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On coupling of vertical and longitudinal dynamics of unsuspended bicycles

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1 INTRODUCTION

Even though the majority of fatal bicycle accidents involves cars, still a relevant amount of fatalities can be traced back to single sided accidents [1]. This highlights the fact that the limits of vehicle dynamics are a serious threat for bicycle riders, which requires in-depth analysis. Especially braking dynamics are an important field of interest, since it is closely related to safety. Braking performance is in general defined by tire-road friction forces, which directly depend on wheel load. Most bicycles that are used as mean of transportation do not have any rear suspension and in many cases neither a suspension fork. In this context it is interesting to investigate the coupling of vertical dynamics and longitudinal dynamics in order to reveal its impact on cycling safety. Previous research was either focused on a separate treatment of braking [2][3] and suspension [4][5] of bicycles.

In this work a model of vertical dynamics is derived and compared to measurements recorded on cobblestone via an instrumented bicycle. In the next step the model is extended to cover also longitudinal dynamics and again validated. In conclusion it is shown how road induced oscillations affect the road-tire friction condition during braking.

2 VERTICAL DYNAMICS

In the automotive field it is common to use a quarter-car model as in Figure 1 to describe vertical dynamics. The tire is represented by a spring stiffness k_t between road and unsprung mass m. The vehicle suspension is modeled by the spring and damper constants k and c between unsprung and body mass M.

In the case of an unsuspended bicycle the whole vehicle mass contributes to unsprung mass m and the only suspension that the vehicle inherits is the vertical tire stiffness k_t. Nevertheless, the rider can be considered an biomechanical spring-damper system, so the quarter-car body mass corresponds to a portion of the rider mass. It should be noted that this is a first-principle approximation of the original system and especially the parameters k and c cannot be measured directly. They are fitted according to the measured system behavior. The masses are measured by a scale located under the rear wheel of the bicycle measuring wheel load with and without the rider. The vertical tire stiffness can be determined quite precisely directly measuring the vertical tire deflection under various wheel loads. All used parameters are summarized in Table 1.

Cobblestone is a common road-surface for bike tracks and is therefore used as reference for vertical road excitation in this work. A stochastic cobblestone-shaped road excitation that is smoothed to account for the tire radius is used

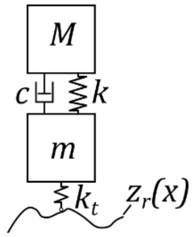

Figure 1: Quarter-car model

unsprung mass	m (kg)	12
rider mass	M (kg)	44
stiffness	c (N/m)	10000
damping	d (Ns/m)	450
vertical tire stiffness	k_t (N/mm)	152.46
wheel radius	r (m)	0.36
vehicle mass	m_v (kg)	100

Table 1: Parameters

to simulate the system. In Figure 2 the result is compared to an acceleration measurement recorded by the instrumented bicycle also on cobblestone. The model shows realistic behavior in both frequency and time domain.

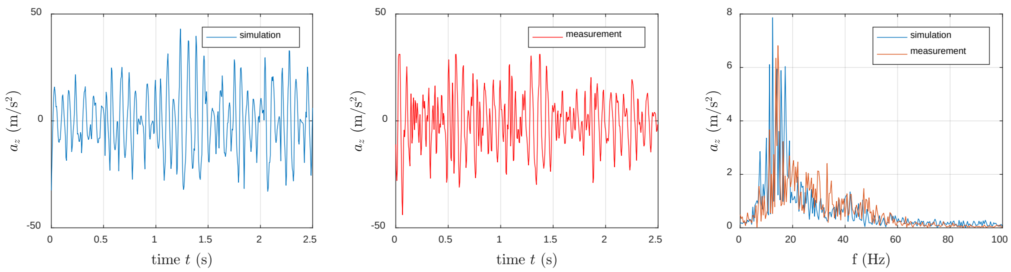

Figure 2: Validation of vertical dynamics in time and frequency domain

3 LONGITUDINAL DYNAMICS

This section focusses on the process of transforming brake torque T_b that is applied on the wheel to a longitudinal brake force F_x. Special attention is directed to the influence of the vertical oscillations described in the former section. The basic effects of longitudinal vehicle dynamics can be assessed using a single-corner model as in Figure 3. It is based on the torque equilibrium around the wheel axis and the longitudinal vehicle force equilibrium

