

## Turnover Rate of Popularity Charts in Neutral Models

Tim Evans

work done with  
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ECCS 2012, Brussels,  
7<sup>th</sup> September 2012



# Turnover Rate of Popularity Charts in Neutral Models

- Neutral Models and Cultural Transmission
- Popularity Charts
- Wright-Fisher model results
- Moran model summary

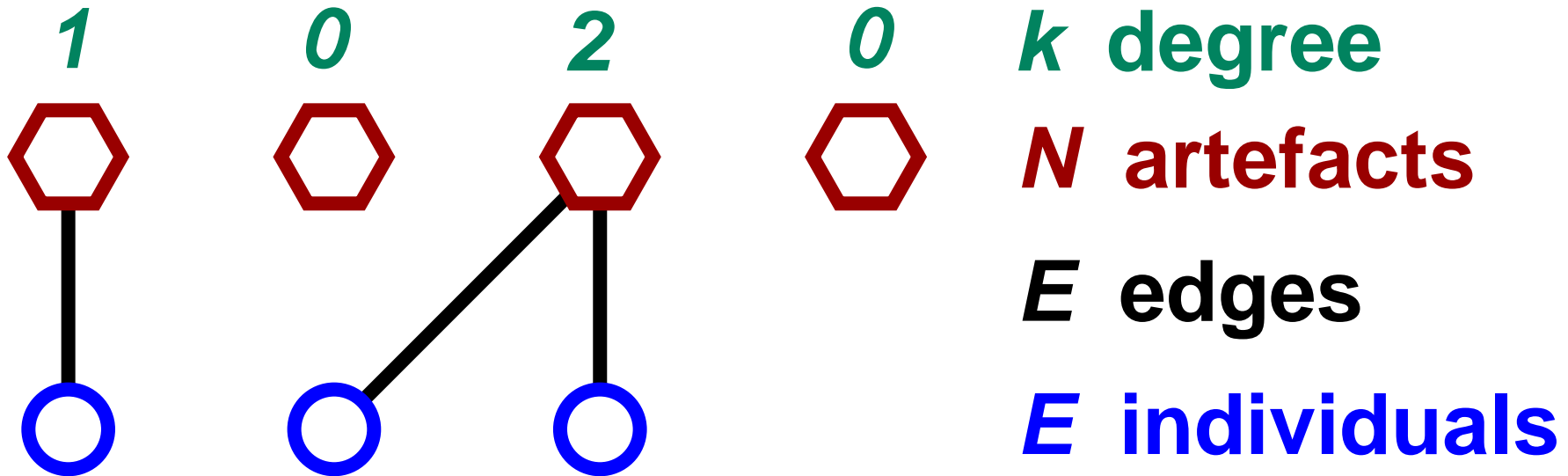
THIS WEEK'S CHARTED SINGLES BY TYPE

Billboard **HOT 100** SINGLES

WEEK	1	2	3	4	5	6	7	8	9	10	TITLE	Artist, Label & Number	Weeks On Chart
27	—	—	—	—	—	—	—	—	—	—	<b>CAN'T BUY ME LOVE</b>	Beatles, Capitol 5150	2
2	3	7	55	—	—	—	—	—	—	—	<b>TWIST AND SHOUT</b>	Beatles, Tattle 9001	4
3	1	1	2	—	—	—	—	—	—	—	<b>SHE LOVES YOU</b>	Beatles, Swan 4132	11
4	2	2	1	—	—	—	—	—	—	—	<b>I WANT TO HOLD YOUR HAND</b>	Beatles, Capitol 5172	12
5	4	3	3	—	—	—	—	—	—	—	<b>PLEASE PLEASE ME</b>	Beatles, Vee Jay 581	10
6	7	19	49	—	—	—	—	—	—	—	<b>SUSPICION</b>	Terry Stafford, Crusader 101	7
7	8	10	13	—	—	—	—	—	—	—	<b>HELLO, DOLLY!</b>	Louis Armstrong, Epy 573	8
16	22	50	—	—	—	—	—	—	—	—	<b>SHOOB SHOOP SONG</b>	Betty Everett, Vee Jay 583	6
9	9	13	22	—	—	—	—	—	—	—	<b>MY HEART BELONGS TO ONLY YOU</b>	Bobby Vinton, Epic 9663	6
10	10	15	20	—	—	—	—	—	—	—	<b>GLAD ALL OVER</b>	Beverly Sills, Epic 9664	8

# The Neutral Model - as a Bipartite Network

- $E$  individual vertices each with one edge connected to  $N$  artefact vertices
- Popularity of artefact is vertex degree  $k$ 
  - $n(k)$  = degree distribution,
  - $p(k) = n(k)/N$  = degree probability distribution

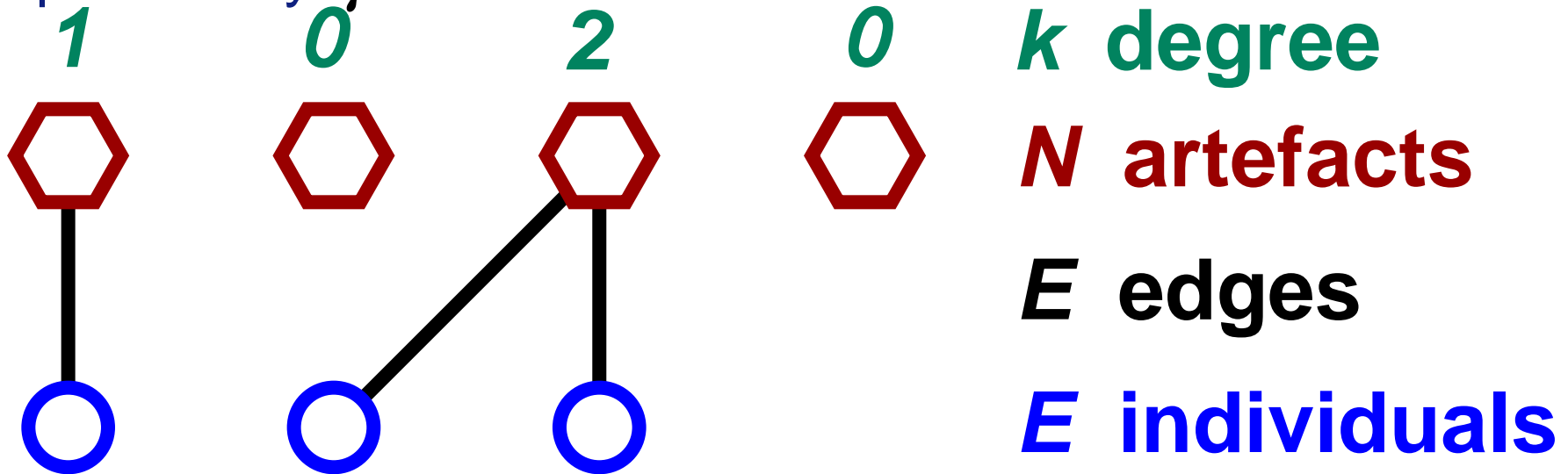


[Evans 2007; Evans & Plato 2007;  
Evans, Plato & You 2010]

