**Non-proportional aerodynamic damping of transmission line**

**conductors under wind loads**

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**Abstract**

Aerodynamic damping is the dominant source of transmission line conductor damping. Inaccurate estimate of aerodynamic damping may lead to an overestimation or underestimation of the resonant response of conductor. Conductor is of notable geometric nonlinearity because of its low rigidity. However, the effects of geometric nonlinearity on aerodynamic damping characteristics of conductor are usually neglected in previous studies. In this paper, the aerodynamic modal damping matrix of conductor under mean wind load is derived, considering the interaction between conductor and wind, and the geometric nonlinearity of conductor as well. Variations of aerodynamic damping ratio and non-proportionality with mean wind speed are investigated through numerical studies. The results show that non-proportionality of conductor’s aerodynamic damping is remarkable in extreme winds. Variations of aerodynamic damping ratios with mean wind speed are nonlinear.

**Introduction**

It is widely approved that aerodynamic damping is the major source of damping and has a great influence on the dynamic wind-induced response of conductors [1-4]. Aerodynamic damping derives from the interaction between wind and structures. By calculating the drag forces on conductors with the relative speed between air and conductors, the aerodynamic damping can be implicitly included [5-6]. Deflection of a conductor subjected to extreme winds may be large, which results in a movement non-parallel to the horizontal incoming wind. In this case, relative wind attack angle is changed, leading to a vertical aerodynamic damping force which may not be neglected. However, most of the previous studies considered only the horizontal relative motion of wind and conductors.

In order to calculate the buffeting response of conductors in frequency domain, attempts were made to quantify the aerodynamic damping explicitly, in the form of aerodynamic damping ratio. Davenport [7] proposed an expression for aerodynamic damping ratio based on modal analysis, indicating that aerodynamic damping ratio is proportional to mean wind speed, and inversely proportional to the mass per unit length and natural frequencies of the conductor. According to Loredo-Souza and Davenport [8], aerodynamic damping can be as high as 60% of the critical damping for lightweight conductors in extreme winds. However, this expression is also based on the premise that the movement of the conductor is parallel to the incoming wind, which is not the case for high wind speed. Moreover, the natural frequencies and mode shapes of the conductor may vary with wind speed because of its geometric nonlinearity. Therefore the expression in [7] cannot quantify the aerodynamic damping accurately. Stengel et al. [9] proposed a modified expression for aerodynamic damping ratio based on aeroelastic wind tunnel tests and numerical simulations assuming the conductor to sway in a pendulum mode, while in fact conductors have various in-plane and out-of-plane modes. Wang et al. [10] proposed another approach for modal aerodynamic damping, revealing the non-proportionality of conductors’ aerodynamic damping. Nevertheless, the effects of damping non-proportionality on the wind-induced dynamic response of conductors have not been emphasized.

In this paper, the aerodynamic modal damping matrix of conductor under mean wind load is derived based on modal analysis procedure, considering the relative motion between conductor and wind in both horizontal and vertical directions. The effects of geometric nonlinearity on conductor’s dynamic characteristics are also taken into account. Variations of aerodynamic damping characteristics with wind speed are investigated through numerical studies. Special attention is paid to the non-proportionality of aerodynamic damping, which may have a non-negligible influence on the wind-induced dynamic response of conductors. The proposed method is applicable to both single-span and multi-span conductors, which may help better understanding the characteristics of aerodynamic damping.

**Interaction between conductor and wind**



Figure 1. Relative motion between wind and conductor

The relative motion between the incoming flow and the conductor is shown in Figure 1. Based on the quasi-steady assumption, the wind-induced drag force *F*D on a segment of conductor can be expressed as





whereis air density; *CD* is the drag coefficient; *A* is the area of conductor segment exposed to wind; *U*r is the relative speed between wind and conductor; and *u* are the mean and fluctuating wind speed, respectively; andare the horizontal and vertical velocity components of the conductor segment, respectively.

The relative wind attack angle can be expressed as



Thus the horizontal and vertical drag forces can be expressed as





Note that the third term in the right side of Equation (4) and the term in the right side of Equation (5) are proportional to and, respectively. They are the aerodynamic damping forces in horizontal and vertical directions, respectively.

**Aerodynamic modal damping**

Conductor is a kind of structure with notable geometric nonlinearity. However, since the turbulence intensity at conductor height is usually small, the fluctuating displacement of conductor is small compared with the mean part [6], therefore the dynamic wind-induced motion of conductor may be approximately regarded as linear motion around the mean wind load state.



Figure 2. Calculation model of conductor

Consider a conductor model with *N* discrete segments as shown in Figure 2. Since the longitudinal displacement of the conductor is small and can be neglected under the action of transverse wind load, each conductor segment is assumed to have 2 degrees of freedom. The equations of motion around the mean wind load state can be expressed as











whereis the fluctuating displacement vector; andare the horizontal and vertical components of the *i*th node, respectively;  and  are the velocity and acceleration vectors, respectively; is the fluctuating wind load vector; is the stiffness matrix of conductor in the mean wind load state; andare the mass matrix and the structural damping matrix, respectively; *D* is the diameter of conductor; is the length of the *i*th conductor segment.

By substituting Equations (9)、(10) into Equation (6), the equations of motion can be rewritten as



where









Thus the aerodynamic damping matrix  is expressed by Equation (14).

By mode superposition, the fluctuating displacement can be calculated as







where is the generalized displacement vector of conductor;  is the mode shape matrix of conductor; is the *j*th mode shape vector.

