An Improved Simulated Annealing Algorithm for Solving Complex Water Distribution Networks

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ABSTRACT

Optimising the design of water distribution networks (WDNs) is a well-known problem that has been studied by numerous researchers. This work proposes a heuristic based on simulated annealing and improved by using concepts from the cross-entropy method. The proposed optimization approach is presented and used in two case studies of different complexity. The results show not only a fall in the computational effort of the new approach relative to simulated annealing but also include a comparison with other heuristic results from the literature, used to solve the same problems.

Keywords: water distribution networks, simulated annealing, cross-entropy

1 BACKGROUND

The literature on the optimal design of water distribution networks (WDNs) contains numerous works that propose heuristic methods of which the following are the most popular: genetic algorithms [1], tabu search [2], shuffled frog-leaping algorithm [3], ant colony optimisation [4], simulated annealing [5] and cross-entropy [6]. All are used to find global or near global optimal solutions to WDN problems. Furthermore, with the exponential improvement in the performance of supercomputers, which nowadays can have thousands or even millions of processor cores, it is possible to perform the complete enumeration of all possible solutions to identify the global optimum of benchmark problems or the true Pareto front of the problem (Wang et al. [7]). As stated in the position paper by Maier et al. [8], metaheuristics methods have been successfully used in a range of problems and a variety of situations, but many of the case studies have been academic and fail when it comes to the complexities and uncertainties of real case studies. To ensure that the success of these metaheuristics can be replicated when applied to real problems, Maier et al. [8] showed that these algorithms should be able to find near-optimal solutions in reasonable time. Therefore, for complex real-size networks there is still a long way to go in devising optimization methods able to find optimal or near-optimal solutions within reasonable computing time.

The objective of this work is to improve an already well-established metaheuristic method, simulated annealing (SA), to reduce the computational effort required to achieve near-optimal solutions for complex real size networks. Simulated annealing was proposed by [9] and was used in the optimization of water distribution networks by [5]. The main advantages of SA are that it can deal with combinatorial problems with many constraints, escape from local optima during the search and has theoretical proof of convergence to the global optimum. However, there is a clear trade-off between the quality of results and the time required to achieve the solutions. In this work we propose using cross-entropy method (CE) features to speed up the SA while maintaining its strengths [10].

2 METHODOLOGY

2.1 Optimization model

This work treats the problem of minimizing the design cost of WDNs where pipe diameters are the decision variables, and are selected from a set of possible commercial diameters. The optimization model is structured by the minimum cost objective of (1) and the constraints are represented by (2) to verify minimum heads at nodes, (3) to use a set of commercial diameters and (4) the assignment of one commercial diameter per pipe.

$$Ct = \min \sum_{i=1}^{NPI} \left(Cp_i(Dc_i) \times L_i \right) \tag{1}$$

$$H_n \geq H_{min} \qquad \forall n \in NN \tag{2}$$

$$Dc_i = \sum_{d=1}^{ND} YD_{d,i} . Dcom_{d,i} \quad \forall i \in NPI \tag{3}$$

$$\sum_{d=1}^{ND} YD_{d,i} = 1 \quad \forall i \in NPI \tag{4}$$

Where,

Ct – total investment cost (*USD*)

NPI – number of pipes in the network

$Cp_i(Dc_i)$ – unit cost of pipe *i* as function of the commercial diameter Dc_i adopted (*USD/m*)

Dc_i – commercial diameter of pipe i (*mm*)

L_i – length of pipe *i* (*m*)

H_n – head at node *n* (*m*)

$Hmin$ – minimum head (*m*)

NN – number of nodes

ND – number of commercial diameters

$YD_{d,i}$ – binary variable to represent the use of diameter *d* in pipe *i*

$Dcom_{d,i}$ – commercial diameter *d* assigned to pipe *i*

EPANET (Rossman, [11]) hydraulic simulator is used to solve the mass and energy conservation laws and to check the hydraulic feasibility of WDN solution,. This is the most frequently used hydraulic solver in the literature.

