

# Superresolution far-field imaging of complex objects using reduced superoscillating ripples: supplementary material

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Published 15 September 2017

This document provides supplementary information to “Superresolution far-field imaging of complex objects using reduced superoscillating ripples,” <https://doi.org/10.1364/OPTICA.4.001126>. In the above-mentioned work, it is stated that some comparison studies are done on the imaging capabilities of two other superoscillation waveforms; some further details of the comparisons are presented here. Result of the comparison simulations shows that having low ripples near the main beam of the PSF is important for a coherent imaging system to accurately image complex objects. Otherwise, resolution is compromised, even if the PSF's main beam is narrower than the diffraction limit. The imaging capability of an apodized PSF is also included.

<https://doi.org/10.6084/m9.figshare.5297500>

## 1. COMPARISON

Two superoscillation waveforms, reported in [1] and [2], are compared with the superoscillating ripple. The first is chosen because the same design method was used as this paper. The second is chosen because it is another superoscillation waveform which seems to generate very little sideband structure.

In [1], a PSF is designed with 63 zeros, 10 of which are constrained in a ROI with halfwidth of  $2\lambda/NA$  (see fig. S1). The original waveform data is obtained and used to simulate the resulting output image and compare to the imaging and sensitivity results of our proposed PSF presented in the paper. The results in fig. S2 show that simply having a sub-diffraction main beam does not guarantee superresolution imaging of complex objects; rather, the PSF's ROI ripples also affect image quality.

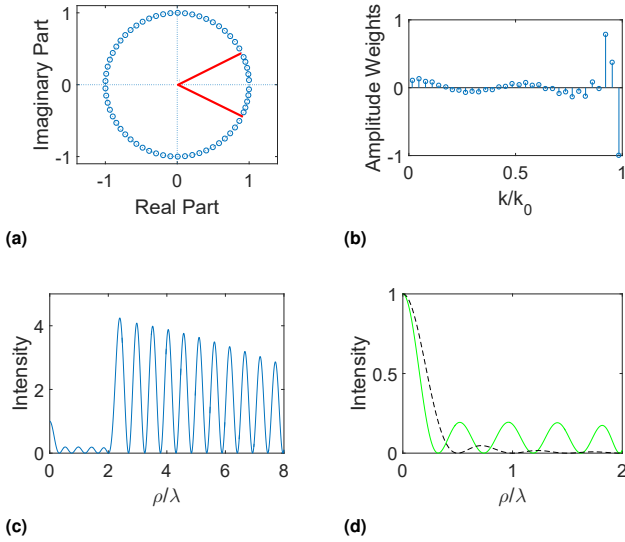
A waveform that approximates the superoscillation needle seen in [2] is constructed using interpolation with a basis of Bessel functions (fig. S3), with an adjusted  $R^2$  of 0.9939. For ease of comparison, after interpolation the wavelength and numerical aperture are changed from the values in [2] to the values used in this paper, and the waveform is scaled accordingly. Simulation results in fig. S4 show that it also images poorly, with

considerable ringing noise. Just as in the case of the first comparison, we believe the significantly-increased ripples nearest to the main beam of the PSF is generating the spurious signals which degrade the final images.

The result of the comparison study emphasizes the importance of having low ripples in the region near the main beam of the PSF in order to be able to implement a robust superoscillation-based imaging system. A major focus of superoscillation wave-form design for imaging purposes, then, should be on reducing the ripples in the PSFs.

Whether conventional apodization can lead to resolution improvement for complex objects, similar to the superoscillation waveform presented in this work, is investigated. An apodized PSF is software coded by convolving a Gaussian kernel with a standard deviation of  $26.0\mu\text{m}$  with the diffraction limited 2D PSF. As can be seen in fig. S5, the maximum sidelobe level of the apodized waveform is comparable to the ripples of the superoscillating waveform studied in this work. Typical behaviours of an apodized waveform can be seen: lower ripples, but wider main beam, than the diffraction limit. The simulated apodized PSF is then used to generate images of the same set of objects used to obtain the diffraction-limited images. Rings of noise

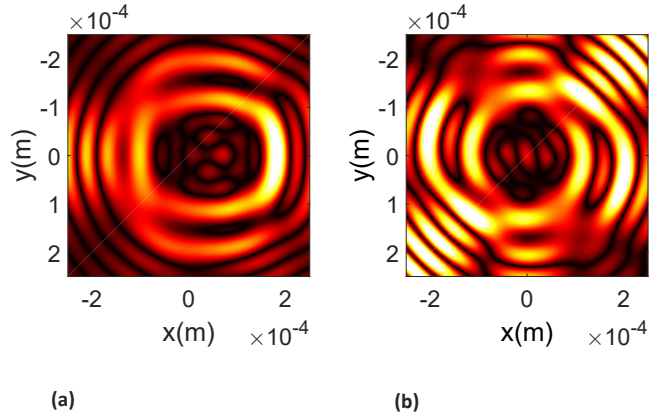
around the objects are reduced; however, the resolution loss due to the widened main beam cancels out any resolution gain due to the reduced sidebands, and the resolution seen in fig. S6 is slightly worse than the diffraction limit.



