 0.1 }	1	.98	1	.58	1	.55	.96	.43	.41	.20	.44	.20	.99	1	.93	.15
{ .3 ; .3 ; 450 ; 0.5 }	1	.99	1	.90	1	.90	1	.85	.22	.14	.22	.14	1	1	1	.00
{ .3 ; .3 ; 450 ; 1.0 }	1	.98	1	.95	1	.95	1	.93	.18	.12	.18	.12	1	1	1	.00
{ .3 ; .3 ; 450 ; 2.0 }	1	.98	1	.97	1	.97	1	.96	.14	.11	.14	.11	1	1	1	.00
{ .3 ; .5 ; 150 ; 0.1 }	1	.97	.99	.40	.98	.39	.84	.27	.37	.19	.41	.18	.96	1	.84	.30
{ .3 ; .5 ; 150 ; 0.5 }	1	.97	1	.85	1	.84	1	.75	.30	.18	.30	.18	1	1	.99	.01
{ .3 ; .5 ; 150 ; 1.0 }	1	.97	1	.92	1	.92	1	.87	.21	.13	.21	.13	1	1	1	.00
{ .3 ; .5 ; 150 ; 2.0 }	1	.98	1	.96	1	.96	1	.94	.17	.11	.17	.11	1	1	1	.00
{ .3 ; .5 ; 300 ; 0.1 }	1	.98	1	.51	1	.51	.93	.35	.41	.20	.45	.20	.99	1	.92	.19
{ .3 ; .5 ; 300 ; 0.5 }	1	.98	1	.88	1	.88	1	.84	.25	.16	.25	.16	1	1	1	.00
{ .3 ; .5 ; 300 ; 1.0 }	1	.98	1	.94	1	.94	1	.91	.18	.12	.18	.12	1	1	1	.00
{ .3 ; .5 ; 300 ; 2.0 }	1	.98	1	.97	1	.97	1	.96	.14	.11	.14	.11	1	1	1	.00
{ .3 ; .5 ; 450 ; 0.1 }	1	.98	1	.59	1	.58	.96	.43	.42	.21	.45	.20	.99	1	.94	.13
{ .3 ; .5 ; 450 ; 0.5 }	1	.98	1	.90	1	.90	1	.85	.21	.14	.21	.14	1	1	1	.00
{ .3 ; .5 ; 450 ; 1.0 }	1	.98	1	.96	1	.96	1	.93	.17	.12	.17	.12	1	1	1	.00
{ .3 ; .5 ; 450 ; 2.0 }	1	.99	1	.98	1	.97	1	.96	.13	.11	.13	.11	1	1	1	.00

Notes. See Table 1.

Table 14: Two factors DGP2. Testing results for $T = 5, S = 10$ and $\sigma^2_\varepsilon = 0.9$. For $\zeta_0 = 1$.