2.2 Simulated annealing

Simulated annealing is a stochastic technique proposed by [9] based on the analogy between the way a metal cools to turn into a crystalline structure with minimum energy state and the search for a minimum objective function solution in an optimization problem. The process considers random moves that generate candidate solutions from the neighborhood of current solutions [5]. These random moves are always accepted if the candidate solution has a better value for the objective function than current solution. However, worse candidate solutions can also be accepted with a certain probability, computed by the Metropolis criterion [12], that is depending on a temperature parameter of the method and on the degree of quality decrease (in terms of the objective function value) of the candidate solution relative to the current solution. The temperature parameter is set to a high value at the start of the process and decreases during the optimization, which reduces the probability of accepting candidate solutions worse than current solutions. The process ends when temperatures are sufficiently low. SA was used not only in the optimization of WDNs of a single objective but also in multiobjective problems [13]. As stated before, in SA there is a clear trade-off between the quality of results and the time required for convergence, and therefore a well established balance is needed in the SA search behavior between the exploitation of the domain of the problem and the intensification in certain parts of the solution space to try to find the global optimal solution. To overcome these difficulties, we propose making use of cross-entropy concepts to speed up SA by driving the search to the most promising regions of the solution space. The main purpose is to accelerate the convergence of the optimization process so that high quality solutions can be obtained for fewer iterations than are needed in the original SA process.

2.3 Cross-entropy

Cross entropy is an adaptive method originating in an algorithm of rare-event simulation based on variance minimization. It was developed by Rubinstein [14] and applied by [6] to the optimization of WDN. This method makes use of a probability matrix that stores the probabilities of choosing diameter sizes for each pipe in the network. These probabilities are used to generate new solutions during the optimization process. In this method the best solutions from a set of solutions stored in a solution list are used to update the probability matrix. The process ends when in subsequent iterations the probability matrix remains unchanged and converges to a degenerated case, this means that all probabilities of choosing a diameter are close to zero except one diameter for each network pipe that has a probability close to one. These diameter sizes with probabilities close to one provide the final optimal solution from the CE method. A deeper study of this method applied to WDNs can be found in [6].

2.4 The SACE method

This work proposes an optimization method based on simulated annealing improved by using information provided by cross-entropy in a new optimization tool that we called SACE, simulated annealing and cross-entropy method. The flowchart of the algorithm shown on Fig. 1 better explains the steps of this method. In this figure, the original SA structure steps are green and the changes implemented on the original SA by cross-entropy concepts are orange. The SACE method starts by inputting the WDN data and the available commercial diameters for the network design, then a set of solutions is randomly generated so that an initial probability matrix of the CE method can be computed. This is done by generating and saving a set solutions in a solution list. The list is then used to compute the probability matrix by using the best solutions from this list and checking how many times the diameter sizes occur in the set of the best solutions.