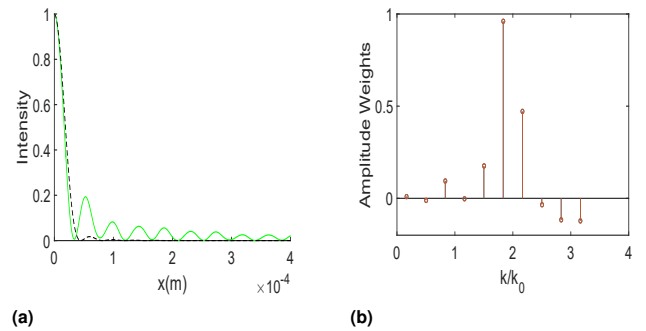
**Fig. S1.** Waveform design from [1], with 63 zeros, 10 of which are in a ROI with half width  $2\lambda/NA$ . (a): Distribution of zeros in the complex  $z$  plane. (b): the amplitude weights. (c): Resulting 2D waveform, including the sidebands. (d): resulting 2D waveform in the ROI, with main beam (green solid line) narrower than the diffraction limit (black dotted line).

## REFERENCES

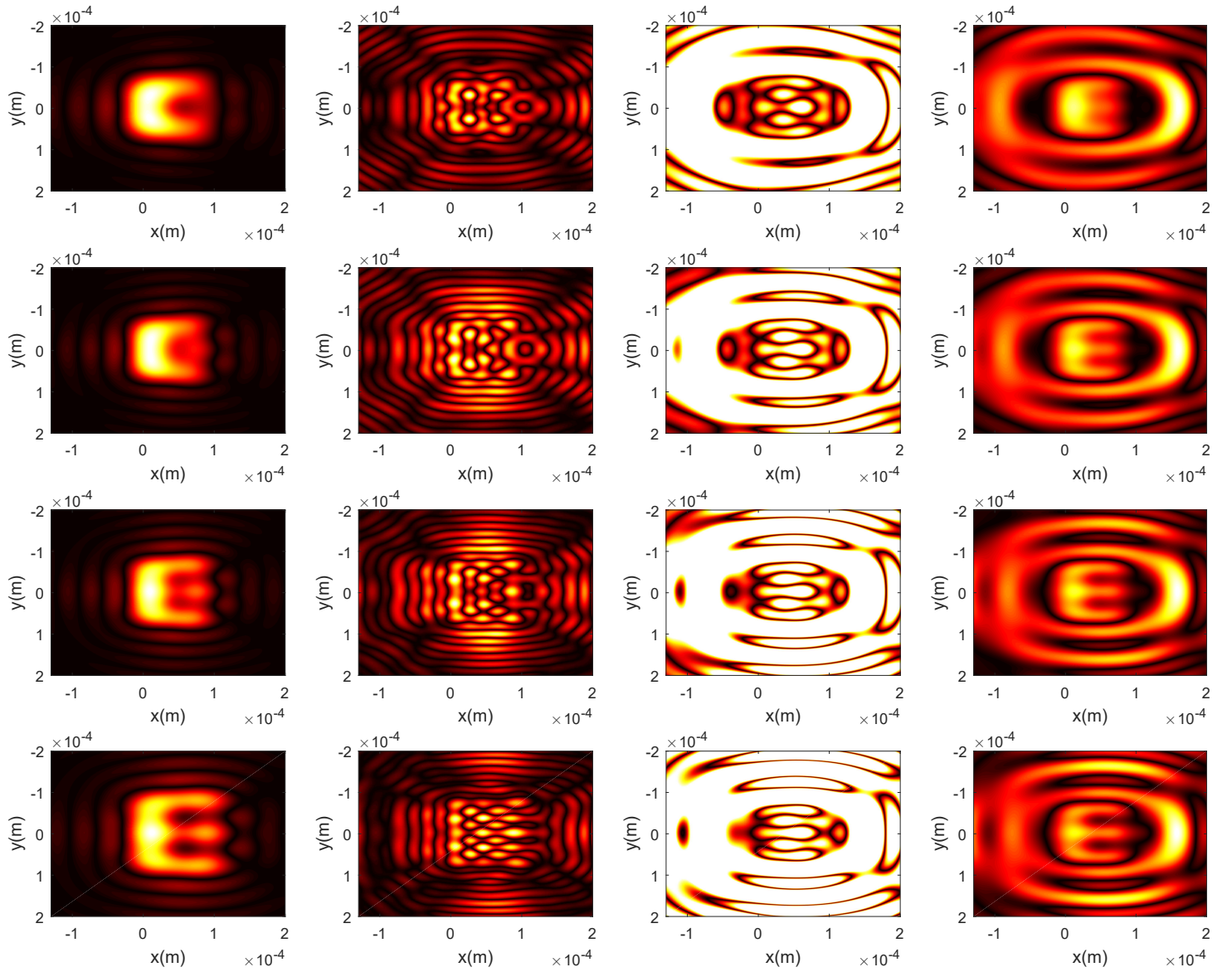
1. A. M. H. Wong and G. V. Eleftheriades, "An optical super-microscope for far-field, real-time imaging beyond the diffraction limit," *Sci. Rep.* **3**, 1715 (2013).
2. G. Yuan, E. T. F. Rogers, T. Roy, G. Adamo, Z. Shen, and N. I. Zheludev, "Planar super-oscillatory lens for sub-diffraction optical needles at violet wavelengths," *Sci. Rep.* **4**, 6333 (2014).