# How Individuals choose in a neutral model

Individuals choose a new artefact in two ways:-

- COPY the choice made by another individual (inheritance)  
probability  $(1-\mu)$
- INNOVATE by choosing a new artefact at random (mutation)  
probability  $\mu$

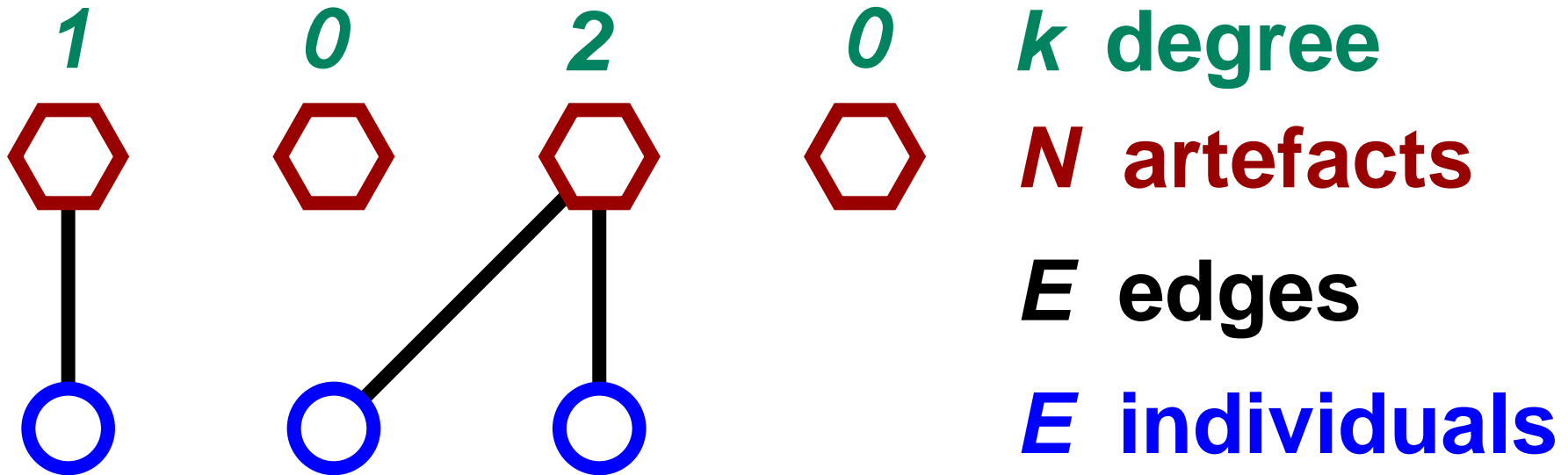


[Evans 2007; Evans & Plato 2007;  
Evans, Plato & You 2010]

# Wright-Fisher Model and Moran Model

Many variations for update rules:-

- Wright-Fisher model - all individuals update simultaneously
- Moran model – only one individual updates at each step
- Choices driven by simplicity and/or reality



[Evans 2007; Evans & Plato 2007;  
Evans, Plato & You 2010]

# Relationship to Other Systems

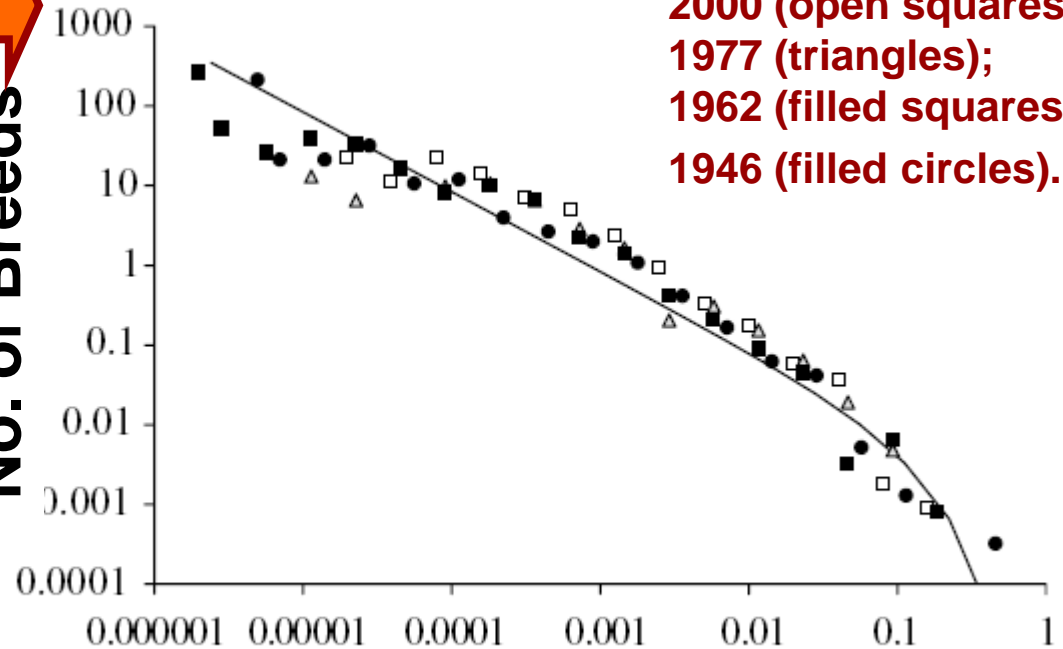
- Genes not Memes [Fisher-Wright & Moran models]
- Speciation in Ecology ['Tangled Nature', Christensen et al 2002]
- Network Rewiring [Evans & Plato, 2007]
- Statistical Physics Models [Blythe & McKane, 2007]
- **Cultural Transmission** [Bentley et al, 2004]
- Language Change [Baxter et al, 2006]
- Minority Game strategies [Clemson & Evans 2012]
- Opinion formation [Lambiotte et al. 2007]
- Family Names [Zanette & Manrubia, 2001]