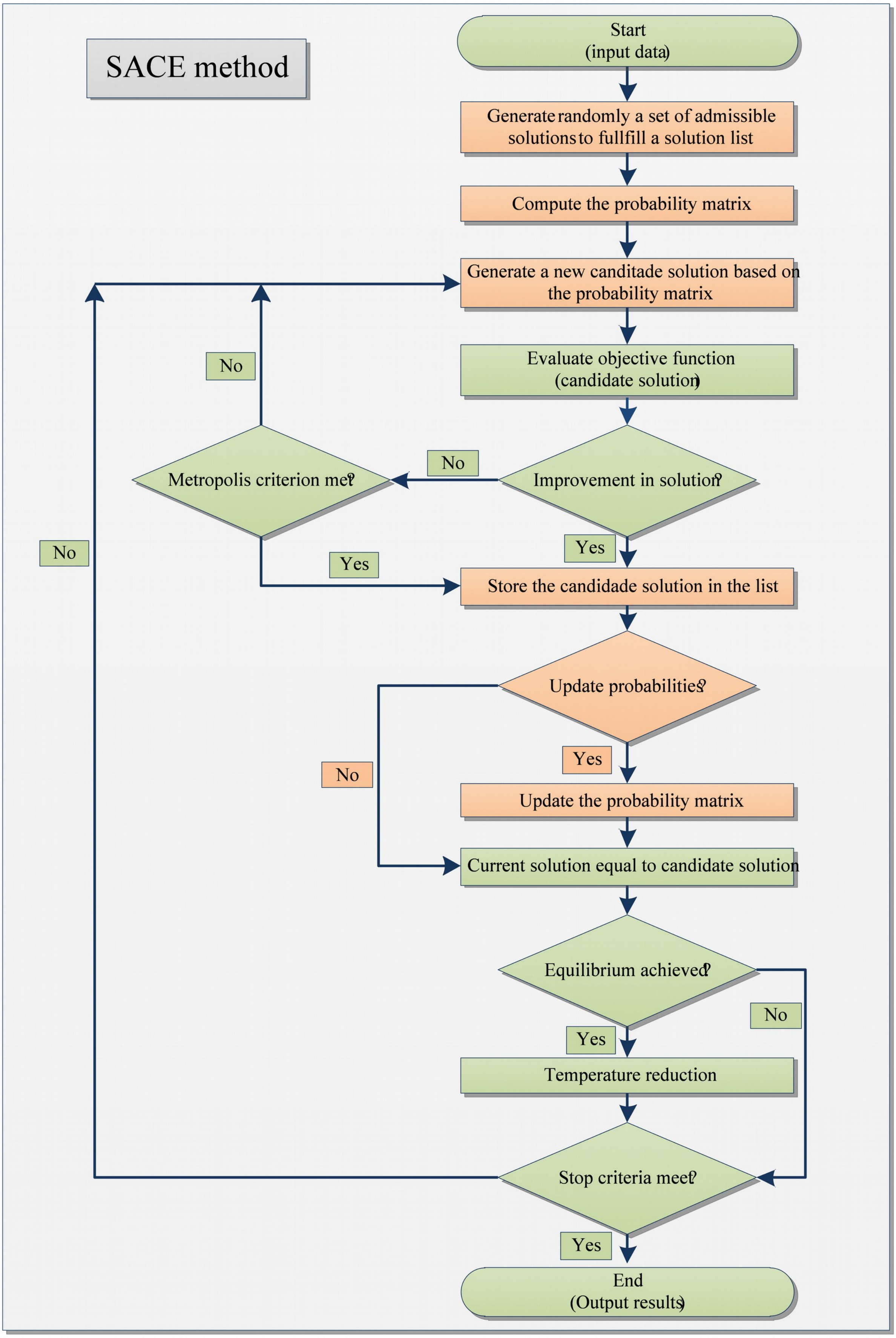

Figure 1. General structure of SACE method (adapted from [15]*)*

After computing the probability matrix (Fig. 1), the process continues by generating a new admissible candidate solution in the neighborhood of the current solution taking into account the information from this probability matrix. Then the objective function is evaluated and the candidate solution can be accepted or rejected by the Metropolis criterion. If it is accepted, this solution will be used as the starting point (current solution) for the next iteration and the solution is stored in the list. When a set of solutions are saved in the list, the probability matrix is updated. If it is rejected, the original current solution will be used. After a number of iterations are performed to reach equilibrium at a temperature level, the cooling process is performed by decreasing the temperature parameter. The process continues until a stop criterion is achieved. The results are presented at the end.

3 RESULTS

3.1 Case studies

The SACE is applied to two benchmark least-cost optimization problems used by numerous researchers in literature so that it could be possible to compare the results obtained with this method. A simple network, Hanoi, proposed by [16] is case study 1 (CS1) and a larger real life network studied by [17] is case study 2 (CS2). The Hanoi network is a gravity feed network with 1 fixed-level reservoir consisting of 34 pipes and 31 demand nodes. For the design of this network there are 6 commercial pipe diameters available, from 304.8 to 1 016 mm. The second case study is the real Balerma network with 4 fixed-head reservoirs, 454 pipes and 443 demand nodes and 10 commercial diameters from 113 to 581.8 mm.