	L_0		L_1^{FE}		L_2^{FE}		L_1		L_2		$\hat{L}$					
$\{\rho;\sigma^2_\mu;N;\sigma_f\}$	J	t	J	t	J	t	J	t	J	t	J	t	W	h0	h1	$\#L=1$
{ .0 ; .3 ; 150 ; 0.1 }	1	.96	.94	.29	.92	.28	.69	.19	.30	.16	.36	.16	.91	1	.75	.49
{ .0 ; .3 ; 150 ; 0.5 }	1	.95	1	.79	1	.78	1	.68	.33	.19	.33	.19	1	1	.98	.01
{ .0 ; .3 ; 150 ; 1.0 }	1	.96	1	.90	1	.90	1	.84	.25	.15	.25	.15	1	1	.99	.00
{ .0 ; .3 ; 150 ; 2.0 }	1	.96	1	.95	1	.94	1	.91	.19	.14	.19	.14	1	1	1	.00
{ .0 ; .3 ; 300 ; 0.1 }	1	.97	.99	.40	.98	.40	.84	.27	.36	.18	.42	.17	.97	1	.84	.35
{ .0 ; .3 ; 300 ; 0.5 }	1	.96	1	.84	1	.84	1	.76	.28	.16	.28	.16	1	1	.99	.00
{ .0 ; .3 ; 300 ; 1.0 }	1	.97	1	.92	1	.92	1	.89	.21	.13	.21	.13	1	1	1	.00
{ .0 ; .3 ; 300 ; 2.0 }	1	.98	1	.96	1	.96	1	.94	.16	.11	.16	.11	1	1	1	.00
{ .0 ; .3 ; 450 ; 0.1 }	1	.98	1	.47	.99	.47	.91	.32	.42	.19	.47	.19	.98	1	.89	.26
{ .0 ; .3 ; 450 ; 0.5 }	1	.98	1	.87	1	.87	1	.82	.26	.16	.26	.16	1	1	.99	.00
{ .0 ; .3 ; 450 ; 1.0 }	1	.98	1	.94	1	.94	1	.91	.19	.13	.19	.13	1	1	1	.00
{ .0 ; .3 ; 450 ; 2.0 }	1	.98	1	.97	1	.96	1	.96	.15	.12	.15	.12	1	1	1	.00
{ .0 ; .5 ; 150 ; 0.1 }	1	.95	.94	.29	.93	.28	.70	.19	.31	.16	.38	.15	.91	.99	.75	.49
{ .0 ; .5 ; 150 ; 0.5 }	1	.96	1	.78	1	.78	1	.68	.33	.17	.33	.17	1	.99	.98	.01
{ .0 ; .5 ; 150 ; 1.0 }	1	.96	1	.90	1	.89	1	.83	.24	.15	.24	.15	1	1	.99	.00
{ .0 ; .5 ; 150 ; 2.0 }	1	.96	1	.95	1	.95	1	.92	.18	.13	.18	.13	1	1	1	.00
{ .0 ; .5 ; 300 ; 0.1 }	1	.97	.99	.40	.98	.40	.85	.27	.38	.17	.45	.17	.97	1	.85	.35
{ .0 ; .5 ; 300 ; 0.5 }	1	.97	1	.84	1	.85	1	.77	.28	.16	.28	.16	1	1	.99	.00
{ .0 ; .5 ; 300 ; 1.0 }	1	.97	1	.92	1	.92	1	.88	.20	.14	.20	.14	1	1	1	.00
{ .0 ; .5 ; 300 ; 2.0 }	1	.98	1	.96	1	.96	1	.93	.16	.12	.16	.12	1	1	1	.00
{ .0 ; .5 ; 450 ; 0.1 }	1	.98	1	.46	.99	.46	.91	.32	.40	.18	.46	.18	.98	1	.89	.26
{ .0 ; .5 ; 450 ; 0.5 }	1	.97	1	.88	1	.87	1	.80	.26	.16	.26	.16	1	1	.99	.00
{ .0 ; .5 ; 450 ; 1.0 }	1	.97	1	.94	1	.94	1	.90	.19	.12	.18	.12	1	1	1	.00