# Cultural Transmission Data

- Registrations of pedigree dogs
- Baby name registrations
- Music charts
- Archaeological pot shards



No. of Breeds



Frequency of registrations of  
each breed of pedigree dog

See Neiman (1995); Bentley, Maschner (2000,2001);  
Bentley, Hahn, Shennan (2004); Bentley, Shennan (2003,2005);  
Hahn, Bentley (2003); Herzog, Bentley, Hahn (2004);  
Bentley, Lipo, Herzog, Hahn (2007).

# Old Models, New Questions

Cultural transmission context for neutral models can produce new questions

- What if can only copy from neighbour in a ***social*** network? [Evans, Plato & You, 2010; Omerod talk on Thursday]
- Measuring innovation rates from data
- How do we change innovation rate?
- **Turnover in Popularity** (statistics of rank)



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Billboard **HOT 100** SINGLES

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⑤ 4	3	3	—	<b>PLEASE PLEASE ME</b>	Beatles, Vee Jay 581	10
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★ 16	22	50	—	<b>SHOOP SHOOP SONG</b>	Betty Everett, Vee Jay 583	6
⑨ 9	13	22	—	<b>MY HEART BELONGS TO ONLY YOU</b>	Bobby Vinton, Epic 9663	6
⑩ 10	15	20	—	<b>GLAD ALL OVER</b>	Beverly Sills, Epic 9664	8

# Popularity Lists

- Rank artefacts by popularity (degree)
- Take list of top  $y$  to form popularity list

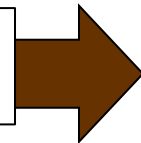
Rank	Artefact
1	A
2	B
3	C
4	D
5	E
6	F
7	G



# Turnover Rates in Popularity Lists

- Evolve system
- Update top  $y$  list
- Turnover  $z$  is number of new artefacts in top  $y$  *plus* number leaving top  $y$

Here  $z=2$



Rank	Artefact		Rank	Artefact
1	A	→	1	A
2	<b>B</b>	↘	2	C
3	C	→	3	D
4	D	→	4	<b>F</b>
5	E	↘	5	<b>B</b>
6	<b>F</b>	↘	6	E
7	G	↘	7	H

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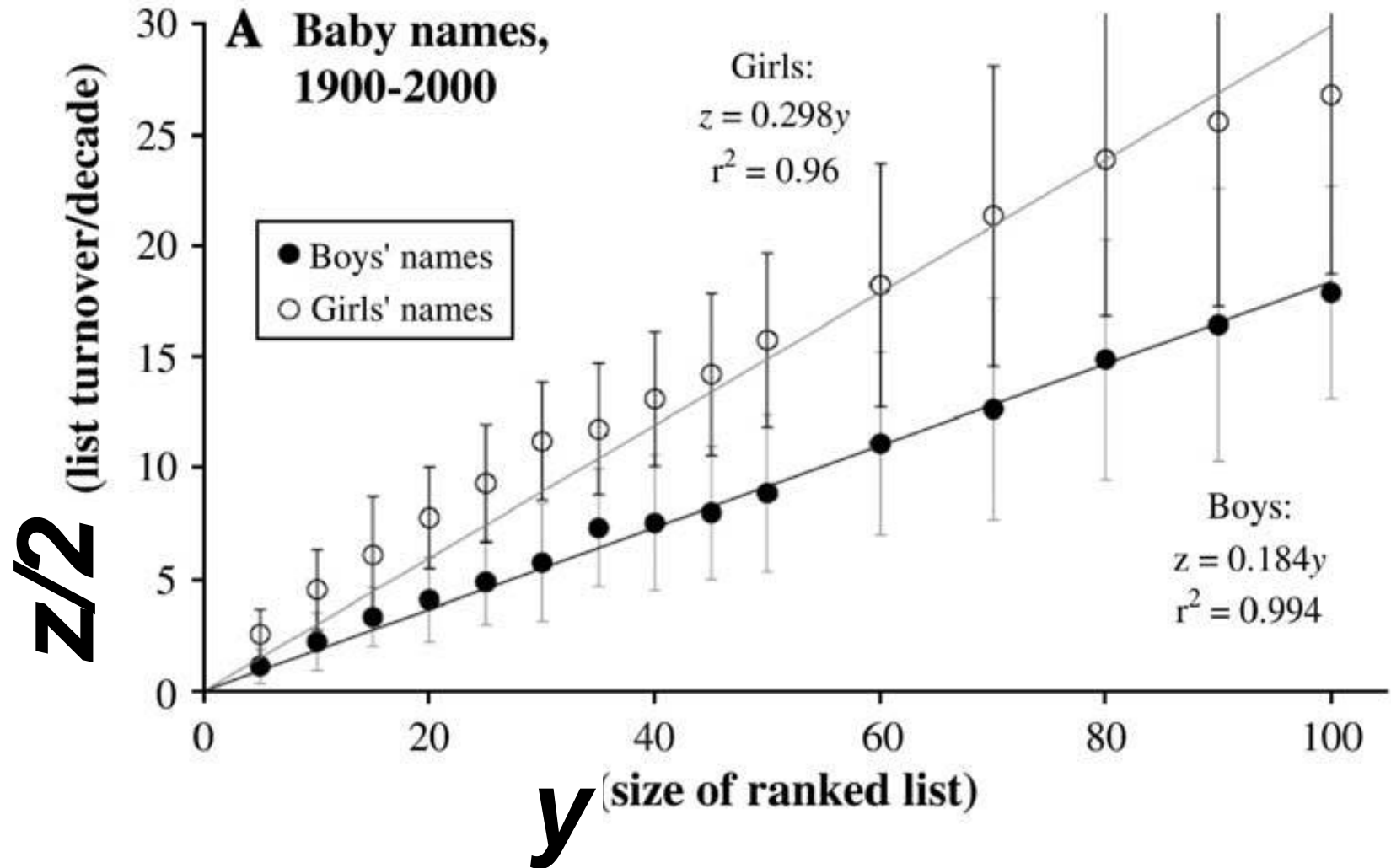
Billboard