3.2 Results and comparisons

The performance of the developed SACE method is compared with that of the SA in Table 1. The values obtained with the SACE and SA methods are determined for 25 runs for different sets of random numbers for the CS1 and CS2 problems.

Table 1. Comparisons of results for CS1 and CS2 obtained by SACE and SA optimization methods

Case study	Optimization method	Minimum ($\times10^6$)	Maximum ($\times10^6$)	Average ($\times10^6$)	Average number of iterations	Average time (seconds)
CS1	SACE	6.08	6.33	6.19	31 921	11
	SA	6.08	6.33	6.12	134 154	28
CS2	SACE	1.93	2.08	2.01	1 216 635	3 153
	SA	1.93	2.06	2.02	4 966 904	8 577

The results are compared in terms the minimum, maximum and average cost function values for each method. The last two columns of Table 1 show the average number of iterations and the running time. Both methods were run with the same input data, using the same hydraulic simulator and the same computer (Intel Core i5 2.5GHz), and so it is possible to compare the running times. The results show that for CS1 these methods arrive at the same minimum solution of 6.08×10^6 (USD). However, the SACE method reaches this solution cost for a smaller average number of iterations (31 921) compared with SA (134 154) and in the shorter running time of 11s, as opposed to the 28s taken by SA. In the literature, this minimum cost solution is also achieved by [6], who used a cross-entropy optimization method. It those authors take 97 000 iterations to reach the minimum cost solution. The work of [17], too, found the same optimal solution but it took 150 000 iterations to do so. Relative to the maximum and average solution cost found over 25 different runs, maximum is the same and the average costs of SACE are slightly worse (around 1%) than the SA method.

For the real network case study, CS2, the results from table 1 show that SACE reaches the minimum of 1.93×10^6 (€) as SA. It does so using small average number of evaluations (1 216 635) compared with SA (4 966 904), and takes less average computing time (3 153s) than the computing time (8 577s) taken by SA. Regarding the maximum and average solution cost found over 25 different runs, the maximum value of SACE is slightly worse (less than 1%) and average cost is almost equal compared with the SA results. In the literature, the GENOME method proposed by [17] achieves the best solution after 10×10^6 iterations and the NSGA proposed by [18] does so after 10.64×10^6 iterations.

In fact, the main aim of this study was to reduce the computation effort of the SA method while maintaining its level of reliability in finding optimal solutions. To compare the convergence speed of SACE and SA, we present in Fig. 2 the objective functions' values obtained during the search process.