The equations of motion in normal coordinates can be expressed as



where andare the generalized stiffness matrix and generalized mass matrix, respectively; and are the generalized structural and aerodynamic damping matrix, respectively. Thus the generalized aerodynamic damping matrix can be calculated as



in which the element in the *j*th row, *k*th column is expressed as



It can be noted from (20) that the non-diagonal elements of are non-zero, indicating that the aerodynamic damping of conductor under wind loads is non-proportional damping.

The aerodynamic damping ratio can be calculated as



where is the *j*th generalized mass; *m* is the mass per unit length;  is the *j*th natural frequency.

**Results and discussions**

A single span transmission line conductor is considered, as shown in Figure 3. Main physical parameters of the conductor are: *D* = 30.00mm, *E* = 63GPa, *m* = 1.642kg·m-1, *CD* = 0.95.



Figure 3. Geometric parameters of the calculation line

A finite element model of the conductor is established with the software ANSYS. The element type for the conductor is LINK10. The conductor is divided into segments with a length of 10m.

The exposure category of the site is assumed to be Category B, which is specified in GB 50009-2012 [11]. The mean wind speed profile can be calculated by



where  is the mean wind speed at 10m height.

Non-linear static analysis is conducted to calculate the mean wind-induced response of conductor. The natural frequencies and mode shapes of conductor under mean wind load can be obtained by modal analysis, which are shown in Table 1. Then the aerodynamic modal damping can be calculated by Equation (19).

In order to investigate the aerodynamic damping characteristics under different wind speed conditions, the calculation cases are set as = 5m/s ~ 50m/s.

The first six natural frequencies and mode shapes in the case of = 25m/s are shown in Table 1.

Elements in the first six rows and the first six columns of in the case =25m/s are shown in Table 2. It can be inferred that the non-diagonal elements are in the same order as the diagonal ones, which means that the non-proportionality of aerodynamic damping in this case is high.

|  |  |  |  |
| --- | --- | --- | --- |
| Mode | Natural frequency | Mode shape | Description |
| 1 | 0.12969 |  | 1st out-of-plane |
| 2 | 0.25731 |  | 2nd in-plane |
| 3 | 0.25917 |  | 2nd out-of-plane |
| 4 | 0.33028 |  | 1st in-plane |
| 5 | 0.38889 |  | 3rd out-of-plane |
| 6 | 0.44697 |  | 3rd in-plane |

Table 1. Natural frequencies and mode shapes of the conductor

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *k*=1 | *k*=2 | *k*=3 | *k*=4 | *k*=5 | *k*=6 |
| *j*=1 | 0.484 | 0 | 0 | 0.127 | -0.005 | 0.100 |
| *j*=2 | 0 | 0.488 | 0.163 | 0 | 0 | 0 |
| *j*=3 | 0 | 0.163 | 0.488 | 0 | 0 | 0 |
| *j*=4 | 0.127 | 0 | 0 | 0.495 | 0.099 | -0.009 |
| *j*=5 | -0.005 | 0 | 0 | 0.099 | 0.474 | -0.129 |
| *j*=6 | 0.100 | 0 | 0 | -0.009 | -0.129 | 0.507 |

Table 2. Elements of aerodynamic modal damping matrix (=25m/s)

In order to compare the non-proportionality of aerodynamic damping in different wind speed cases, the index of damping non-proportionality defined in [12] is utilized, which can be calculated by



where  and  are the maximum and minimum eigenvalues of , respectively; and



It can be inferred from (23) that .  denotes that aerodynamic damping is proportional. The larger is, the more non-proportional the aerodynamic damping is.



Figure 4. Variation of aerodynamic damping non-proportionality index with reference wind speed

Figure 4 shows the variation of aerodynamic damping non-proportionality index  with reference wind speed . It can be inferred that the non-proportionality of conductor aerodynamic damping is low when  is low. increases as  increases when ≤ 25m/s. The non-proportionality of aerodynamic damping reaches its maximum when = 25m/s. And  decreases with the increase of  when ≥ 25m/s.

Figure 5 shows the variation of aerodynamic damping ratio with reference wind speed. The variations of aerodynamic damping ratios of the first and the third modes are similar, as these modes are both out-of-plane modes. However, the variation of aerodynamic damping ratio of the second mode is different from those of the first and the third modes, for the second mode is an in-plane mode. When 5m/s ≤≤ 50m/s, the aerodynamic damping ratios of the first and the third modes reach their maximums at = 25m/s, while the aerodynamic damping ratio of the second mode is always increasing with the increase of . Variation of each aerodynamic damping ratio with  is non-linear.



Figure 5. Variation of aerodynamic damping ratio with reference wind speed

**Conclusions**

In this paper, the aerodynamic modal damping matrix of conductor under mean wind load is derived based on modal analysis procedure, considering the relative motion between conductor and wind in both horizontal and vertical directions. The effects of geometric nonlinearity on conductors’ dynamic characteristics are also taken into account. Variations of aerodynamic damping characteristics with wind speed are investigated through numerical studies.

The non-proportionality of conductor aerodynamic damping increases at first and then decreases with the increase of mean wind speed.

Aerodynamic damping ratios vary non-linearly with mean wind speed. Variation laws of aerodynamic damping ratios of in-plane modes and out-of-plane modes are different. Generally, aerodynamic damping ratios of all modes are high when the mean wind speed is high.

The geometric nonlinearity has a great influence on the aerodynamic damping characteristics of conductor. Ignoring the effects of geometric nonlinearity may lead to an inaccurate estimate of aerodynamic damping.

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