Award

★	27	—	—	—	CAN'T BUY ME LOVE	Beatles, Capitol 5150	2
2	3	7	55	—	TWIST AND SHOUT	Beatles, Tattle 9901	4
3	1	1	2	—	SHE LOVES YOU	Beatles, Swan 4150	11
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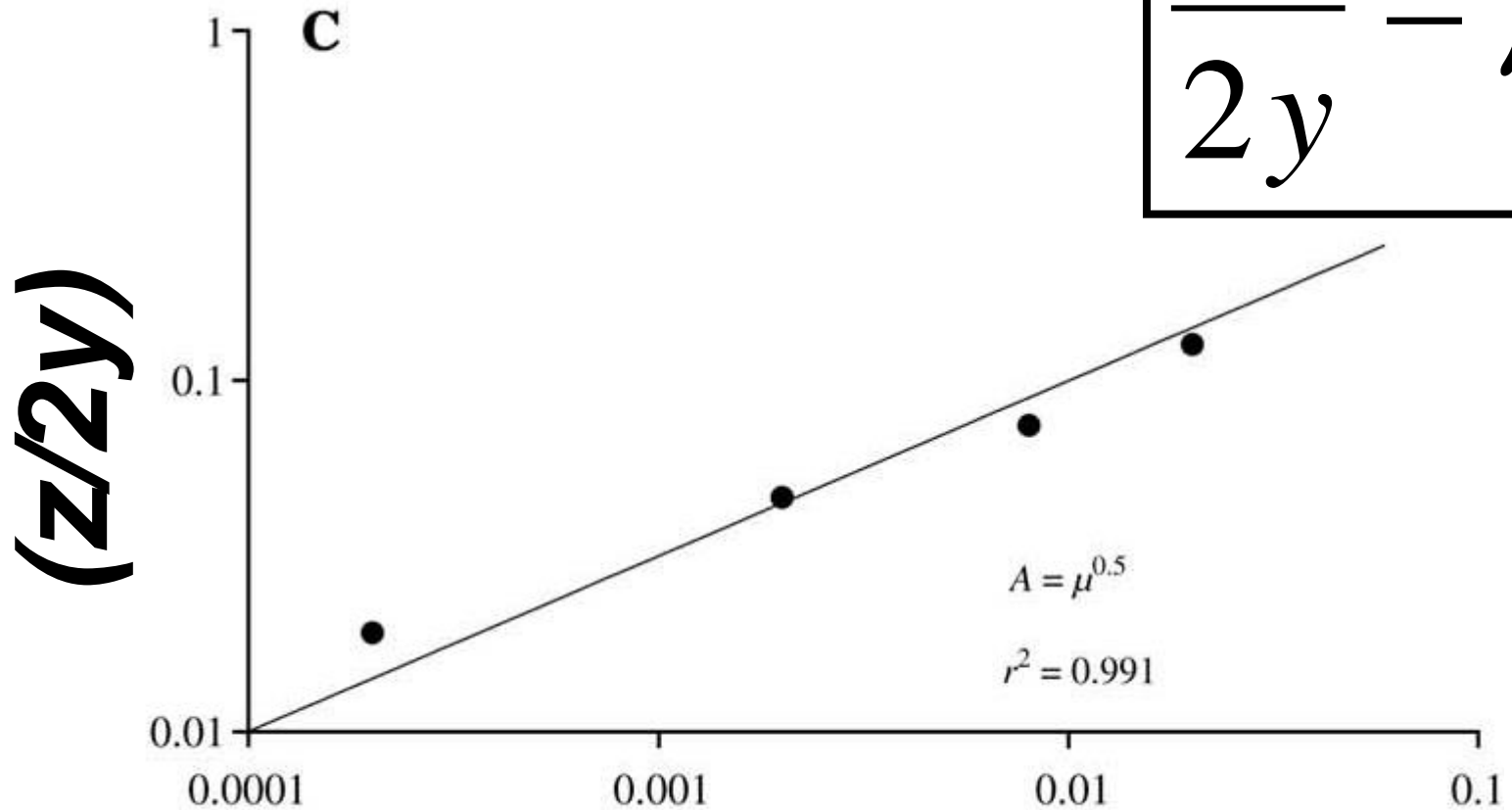
# Linear relationship

[BLHH = Bentley, Lipo, Herzog, & Hahn 2007]



Scaling with innovation rate  $\mu$

$$\frac{z}{2y} = \mu^{0.5}$$



Computer  
Wright-Fisher Model  $\mu$

[BLHH =  
Bentley et al 2007]