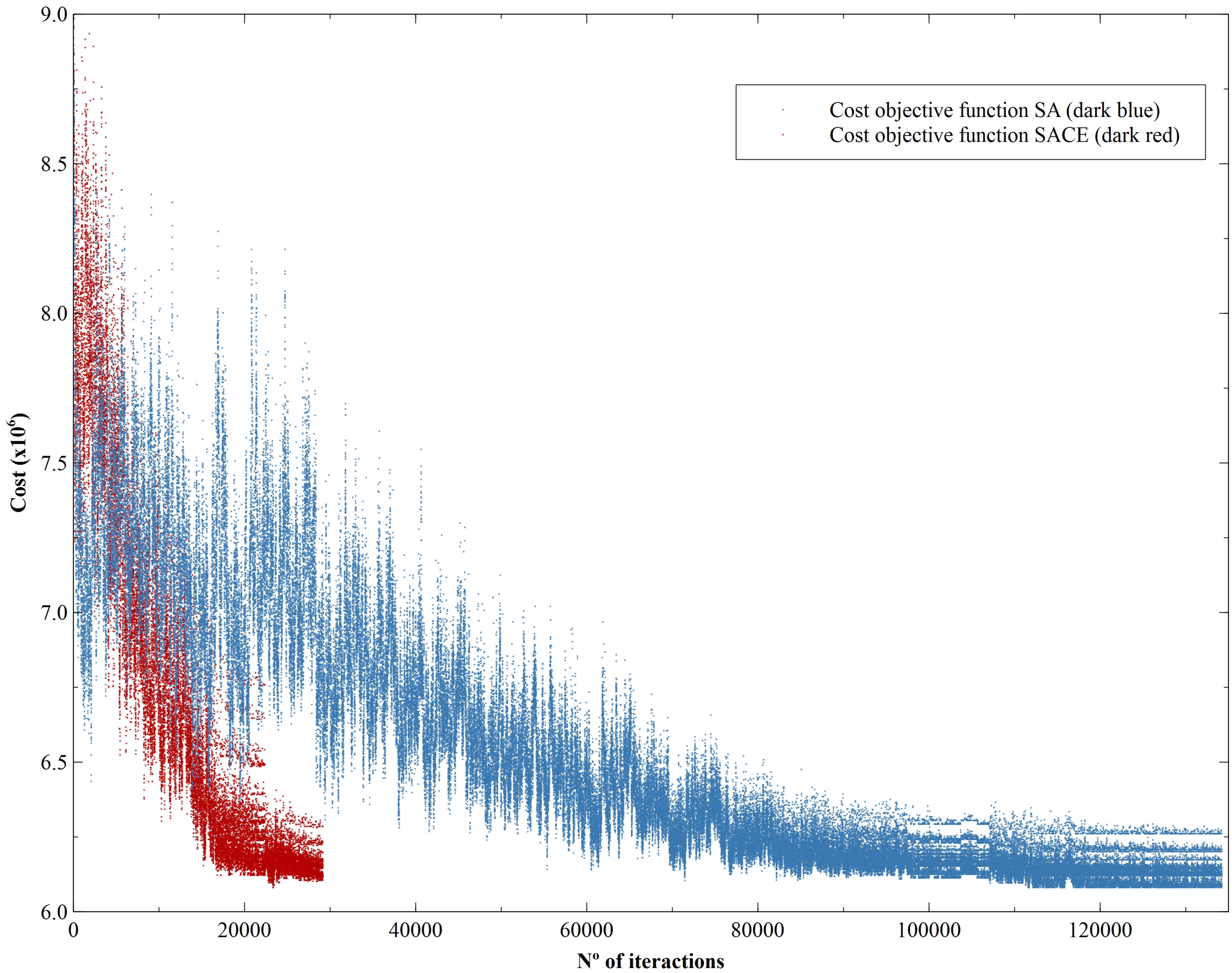

Figure 2. Comparison of the convergence of SA and SACE for CS1 (Hanoi network)

From Fig.2 it is possible to conclude that SACE has a faster convergence than SA. This is because it stores the information to guide the search to the most promising areas of the solution space. In fact, the SACE method arrives at solutions of almost 6.1×10^{6} (USD) in fewer than 20 000 iterations whereas SA needs more than 100 000 iterations to reach this solution cost level.

4 CONCLUSIONS

This work contributes to the optimal design of water distribution networks by proposing a new optimization method, SACE, which is based on simulated annealing and has been improved with concepts from cross-entropy. The simulated annealing technique is able to escape from local optimums and has theoretical proof of convergence, but there is a clear trade-off between the quality of the final solutions and the computation effort. Therefore, our work proposes using cross-entropy by storing solutions during the annealing search so that probabilities can be computed and used to guide the search to the most promising areas of the solution space of the WDN design problem. This is very important, because for very complex WDNs and for complex models dealing with uncertainty, multiple scenarios might need to be evaluated and the computing time could be prohibitive, thus it can be very useful to speed up the convergence to the optimal solutions. The SACE method was applied to two case studies and the results show that it was possible to achieve to the same optimal solution, and with the practically same level of reliability as offered by the SA method, but using fewer iterations and taking less computing time than SA. The comparison of the results with those reported in other works in the literature also demonstrates the good performance of the SACE method. In future analyses, the SACE method will be applied to solve more complex optimization WDN models that take into account future uncertainty, multiple scenarios and phased approaches.

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