{ .0 ; .5 ; 450 ; 2.0 }	1	.98	1	.97	1	.97	1	.95	.15	.11	.15	.11	1	1	1	.00
{ .3 ; .3 ; 150 ; 0.1 }	1	.96	.94	.31	.93	.30	.70	.21	.31	.16	.36	.17	.92	1	.75	.47
{ .3 ; .3 ; 150 ; 0.5 }	1	.96	1	.79	1	.79	1	.70	.35	.19	.35	.19	1	1	.98	.01
{ .3 ; .3 ; 150 ; 1.0 }	1	.96	1	.90	1	.89	1	.84	.25	.15	.25	.15	1	1	1	.00
{ .3 ; .3 ; 150 ; 2.0 }	1	.97	1	.94	1	.94	1	.92	.19	.13	.19	.13	1	1	1	.00
{ .3 ; .3 ; 300 ; 0.1 }	1	.97	.99	.40	.98	.40	.85	.27	.37	.18	.44	.17	.97	1	.85	.33
{ .3 ; .3 ; 300 ; 0.5 }	1	.97	1	.85	1	.85	1	.79	.30	.16	.30	.16	1	1	.99	.01
{ .3 ; .3 ; 300 ; 1.0 }	1	.97	1	.92	1	.93	1	.88	.22	.15	.22	.15	1	1	1	.00
{ .3 ; .3 ; 300 ; 2.0 }	1	.98	1	.96	1	.96	1	.94	.15	.11	.15	.11	1	1	1	.00
{ .3 ; .3 ; 450 ; 0.1 }	1	.98	.99	.49	.99	.49	.91	.35	.41	.19	.46	.19	.99	1	.90	.26
{ .3 ; .3 ; 450 ; 0.5 }	1	.97	1	.87	1	.87	1	.81	.27	.16	.27	.15	1	1	1	.00
{ .3 ; .3 ; 450 ; 1.0 }	1	.98	1	.93	1	.93	1	.91	.19	.14	.19	.14	1	1	1	.00
{ .3 ; .3 ; 450 ; 2.0 }	1	.98	1	.97	1	.97	1	.95	.15	.13	.15	.13	1	1	1	.00
{ .3 ; .5 ; 150 ; 0.1 }	1	.96	.95	.30	.94	.30	.71	.20	.32	.17	.38	.16	.91	1	.75	.47
{ .3 ; .5 ; 150 ; 0.5 }	1	.96	1	.80	1	.79	1	.69	.33	.18	.33	.18	1	1	.98	.01
{ .3 ; .5 ; 150 ; 1.0 }	1	.96	1	.88	1	.89	1	.85	.25	.15	.25	.15	1	1	1	.00
{ .3 ; .5 ; 150 ; 2.0 }	1	.97	1	.93	1	.94	1	.91	.19	.13	.19	.13	1	1	1	.00
{ .3 ; .5 ; 300 ; 0.1 }	1	.97	.99	.42	.98	.41	.85	.27	.38	.19	.44	.19	.97	1	.85	.33
{ .3 ; .5 ; 300 ; 0.5 }	1	.97	1	.85	1	.85	1	.78	.27	.15	.27	.16	1	1	.99	.00
{ .3 ; .5 ; 300 ; 1.0 }	1	.97	1	.93	1	.92	1	.88	.21	.14	.21	.14	1	1	1	.00
{ .3 ; .5 ; 300 ; 2.0 }	1	.98	1	.96	1	.96	1	.93	.17	.12	.17	.12	1	1	1	.00
{ .3 ; .5 ; 450 ; 0.1 }	1	.98	1	.50	.99	.49	.90	.34	.40	.19	.45	.19	.99	1	.90	.26
{ .3 ; .5 ; 450 ; 0.5 }	1	.97	1	.88	1	.87	1	.81	.25	.16	.25	.16	1	1	1	.00
{ .3 ; .5 ; 450 ; 1.0 }	1	.97	1	.93	1	.94	1	.90	.20	.14	.20	.14	1	1	1	.00
{ .3 ; .5 ; 450 ; 2.0 }	1	.98	1	.97	1	.97	1	.95	.16	.12	.16	.12	1	1	1	.00