# Conjecture of BLHH [Bentley, Lipo, Herzog, Hahn 2007]

**Power  
1**

**Power  
1/2**

$$z = 2y\sqrt{\mu}$$

**Coefficient  
2**

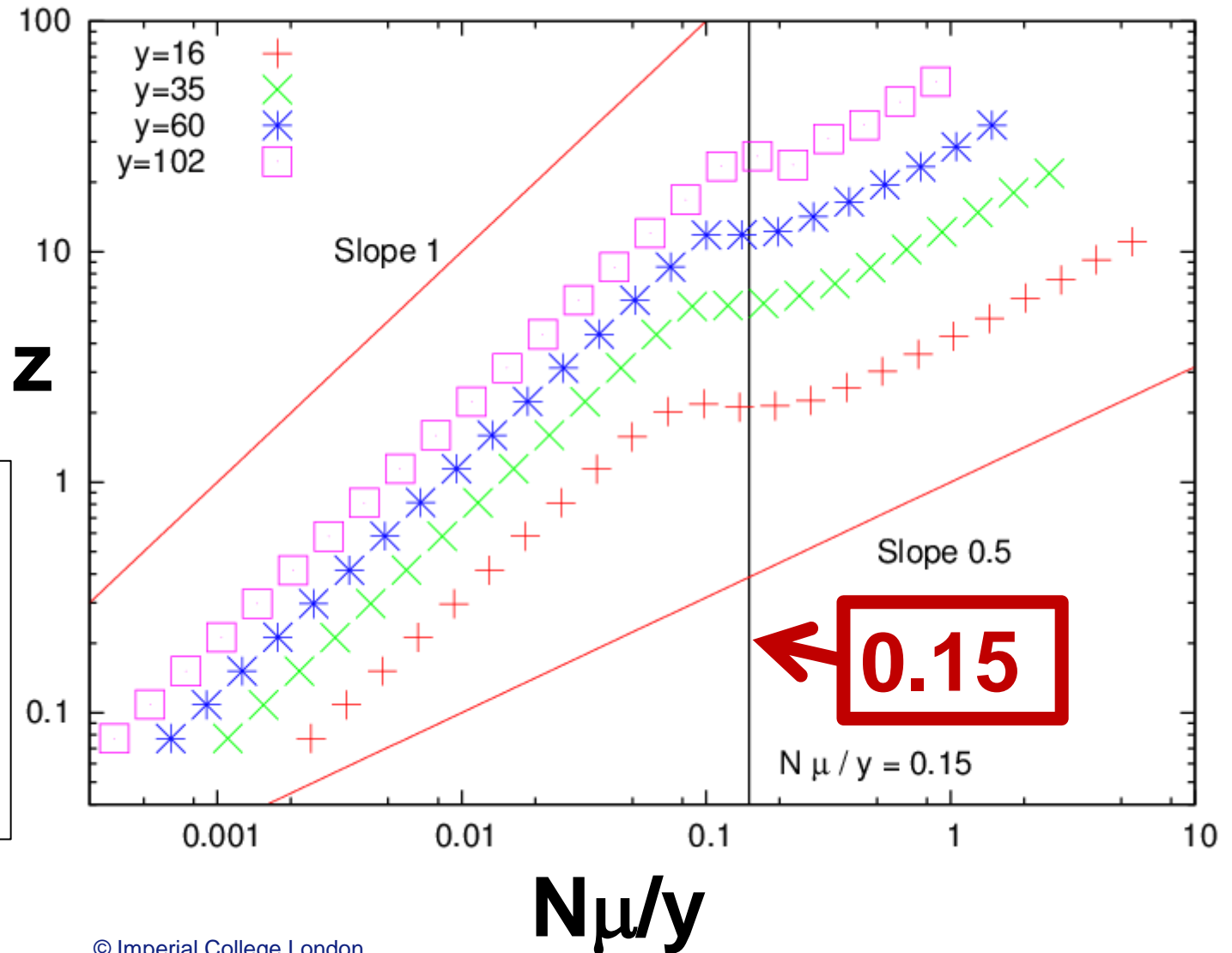
**within context of  
Wright-Fisher Model**

# Testing the Hypothesis

- Update rule
  - Wright-Fisher Model or Moran Model
- Wide range of parameters
  - The value  $N\mu \sim 1$  is a key scale
- Wait  $\tau$  updates for equilibrium
  - We use  $\tau = 4/\mu > \ln(\lambda_2)$  where  $\lambda_2$  is known analytically
- Form popularity lists after  $N$  individual choices
- Make  $T$  measurements of  $\mathbf{z}$ 
  - We aim for  $< 10\%$  error in  $\mathbf{z}$ , using  $T = 50 + \mu^{-1}$