$$J\dot{\omega} = rF_x - T_b$$

$$m_v\dot{v} = F_x,$$

where ω is the wheel speed, r is the wheel radius and m_v denotes the total vehicle mass. The coupling to the vertical vehicle dynamics is defined by the longitudinal tire-road friction $\mu_x = F_x/F_z$ that is considered a function of wheel slip $\lambda = (v - \omega r)/v$.

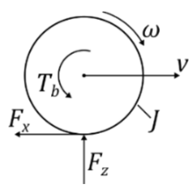

Figure 3: Single-corner model

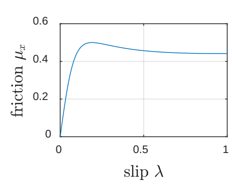

Figure 4: Friction slip curve

A curve that describes typical friction $\mu_x(\lambda)$ is shown in Figure 4. As slip raises the longitudinal friction increases until reaching a peak value. From here on friction decreases until $\lambda = 1$, which denotes a nonrotating, blocked wheel. Also, excessive slip values go along with a decrease in lateral friction which is necessary for stability [6]. The model extension allows simulating brake maneuvers under the influence of vertical vehicle oscillations. A comparison of measurement and simulation is shown in Figure 5. In both cases a constant brake torque is applied to only one wheel. In the measurement this is the rear wheel, whereas the front wheel speed gives the information about the actual vehicle speed. For the simulation the respective front wheel speed is calculated by $\omega_f = v/r$. Of course, no perfect agreement between simulation and measurement can be reached, which is mainly due to the stochastic road excitation. But it can be seen that the first principle modelling approach is capable of simulating the basic effect: Multiple temporary wheel-lock situations occur during the braking process.

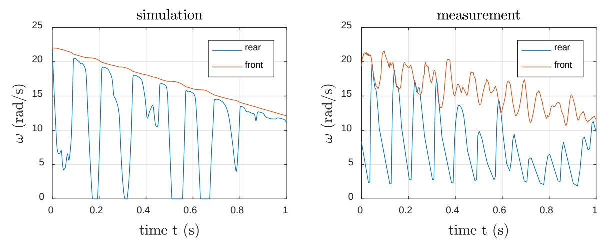

Figure 5: Validation of longitudinal dynamics under influence of vertical oscillations

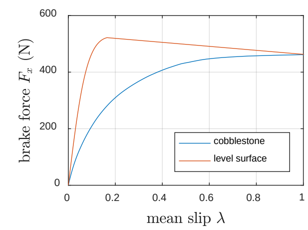

Figure 6: Mean slip vs. constant brake force

4 CONCLUSION

When vertical dynamics can be neglected, either because of level road surface or decent suspension performance, the maximal feasible deceleration is determined by the peak in the slip curve. Also the necessary slip and the involved loss of lateral friction is defined by this curve. The tire-road friction is the primary limit of braking dynamics.

In the presence of relevant vertical oscillations short-term wheel load fluctuations can cause transient wheel-lock, which makes it practical to examine the mean slip that coincides with a certain brake force for a longer period of time. In Figure 5 the result is shown for the coupled vertical dynamics simulated above as well as for level surface. The simulated vertical oscillations have two basic effects: On the one hand peak friction cannot be reached. On the other hand a certain brake force requires a significantly higher mean slip angle. Both aspects can be seen as a serious drawback in terms of safety, since brake distance is increased while stability is reduced.

5 DISCUSSION AND OUTLOOK

The presented study highlights how the absence of a suspension system influences braking dynamics by a simple model that is validated via a reference measurement. It may be seen as an exemplary problem description. A detailed quantization of the effect in various boundary conditions should be provided by future work. Since a continuous adaptation of brake torque to fit the transient fluctuations is an elaborate control task, primarily the efficacy of different suspension layouts shall be studied as a future step to counteract this problem.

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