Notes. See Table 1.

3. The ENEMDU dataset

3.1. Computational Remarks

Unlike the Monte Carlo study where only one set of starting values based on FE estimator was used, in this section we use up to 100 random starting values that are uniformly distributed on $[-10; 10]$ for non-linear estimator, to make sure that the global minimum of the objective function is selected.

3.2. The Linear-log Specification

$$hours_{i,t} = \boldsymbol{\lambda}_s' \boldsymbol{f}_t + \gamma \log wage_{i,t} + \beta z_{i,t} + u_{i,t}, \quad i \in \mathcal{I}_{s,t}. \tag{3.1}$$

Variable	GMMl1	GMMl2	GMMn1	GMMn2
$\log wage$	-6.21***	-3.53***	-3.45**	-3.37**
# kids	0.18	-0.39	-0.41	-0.49
df	58	52	52	38
J	88.29***	57.67	56.25	43.45
BIC_1			**-188.07**	-135.10
BIC_2			**-85.12**	-59.86
Wald(FE)	33.68***			

Table 15: $T = 7$, $S = 10$. Results are based on 2-step estimates using the optimal weighting matrix in the second step. Based only on heads of the household. * indicates statistical significance at the 10% level, **- at the 5% level, and ***- at the 1% level $\gamma_0, \beta_0 = 0$. $J(GMMl0) = 580.90$. BIC_1 and BIC_2 use $N = \sum_{s=1}^{S} \sum_{t=1}^{T} N_{s,t}$ and $N = (1/ST) \sum_{s=1}^{S} \sum_{t=1}^{T} N_{s,t}$ respectively.

4. The FE estimator

4.1. Sufficient Conditions for the FE Estimator

For the model with cohort fixed effects we define

$$u_{i,t} \equiv \nu_{i,t} - \delta_s, \quad i \in \mathcal{I}_{s,t}.$$

The following high-level assumptions are sufficient to prove that the linear GMM estimator $\hat{\boldsymbol{\theta}}_{GMMl}$ is consistent and asymptotically normally distributed.

(FE.1) $N_{s,t} \to \infty, \quad \forall s,t; \exists N \to \infty$ s.t. $N_{s,t}/N \to \pi_{s,t}$ and $0 < \min \pi_{s,t} < \max \pi_{s,t} < \infty$. T and S are fixed (Type I asymptotics).

(FE.2) $u_{i,t}$ are i.h.d. with finite $2+\delta$ moment, for $\delta > 0$, such that $\sqrt{N_{s,t}}\bar{u}_{s,t} \xrightarrow{d} \mathcal{N}(0, \sigma^2_{s,t})$ jointly $\forall s,t$ with $0 < \min \sigma^2_{s,t} \leq \max \sigma^2_{s,t} < \infty$.

(FE.3) There exists a unique true value $\boldsymbol{\theta}_0$.

(FE.4) $\text{rk}(\boldsymbol{M}\boldsymbol{X}_\infty) = K$. The $\boldsymbol{X}_\infty \equiv \text{plim}_{N\to\infty} \boldsymbol{X}$ matrix is deterministic.

(FE.5) $S(T-1) > K$.

Under these assumptions the asymptotically efficient weighting matrix can be consistently estimated by

$$\widehat{\boldsymbol{\Omega}}_{opt} = \left(\boldsymbol{M}\hat{\boldsymbol{\Sigma}}\boldsymbol{M}\right)^+.$$

Here we use the Moore-Penrose pseudoinverse of $\left(\boldsymbol{M}\hat{\boldsymbol{\Sigma}}\boldsymbol{M}\right)$ because the rank of this matrix is given by $\text{rk}\,(\boldsymbol{M}) = (T-1)S$. The typical $(s-1)T+t$ diagonal element of the $\hat{\boldsymbol{\Sigma}}$ matrix is equal to $\hat{q}^2_{s,t} = (N/N_{s,t})\hat{\sigma}^2_{s,t}$ where

$$\hat{\sigma}^2_{s,t} = \frac{1}{N_{s,t}} \sum_{i \in \mathcal{I}_{s,t}} (y_{i,t} - \boldsymbol{x}'_{i,s,t}\hat{\boldsymbol{\theta}}_1)^2 - \left(\frac{1}{N_{s,t}} \sum_{i \in \mathcal{I}_{s,t}} (y_{i,t} - \boldsymbol{x}'_{i,s,t}\hat{\boldsymbol{\theta}}_1)\right)^2, \tag{4.1}$$

which is evaluated at some consistent initial estimator $\hat{\boldsymbol{\theta}}_1$ (e.g. the estimator that replaces $\boldsymbol{\Omega}$ by an identity matrix). Therefore (under Type I asymptotics) if $\hat{\boldsymbol{\theta}}_1 \xrightarrow{p} \boldsymbol{\theta}_0$, one has

$\hat{q}^2_{s,t} \xrightarrow{p} q^2_{s,t} \equiv \sigma^2_{s,t}/\pi_{s,t}$, where $N_{s,t}/N \to \pi_{s,t}$.