# Wright-Fisher Results – transition point

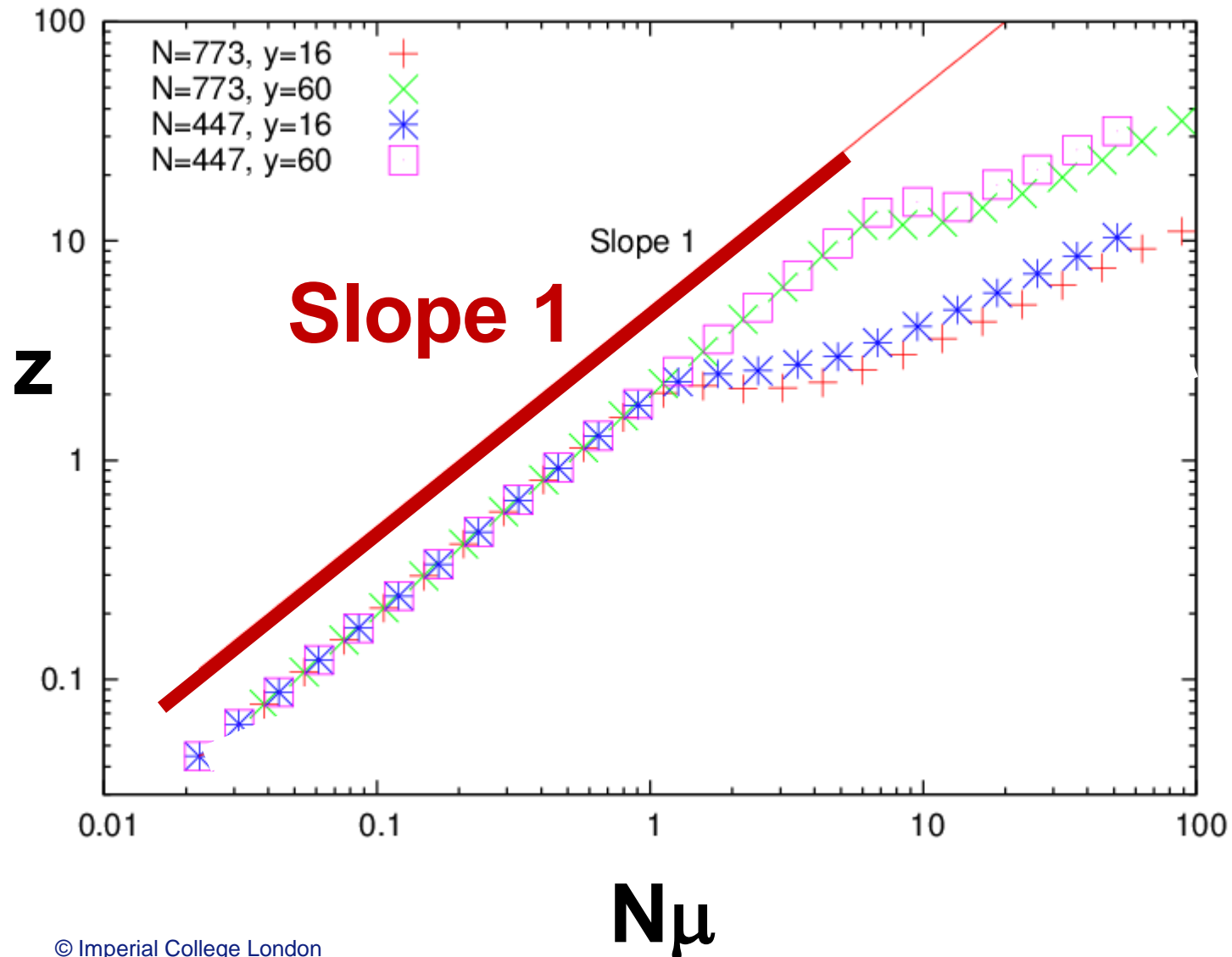


Transition  
at  
 $N\mu =$   
 **$0.15 y$**

$$N\mu < 0.15 \text{ y}$$

# Wright-Fisher Results – Low Innovation

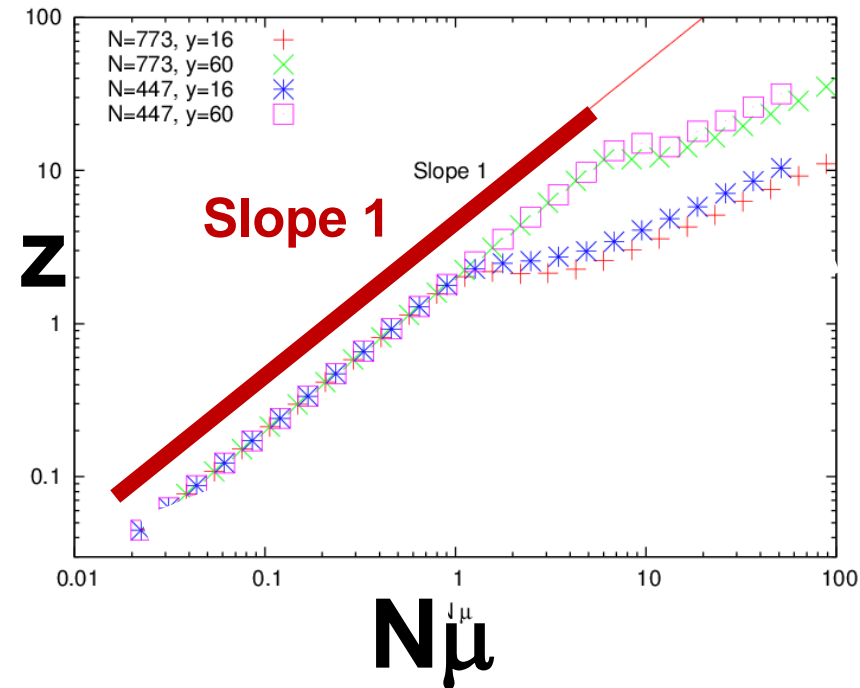
Low  
Innovation  
( $N\mu < 0.15 \text{ y}$ )  
find  
 $z = 2 N\mu$



