Inference in EL++

1 Coverage

Logically, coverage is restricted to $\mathcal{EL}$++: that is, to atomic classes including top and bottom, nominals (enumerated classes containing one individual), conjunctions (intersections of two classes), and existential restrictions over classes and literals.[1]

Within $\mathcal{EL}$++ we allow intersections only when they have two arguments of which the first is either an atomic class or a simple restriction—that is, a restriction over a value that is either an atomic class, a nominal, or a literal. This constraint is imposed in OSE to avoid statements that are structurally ambiguous when verbalised; studies of ontology corpora have shown that these are very rare in practice.

2 Inference model

My aim is to derive all subsumption relationships among the classes that occur in the axioms of an ontology, at any stage of development. In the paper cited above, Baader et al. describe a tractable algorithm for deciding subsumption relationships in $\mathcal{EL}$++; by 'tractable' they mean that a possible relationship can be decided in a time that is polynomial on the number of classes in the ontology, once it has been normalised. To put an ontology in normal form, you define further atomic classes so that each subsumption can be stated in one of the following s ways, where C, D, E are atomic classes, and L is a literal:

$$C \sqsubseteq D \quad C \sqsubseteq \exists P.D \quad C \sqsubseteq \exists P.L \quad C \sqcap D \sqsubseteq E \quad \exists P.C \sqsubseteq D \quad \exists P.L \sqsubseteq C$$

Subsumption relationships are represented by mapping each class to a set of superclasses; these are computed by applying a set of 'completion rules'.

I have implemented a method that I think is essentially the same, as follows.

- Refactor all axioms of the form $C \sqsubseteq D \sqcap E$ to two axioms $C \sqsubseteq D$ and $C \sqsubseteq E$.
- Express all disjointness axioms as subsumption statements of the form $C \sqcap D \sqsubseteq \bot$.

[1] Franz Baader, Sebastian Brand and Carsten Lutz *Pushing the EL envelope* IJCAI 2005.

- Progressively replace all constructed classes $C \sqcap D$ by atomic classes E such that $E \equiv C \sqcap D$ (where C and D are atomic). Do the same for constructed classes of the form $\exists P.C$ or $\exists P.L$ or $\exists P.\{I\}$ (enumerated classes corresponding to individuals).
- In this way represent the ontology by the following: a set $\mathbb{C}$ of atomic classes; a set $\mathbb{D}$ of definitions; and a set $\mathbb{S}$ of subsumption statements, each applying to two classes from $\mathbb{C}$. Here $\mathbb{C}$ will contain all atomic classes both original and defined, and also the classes $\top$ and $\bot$. At first, $\mathbb{S}$ will contain all axioms from the ontology, and all statements of the form $C \sqsubseteq C$, $C \sqsubseteq \top$, and $\bot \sqsubseteq C$, for any C in $\mathbb{C}$.
- Add further subsumption statements (i.e., entailments) to $\mathbb{S}$ by applying completion rules; continue doing this until none of the rules applies.

The completion rules are as follows:

1. **Subclass transitivity**: Add $C \sqsubseteq D$ to $\mathbb{S}$ if you can find E in $\mathbb{C}$ such that $C \sqsubseteq E$ and $E \sqsubseteq D$ are already in $\mathbb{S}$.
2. **Restriction rule**: Add $C \sqsubseteq D$ to $\mathbb{S}$ if you can find definitions $C \equiv \exists P.A$ and $D \equiv \exists P.B$ (for some A, B) in $\mathbb{D}$, and $A \sqsubseteq B$ in $\mathbb{S}$.
3. **Laconic rule**: Add $C \sqsubseteq D$ to $\mathbb{S}$ if you can find in $\mathbb{D}$ a definition of the form $C \equiv D \sqcap E$ or $C \equiv E \sqcap D$ (for some E).
4. **Conjunction rule**: Add $C \sqsubseteq D$ to $\mathbb{S}$ if you can find a definition $D \equiv A \sqcap B$ (for some A, B) in $\mathbb{D}$, and statements $C \sqsubseteq A$ and $C \sqsubseteq B$ in $\mathbb{S}$.
5. **Existential violation**: Add $C \sqsubseteq \bot$ to $\mathbb{S}$ if you can find definition $D \equiv \exists P.A$ in $\mathbb{D}$, and statements $C \sqsubseteq D$ and $A \sqsubseteq \bot$ in $\mathbb{S}$.

Once this is done, you can derive the following information from $\mathbb{S}$:

- If $\mathbb{S}$ contains a statement of the form $C \sqsubseteq \bot$ where C is a nominal (i.e., it represents an individual), the ontology is inconsistent.
- If $\mathbb{S}$ contains a statement of the form $C \sqsubseteq \bot$ where C is an atomic or constructed class, but not a nominal, the class C is unsatisfiable.
- If If $\mathbb{S}$ contains two statements of the form $C \sqsubseteq D$ and $D \sqsubseteq C$, the classes C and D are equivalent.
- If $\mathbb{S}$ contains a statement that is not an axiom of the ontology, it can be reported as an entailment.