4.2. The Hausman Test for Fixed Effects

The Hausman test can be used to test for the presence of unobserved factors (also suggested by Bai (2009) in the context of genuine panels with large N, T). In particular, if $L = L_0 = 1$ then

$$H = N\left(\Delta\hat{\boldsymbol{\theta}}\right)'\left(\widehat{\operatorname{Avar}\hat{\boldsymbol{\theta}}_{GMMn}} - \widehat{\operatorname{Avar}\hat{\boldsymbol{\theta}}_{GMMl}}\right)^{-1}\Delta\hat{\boldsymbol{\theta}} \xrightarrow{d} \chi^2(K) \tag{4.2}$$

holds under the null hypothesis of the fixed effects model being correct. Here $\Delta\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}_{GMMn} - \hat{\boldsymbol{\theta}}_{GMMl}$, while to estimate variance-covariance matrices of estimators we use

$$\widehat{\operatorname{Avar}\hat{\boldsymbol{\phi}}_{GMMn}} = \left(\boldsymbol{D}_{\boldsymbol{\phi}}(\hat{\gamma})'\hat{\boldsymbol{\Omega}}^{1/2}\boldsymbol{M}_{\hat{\boldsymbol{\Omega}}^{1/2}\boldsymbol{D}_{\boldsymbol{\theta}}(\hat{\gamma})}\hat{\boldsymbol{\Omega}}^{1/2}\boldsymbol{D}_{\boldsymbol{\phi}}(\hat{\gamma})\right)^{-1}, \tag{4.3}$$

with $\hat{\boldsymbol{\Omega}} = \left(\boldsymbol{M}(\hat{\boldsymbol{\phi}}_1)\hat{\boldsymbol{\Sigma}}(\hat{\boldsymbol{\theta}}_1)\boldsymbol{M}(\hat{\boldsymbol{\phi}}_1)'\right)^{-1}$ evaluated at a consistent first-step estimator $\hat{\boldsymbol{\gamma}}_1 = (\hat{\boldsymbol{\theta}}_1', \hat{\boldsymbol{\phi}}_1')'$ (under the alternative hypothesis). For the fixed effects estimator the variance-covariance matrix is analogously given by

$$\widehat{\operatorname{Avar}\hat{\boldsymbol{\theta}}_{GMMl}} = \left(\boldsymbol{X}'\boldsymbol{M}\left(\boldsymbol{M}\hat{\boldsymbol{\Sigma}}(\hat{\boldsymbol{\theta}}_{GMMl,1})\boldsymbol{M}\right)^{+}\boldsymbol{M}\boldsymbol{X}\right)^{-1}, \tag{4.4}$$

for $\boldsymbol{M} = \boldsymbol{I}_S \otimes (\boldsymbol{I}_T - (1/T)\boldsymbol{\imath}_T\boldsymbol{\imath}_T')$. Here $\hat{\boldsymbol{\theta}}_{GMMl,1}$ is a first-step consistent estimator (under the null hypothesis). In both cases $\hat{\boldsymbol{\Sigma}}(\cdot)$ can be estimated using the formula in (4.1).

