Appendix B: Numerical discretization of the Advection-Reaction-Diffusion (ARD) equation

The ARD equation (Eq. 1) is used to simulate phytoplankton concentrations (*P*) in a 2D environment. Advection (*U*) in the vertical and horizontal, turbulent diffusion (*D*) and cell sinking (*w*) are modeled implicitly, while the reaction part, growth (*g*), is modeled using an explicit scheme. The temporal and spatial discretization can be written as:

$$\frac{P_{i,j}^{t+1}-P_{i,j}^{t}}{\Delta t} = -\frac{J_{i+1,j}^{t+1}-J_{i,j}^{t}}{\Delta z} + \frac{J_{i,j+1}^{t+1}-J_{i,j}^{t}}{\Delta x} + g(z,t) \qquad \text{(Eq. B1)}$$

where *J* is the total flux in between the spatial grid cells, *t* denotes time; and *i* and *j* denote spatial coordinates in the vertical (*z*) and horizontal (*x*), respectively.

Using the 'upwind scheme' for the advection and a 'central difference' for the diffusion we can insert write down the equation for each spatial point:

$$\begin{aligned}
&\left((-d_{i-1,j}\frac{\Delta t}{\Delta z^2} - u_{i-1,j}\frac{\Delta t}{\Delta z} - w_{i-1,j}\frac{\Delta t}{\Delta z})P_{i-1,j}^{t+1} + (-d_{i,j-1}\frac{\Delta t}{\Delta x^2} - v_{i,j-1}\frac{\Delta t}{\Delta x})P_{i,j-1}^{t+1} \right) \\
&+\left(\left(1+2(d_{i,j}+d_{i+1,j}\right)\frac{\Delta t}{\Delta z^2} + 2(d_{i,j}+d_{i,j+1})\frac{\Delta t}{\Delta x^2} + u_{i,j}\frac{\Delta t}{\Delta z} + v_{i,j}\frac{\Delta t}{\Delta x} + w_{i,j}\frac{\Delta t}{\Delta z})P_{i,j}^{t+1} \right) \qquad \text{(Eq. B2)} \\
&+\left((-d_{i+1,j}\frac{\Delta t}{\Delta z^2} - u_{i+1,j}\frac{\Delta t}{\Delta z} - w\frac{\Delta t}{\Delta z})P_{i+1,j}^{t+1} + (-d_{i,j+1}\frac{\Delta t}{\Delta x^2} - v_{i,j+1}\frac{\Delta t}{\Delta x})P_{i,j+1}^{t+1} \right) = P_{i,j}^{t} + g(z)\Delta t
\end{aligned}$$

The second term in Eq. B2 represents the diagonal of the matrix, while the first term and the third term in Eq. B2 are above and below the diagonal.

Equation B2 can be written in matrix form as:

$$P_{i,j}^{t+1} = AP_{i,j}^{t} \ , \qquad \text{(Eq. B3)}$$

with *A* being a tridiagonal transition matrix.

Inverting the matrix A we can integrate forward in time and solve Eq. B3 in a dynamic time dependent system, which allows for the simulation of the same matrix using the standard time-resolving Eulerian approach (figure B1).

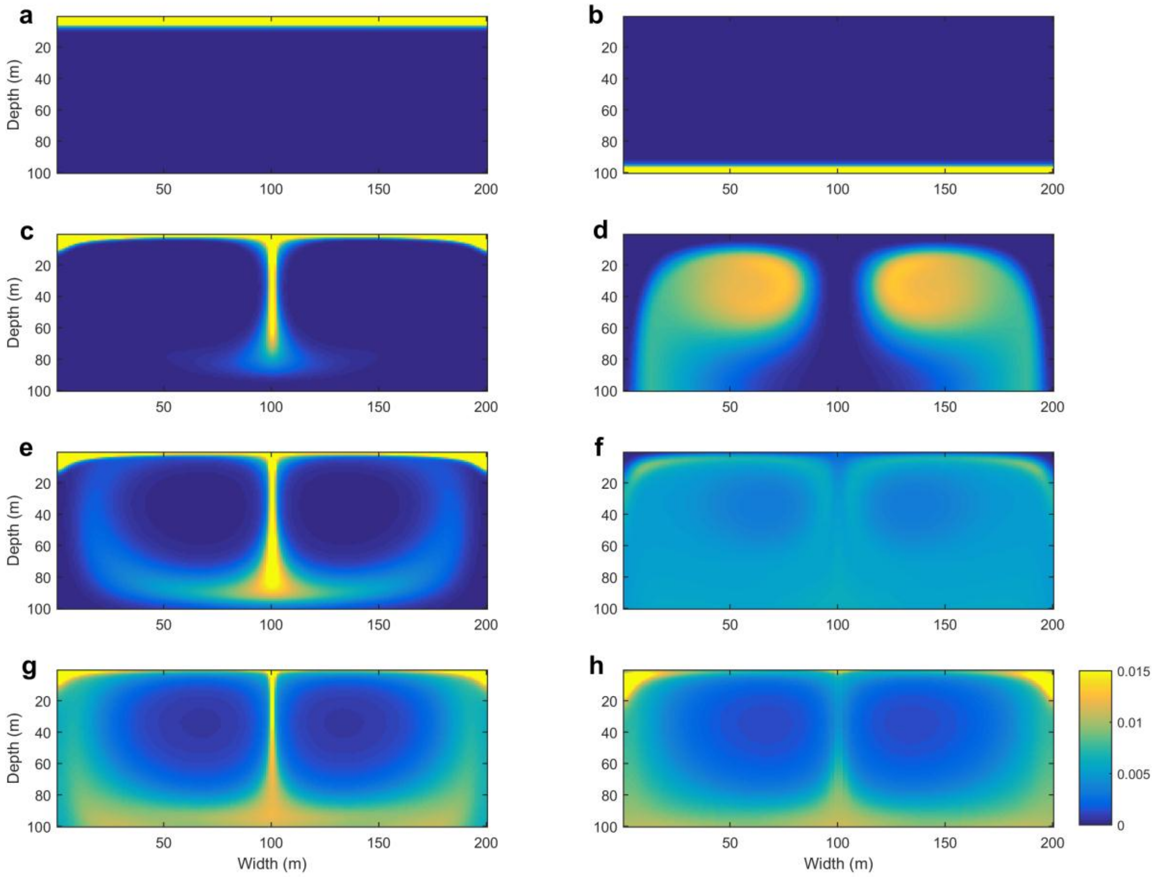

Figure B1: Stable state distribution of phytoplankton distribution using a classical time-resolving Eulerian approach (corresponds to the right eigenvectors presented in figure 2) for neutrally buoyant (left column) and fast sinking (10 m d^{-1}, right column) phytoplankton. The maximum flow field velocities are (a,b) 0 m d^{-1} (c,d) 50 m d^{-1} (e,f) 200 m d^{-1} and (g,h) 1000 m d^{-1}.