$$N\mu < 0.15 y$$

# Wright-Fisher Results – Low Innovation

Rank	Artefact
1	B
2	R
3	T
4	-
5	-
6	-
7	-

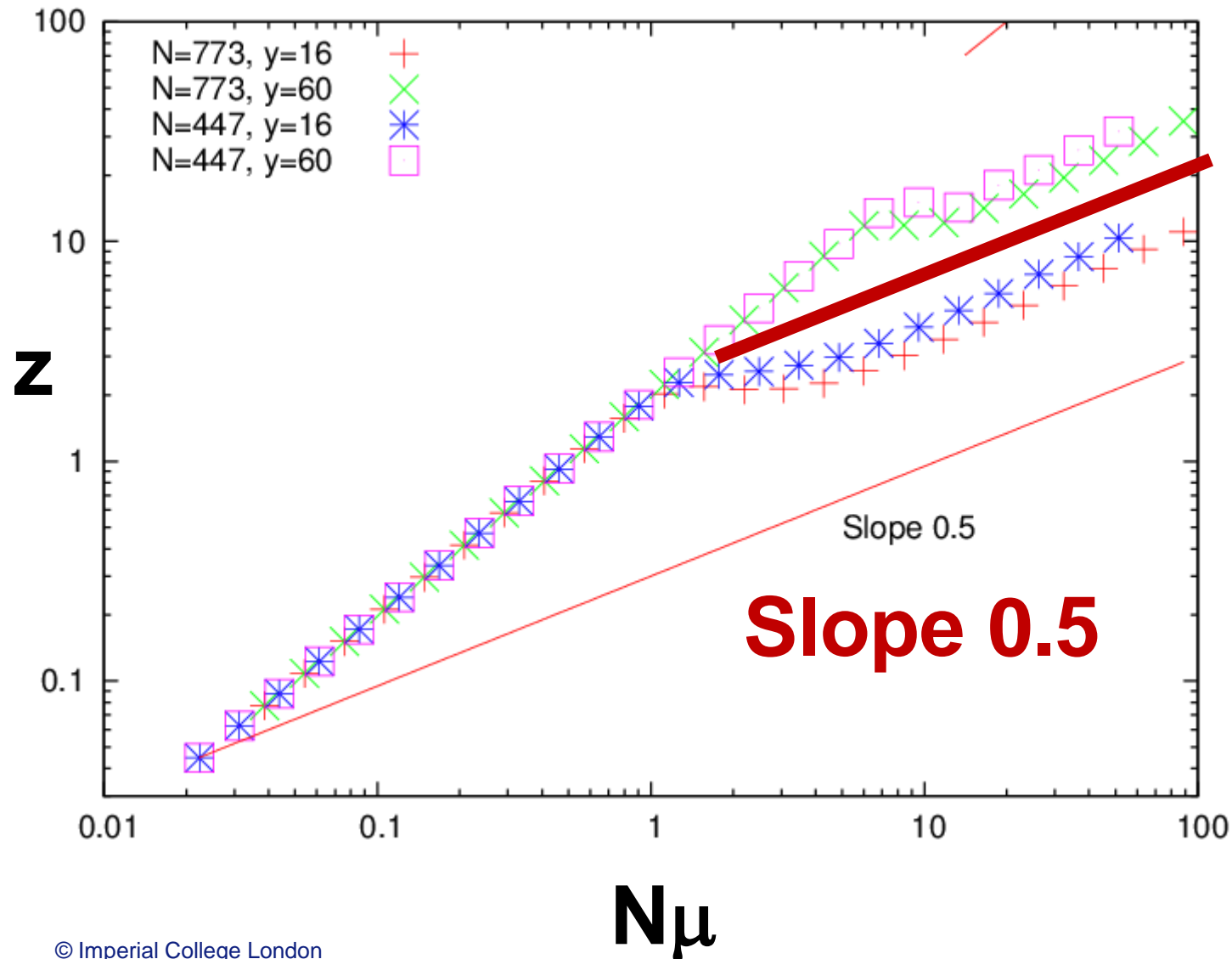


List too long,  $y > y^* = 7N\mu$ , for low innovation where too few artefacts chosen  $\mathbf{z = 2 N\mu}$

$$N\mu > 0.15 y$$

# Wright-Fisher Results – High Innovation

For high  
innovation  
BLHH  
formula  
 $z = 2 y \mu^{0.5}$   
roughly  
right



$$N_{\mu} > 0.15 \text{ y}$$

## Wright-Fisher Results – High Innovation

For  $180 < N < 3993$   
we tried to fit

$$z = A \mu^a y^b N^c$$

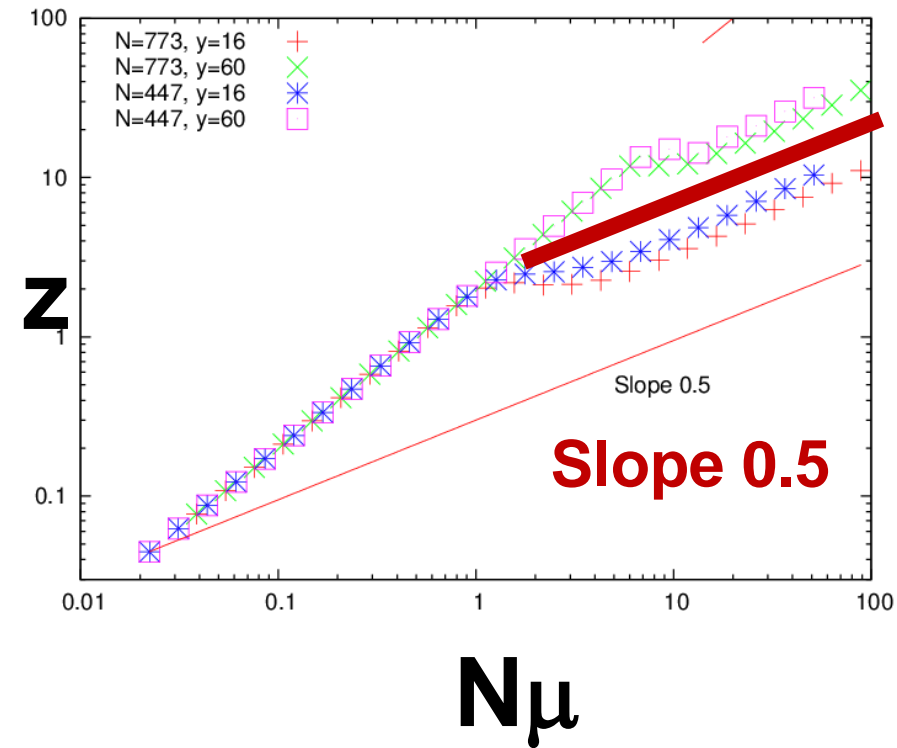
and found [BLHH values]