5. Derivations of Differentials

If we denote transformed equations by $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{\theta}, \boldsymbol{\phi}) = \boldsymbol{M}(\boldsymbol{\phi})(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\theta})$, the objective function can be formulated in the following way

$$f(\boldsymbol{\theta}, \boldsymbol{\phi}) = \frac{1}{2}\boldsymbol{u}'\boldsymbol{\Omega}\boldsymbol{u}.$$

Using the product rule for differentials

$$\mathrm{d}f(\boldsymbol{\theta}, \boldsymbol{\phi}) = \boldsymbol{u}'\boldsymbol{\Omega}\,\mathrm{d}\boldsymbol{u}. \tag{5.1}$$

Here the first differential of the $\boldsymbol{u}$ term can be compactly written

$$\begin{aligned}
\mathrm{d}\boldsymbol{u} &= (\mathrm{d}\boldsymbol{M}(\boldsymbol{\phi}))(\boldsymbol{y}-\boldsymbol{X\theta}) - \boldsymbol{M}(\boldsymbol{\phi})\boldsymbol{X}\,\mathrm{d}\boldsymbol{\theta} \\
&= (\boldsymbol{I}_S \otimes (\mathbf{O}_{T-L}, \mathrm{d}\boldsymbol{\Phi}))(\boldsymbol{y}-\boldsymbol{X\theta}) - \boldsymbol{M}(\boldsymbol{\phi})\boldsymbol{X}\,\mathrm{d}\boldsymbol{\theta} \\
&= \left((\boldsymbol{y}-\boldsymbol{X\theta})' \otimes \boldsymbol{I}_{S(T-L)}\right) \operatorname{vec}(\boldsymbol{I}_S \otimes (\mathbf{O}_{T-L}, \mathrm{d}\boldsymbol{\Phi})) - \boldsymbol{M}(\boldsymbol{\phi})\boldsymbol{X}\,\mathrm{d}\boldsymbol{\theta} \\
&= \left((\boldsymbol{y}-\boldsymbol{X\theta})' \otimes \boldsymbol{I}_{S(T-L)}\right) \boldsymbol{Q}\,\mathrm{d}\boldsymbol{\phi} - \boldsymbol{M}(\boldsymbol{\phi})\boldsymbol{X}\,\mathrm{d}\boldsymbol{\theta} \\
&= \boldsymbol{D}_{\boldsymbol{\phi}}\,\mathrm{d}\boldsymbol{\phi} + \boldsymbol{D}_{\boldsymbol{\theta}}\,\mathrm{d}\boldsymbol{\theta}.
\end{aligned}$$

Here the selection matrix $\boldsymbol{Q}$ is of the following form

$$\boldsymbol{Q} = (((\boldsymbol{I}_S \otimes \mathbf{K}_{TS})(\operatorname{vec}\boldsymbol{I}_S \otimes \boldsymbol{I}_T)) \otimes \boldsymbol{I}_{T-L}) \begin{pmatrix} \mathbf{O}_{[(T-L)^2 \times (T-L)L]} \\ \boldsymbol{I}_{(T-L)L} \end{pmatrix}.$$

For more detailed derivations, see Magnus and Neudecker (2007, p. 56). The second differential of the objective function $f(\boldsymbol{\theta}, \boldsymbol{\phi})$ is

$$\mathrm{d}^2 f(\boldsymbol{\theta}, \boldsymbol{\phi}) = (\mathrm{d}\boldsymbol{u})'\boldsymbol{\Omega}\,\mathrm{d}\boldsymbol{u} + \boldsymbol{u}'\boldsymbol{\Omega}\,\mathrm{d}^2\boldsymbol{u}. \tag{5.2}$$

Note that the second term is asymptotically negligible ($\mathrm{o}_P(1)$) if evaluated at any consistent estimator $\hat{\boldsymbol{\gamma}}$. Hence

$$\mathrm{d}^2 f(\boldsymbol{\theta}, \boldsymbol{\phi}) = (\mathrm{d}\boldsymbol{u})'\boldsymbol{\Omega}\,\mathrm{d}\boldsymbol{u} + \mathrm{o}_P(1). \tag{5.3}$$