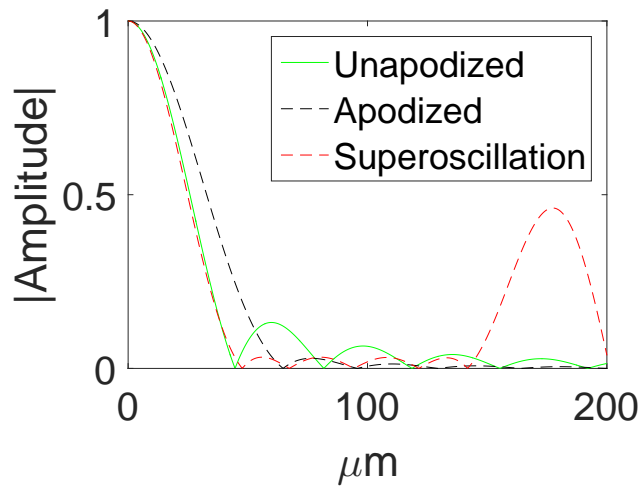
**Fig. S2.** Simulated letter using the PSF designed in Fig. S1. (a) is the simulation of letter 'E' and (b) is the simulation of letter 'N'. Both letters are of the same dimensions as the ones presented in the main text. The images are poorly illuminated and poorly resolved.



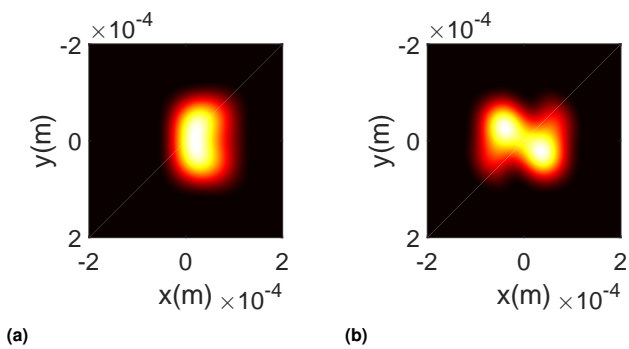
**Fig. S3.** An optical needle PSF, adopted from [2], and scaled to match the wavelength and numerical aperture of our paper. (a): Interpolated waveform (green solid line) to be used as PSF and the diffraction limit (black dashed line); (b): The corresponding weights for a 19 term series of Bessel functions.



**Fig. S4.** Simulation results for imaging the letter 'E'. First column uses a diffraction-limited PSF, the second column uses an optical needle, the third column uses the PSF in [1], and the fourth column uses the waveform presented in this paper. The sizes of the letters, in order from the top to the bottom row, are  $110\mu\text{m} \times 87\mu\text{m}$ ,  $120\mu\text{m} \times 94.5\mu\text{m}$ ,  $130\mu\text{m} \times 102\mu\text{m}$ ,  $140\mu\text{m} \times 110\mu\text{m}$ . The first row has the same dimensions as the result presented in the main text. Ringing effects can be seen for the diffraction-limited cases, but are significantly more noticeable with [1] and [2] because they have ripples which are much larger than the diffraction-limited Airy function. As a result, high resolution images cannot be obtained. Note that the third column is scaled to amplify the ROI, as it is an order of magnitude weaker in magnitude than the sideband rings. The new design, with significantly lower ripples, is much less susceptible to ringing and can consistently produce super-resolution images of various sizes within the designed field of view.



**Fig. S5.** Unapodized diffraction-limited PSF (green solid line) compared with the apodized PSF (black dotted line) and the superoscillating waveform presented in this work.



**Fig. S6.** Simulated images using the apodized waveform. (a) is the simulation of letter 'E' and (b) is the simulation of letter 'N'. The resolution is no better than the diffraction-limited case.