$$A=1.38(2) \quad [2.0]$$

$$a=0.550(2) \quad [0.5]$$

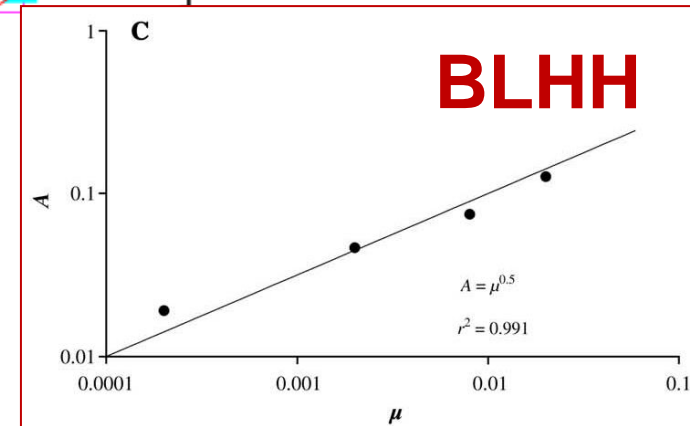
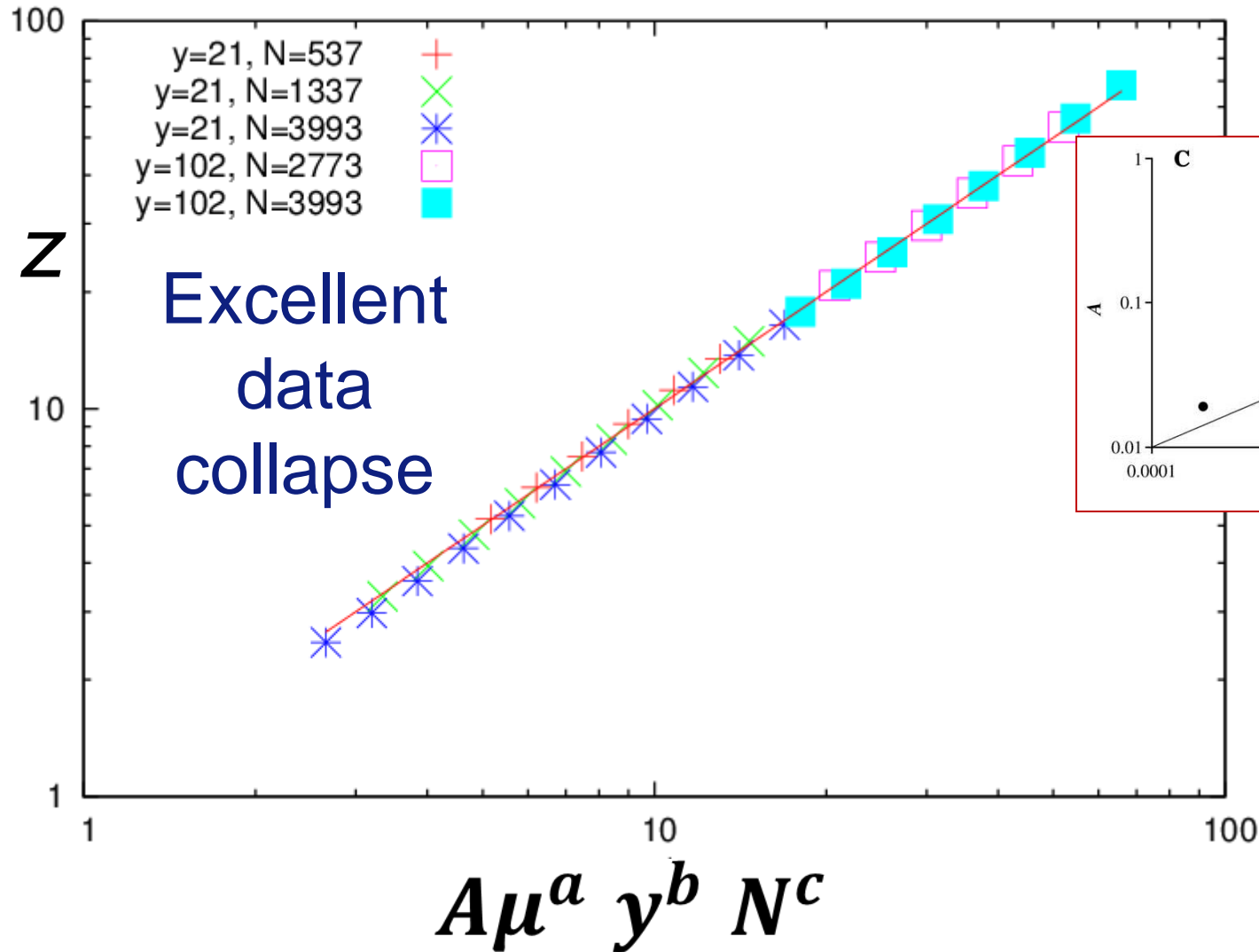
$$b=0.860(1) \quad [1.0]$$

$$c=0.130(2) \quad [0.0]$$



**Weak  $N$   
dependence**

# Wright-Fisher – High Innovation Data Collapse



# Wright-Fisher – Large Populations

- So far  $N \leq 4000$  as in BLHH
- Real data sets are much larger
  - One million births per year for baby names
  - Six hundred thousand dog breed registrations per year
  - Seven thousand new CD issued by major labels per year

So we extended to  $N=100K, 120K, 144K$   
for  $y=200,400$  and  $\mu=0.0012$  and  $0.0024$

## Fisher-Wright – large populations

Now for  **$N = 100K$  to  $144K$**  again fit to

$$z = A \mu^a y^b N^c$$

and found **large N** small N [BLHH values]

$$A = \mathbf{1.79(2)} \quad 1.38(2) \quad [2.0]$$

$$a = \mathbf{0.558(1)} \quad 0.550(2) \quad [0.5]$$

$$b = \mathbf{0.879(1)} \quad 0.860(1) \quad [1.0]$$

$$c = \mathbf{0.091(1)} \quad 0.130(2) \quad [0.0]$$



# Fisher-Wright – Conclusions

- Simple BJHH formula excluded statistically
- Better formulae provided
- Still suggestion of weak dependence on population size N

- *However*  
fluctuations may  
mean differences  
difficult to detect  
in actual data

$$z = A\mu^a y^b N^c$$

<b>A=</b>	<b>1.79(2)</b>	<b>1.38(2)</b>	<b>[2.0]</b>
<b>a=</b>	<b>0.558(1)</b>	<b>0.550(2)</b>	<b>[0.5]</b>
<b>b=</b>	<b>0.879(1)</b>	<b>0.860(1)</b>	<b>[1.0]</b>
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④ 2	2	1	—	<b>I WANT TO HOLD YOUR HAND</b>	Beatles, Capitol 5112	12
⑤ 4	3	3	—	<b>PLEASE PLEASE ME</b>	Beatles, Vee Jay 581	10
⑥ 7	19	49	—	<b>SUSPICION</b>	Terry Stafford, Crusader 101	7
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# Moran Model

- Found similar transition but now at  $y^* = N\mu^{3/2}$  (for FW  $y^* = N\mu/0.15$ )
- Found same low innovation behaviour
- Could not get simple power law fits to work
- Formula of Erikson et al 2010 works better but assumptions used not clearly satisfied (is 5.83 infinity?)

# THANKS

- Neutral Models and Cultural Transmission
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For papers, talks and other material search for  
***Tim Evans Networks***  
or look at  
***netplexity.org***

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4	2	2	1	—	—	—	—	—	—	—	I WANT TO HOLD YOUR HAND	Beatles, Capitol 5172	12
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6	7	19	49	—	—	—	—	—	—	—	SUSPICION	Terry Stafford, Crusader 101	7
7	8	10	13	—	—	—	—	—	—	—	HELLO, DOLLY!	Lucie Arnaz, Epic 573	8
16	22	50	—	—	—	—	—	—	—	—	SHOOP SHOOP SONG	Betty Everett, Vee Jay 583	6
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# Bibliography

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## Primary sources for this talk:-

Evans, T.S. & Giometto, A., “*Turnover Rate of Popularity Charts in Neutral Models*”, **2011** [[arxiv:1105.4044](https://arxiv.org/abs/1105.4044)]

Bentley, R. A.; Lipo, C. P.; Herzog, H. A. & Hahn, M. W., “*Regular rates of popular culture change reflect random copying*”, *Evolution and Human Behavior*, **2007**, 28, 151-158.

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## Other work in this talk by TSE and on this topic:-

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