Plugging in the value of $\mathrm{d}\boldsymbol{u}$ (ignoring the $\mathrm{o}_P(1)$ term),

$$\begin{aligned}
\mathrm{d}^2 f(\boldsymbol{\theta}, \boldsymbol{\phi}) &= (\mathrm{d}\boldsymbol{u})'\boldsymbol{\Omega}\,\mathrm{d}\boldsymbol{u} \\
&= (\mathrm{d}\boldsymbol{\phi})'\boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})(\mathrm{d}\boldsymbol{\phi}) + (\mathrm{d}\boldsymbol{\theta})'\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})(\mathrm{d}\boldsymbol{\theta}) \\
&\quad + 2(\mathrm{d}\boldsymbol{\phi})'\boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})(\mathrm{d}\boldsymbol{\theta}).
\end{aligned}$$

Combining all results we obtain formulas for the score ($\nabla(\boldsymbol{\gamma})$) and the Hessian ($\boldsymbol{\mathcal{H}}(\boldsymbol{\gamma})$):

$$\nabla(\boldsymbol{\gamma}) = \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})' \\ \boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})' \end{pmatrix} \boldsymbol{\Omega u},$$

$$\boldsymbol{\mathcal{H}}(\boldsymbol{\gamma}) = \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma}) & \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma}) \\ \boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma}) & \boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})' \\ \boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})' \end{pmatrix} \boldsymbol{\Omega} \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{\gamma})' \\ \boldsymbol{D}_{\boldsymbol{\phi}}(\boldsymbol{\gamma})' \end{pmatrix}'.$$

Under Assumptions **(A.1)-(A.6)** the asymptotic distribution is

$$\sqrt{N}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0) \xrightarrow{d} \operatorname*{plim}_{N\to\infty} \left(\begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\theta}} & \boldsymbol{D}_{\boldsymbol{\theta}}'\boldsymbol{\Omega}\boldsymbol{D}_{\phi} \\ \boldsymbol{D}_{\boldsymbol{\phi}}'\boldsymbol{\Omega}\boldsymbol{D}_{\boldsymbol{\theta}} & \boldsymbol{D}_{\boldsymbol{\phi}}'\boldsymbol{\Omega}\boldsymbol{D}_{\phi} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}' \\ -\boldsymbol{D}_{\boldsymbol{\phi}}' \end{pmatrix} \right) \boldsymbol{\Omega M}(\boldsymbol{\phi}_0)\boldsymbol{\Sigma}^{1/2}\boldsymbol{\xi}.$$

Furthermore, similarly to any standard GMM problem the (conditional) variance of $\sqrt{N}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0)$ is minimized at

$$\boldsymbol{\Omega}_{opt} = \left(\boldsymbol{M}(\boldsymbol{\phi}_0)\boldsymbol{\Sigma}\boldsymbol{M}(\boldsymbol{\phi}_0)'\right)^{-1}.$$

Therefore, the asymptotic variance with $\boldsymbol{\Omega}_{opt}$ as a weighting matrix equals

$$\operatorname{Avar}\hat{\boldsymbol{\gamma}} = \left[\operatorname*{plim}_{N\to\infty} \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}' \\ \boldsymbol{D}_{\boldsymbol{\phi}}' \end{pmatrix} \left(\boldsymbol{M}(\boldsymbol{\phi}_0)\boldsymbol{\Sigma}\boldsymbol{M}(\boldsymbol{\phi}_0)'\right)^{-1} \begin{pmatrix} \boldsymbol{D}_{\boldsymbol{\theta}}' \\ \boldsymbol{D}_{\boldsymbol{\phi}}' \end{pmatrix}' \right]^{-1}.$$

Finally, the same result holds if evaluated at any weighting matrix $\hat{\boldsymbol{\Omega}}$ such that $\hat{\boldsymbol{\Omega}} \xrightarrow{p} \boldsymbol{\Omega}_{opt}$.

6. References

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